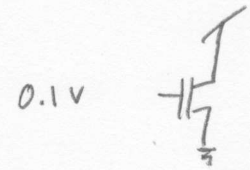
Determine the current flowing in each transistor.

a)



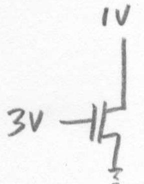
$V_g = 0.1V < V_T \Rightarrow$ Sub V_T operation

$V_{ds} = V_{dd} - 0V > 100mV \Rightarrow$ Saturation

$$I = I_0 e^{\kappa V_g/U_T} \cancel{e^{-V_s/U_T}}^{\,1} e^{V_d/V_A} =$$

$$= (1pA)\, e^{(0.65)(0.1V)/(0.0259V)}\, e^{5V/50V} = 13.5943\,pA$$

b)



$V_g - V_s = 3V > V_T \Rightarrow$ Above V_T operation

$V_d - V_s = 1V$

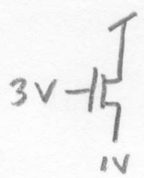
$V_{ov} = V_{gs} - V_T = 3V - 0.7V = 2.3V$

$V_{ds} < V_{ov} \Rightarrow$ Ohmic operation

$$\therefore I = K\left[(V_{gs} - V_T)V_{ds} - \tfrac{1}{2}V_{ds}^2\right] =$$

$$= 100\mu A/V^2 \left[(2.3V)(1V) - \left(\tfrac{1}{2}\right)(1V)^2\right] = 180\mu A$$

c)



$V_{gs} = 2V > V_T \Rightarrow$ Above V_T Operation

$V_{ds} = 4V > V_{gs} - V_T = 1.3V \Rightarrow$ Saturation

But what is the actual V_T (we simply used V_{T0} thus far)

Body Effect $\rightarrow V_T = V_{T0} + \gamma\left(\sqrt{V_{SB} + |2\phi_F|} - \sqrt{|2\phi_F|}\right)$

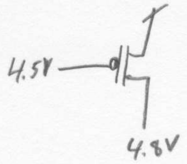
$$V_T = 0.7V + 0.45\,V^{1/2}\left(\sqrt{(1V) + |0.9V|} - \sqrt{|0.9V|}\right) = 0.8934V$$

Go back and check if our assumptions were correct

$V_{gs} = 2V > V_T$, $V_{ds} = 4V > V_{gs} - V_T = 1.1067V$

$$I = \frac{K}{2}\left(V_{gs} - V_T\right)^2\left(1 + \frac{V_{ds}}{V_A}\right) = \frac{100\mu A/V^2}{2}\left(1.1067V\right)^2\left(1 + \frac{5V}{50V}\right) = 67.3632\mu A$$

d)



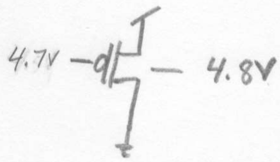
$|V_g - V_s| = 0.5V < |V_T| \Rightarrow$ sub V_T operation

$V_s - V_d = 200mV \Rightarrow$ Saturation

$$I = I_o e^{\kappa(V_w - V_g)/U_T} \, \underbrace{\cancel{e^{-(V_w - V_s)/U_T}}}_{1} \, e^{(V_w - V_d)/V_A}$$

$$I = (1pA)\, e^{(0.65)(0.5V)/0.0259V} \, e^{0.2V/50V} = 282.73\,nA$$

e)



$|V_g - V_s| = 0.3V < |V_T| \Rightarrow$ sub V_T operation

$V_s - V_d = 5V \Rightarrow$ saturation

However $V_w = 4.8V$ (back-gate effect)

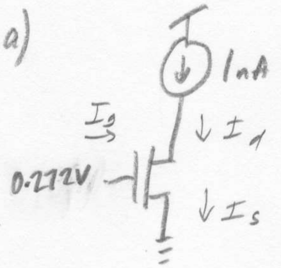
$\rightarrow$ everything is with respect to V_w

$$I = I_o e^{\kappa(V_w - V_g)/U_T} \, e^{-(V_w - V_s)/U_T} \, e^{(V_w - V_d)/V_A}$$

$$I = (1pA)\, e^{(0.65)(0.1V)/0.0259V} \, e^{0.2V/0.0259V} \, e^{4.8V/50V} = 30.57\,nA$$

Determine the DC operating point of each transistor

a)



$V_g - V_s = 0.275V < V_T \Rightarrow$ sub V_T operation

Start with the assumption of saturated operating (we will verify this)

$$I = I_o e^{\kappa V_g / U_T} e^{V_d / V_A} = 1nA = I_{bias}$$

V_g is known

$$\ln \frac{I_{bias}}{I_o} = \frac{\kappa V_g}{U_T} + \frac{V_d}{V_A}$$

$$V_d = V_A \ln \frac{I_{bias}}{I_o} - \frac{\kappa V_g}{U_T} V_A$$

$$= 50V \ln\left(\frac{1nA}{1pA}\right) - \frac{(0.65)(0.272V)}{0.0259V}(50V)$$

$$= 4.0750V$$

✓ this is in saturation

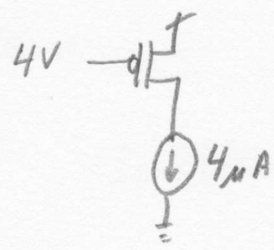
$I_g = 0$ (no gate current)

$I_D = I_s$

DC Operating Point

$V_g = 0.275V$	$I_g = 0$
$V_s = 0V$	$I_s = 1nA$
$V_d = 4.0750V$	$I_d = 1nA$

b)



$|V_{gs}| = 1V > |V_T|$

Find the current value at the transition point between ohmic + saturation

(we do not know if this transistor is operating in ohmic or saturation)

$$I = \frac{K}{2}\left(|V_{gs}| - V_T\right)^2 = \frac{100\,\mu A/V^2}{2}\left(0.3V\right)^2 = 4.5\,\mu A$$

This current is greater than the current source

$\therefore$ The transistor must be operating in the ohmic regime to reduce the current to exactly $4\,\mu A$.

$$I_{bias} = \frac{K}{2}\left[\left(|V_{gs}| - V_T\right)^2 - \left(|V_{gd}| - V_T\right)^2\right]$$

$$|V_{gd}| = \sqrt{\frac{-2I_{bias}}{K} + \left(|V_{gs}| - V_T\right)^2} + V_T$$

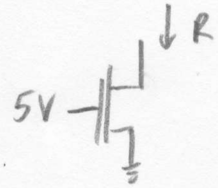
$$|V_{gd}| = \sqrt{\frac{-(2)\,4\mu A}{100\,\mu A/V^2} + (0.3)^2} + 0.7V = 0.8V$$

$\therefore V_d = V_g + 0.8V = 4.8V$

DC Operting Point

$V_g = 4V$	$I_g = 0$
$V_s = 5V$	$I_s = 4\,\mu A$
$V_d = 4.8V$	$I_D = 4\,\mu A$

a) What value of W causes the MOSFET to have a resistance of 100Ω?



Given $C_{ox} = 347 nF/cm^2$

$L = 1\mu m$

The transistor is operating in the deep ohmic region

$$I_D = \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_T) V_{ds}$$

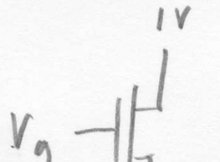
$$R = \frac{V_{ds}}{I_D}$$

$$\therefore R = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{gs} - V_T)}$$

$$W = \frac{L}{(R) \mu_n C_{ox} (V_{gs} - V_T)} =$$

$$= \frac{1\mu m}{(100\Omega)(1460\ cm^2/Vs)(347\ nF/cm^2)(4.3V)} = 4.59\mu m$$

b) Determine the bias voltage required to create a $2.5k\Omega$ resistance, assuming $K = 100\mu A/V^2$.



This needs to be operating in above V_T to generate this resistance ($V_{ds} = 1V \Rightarrow$ will not be able to be sub V_T ohmic)

$V_{gs} > V_{ds} + V_T$ (or greater to put this transistor into the deep ohmic region)

Deep Ohmic (Assumption - we will validate later)

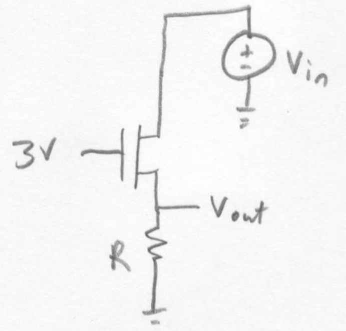
$$R = \frac{1}{K(V_{gs} - V_T)} \Rightarrow V_{ov} = \frac{1}{KR} = \frac{1}{(100\mu A/V^2)(2.5k\Omega)} = 4V$$

$\therefore V_{gs} - V_T = 4V \Rightarrow V_{gs} = 4.7V$ This would put the transistor into deep ohmic

Determine V_{out} when sweeping V_{in} from ground to V_{dd}

Assume $\gamma = \lambda = 0$

$I_0 = 0$ (i.e. no sub V_T current)



In ohmic region when

$$V_g - V_{out} - V_T > V_d - V_{out}$$

$$3V - \cancel{V_{out}} - V_T > V_{in} - \cancel{V_{out}}$$

$$3V - V_T > V_{in}$$

$$V_{in} < 2.3V \Rightarrow \text{Ohmic}$$

$\therefore$ For $V_{in} < 2.3V \rightarrow$ Ohmic

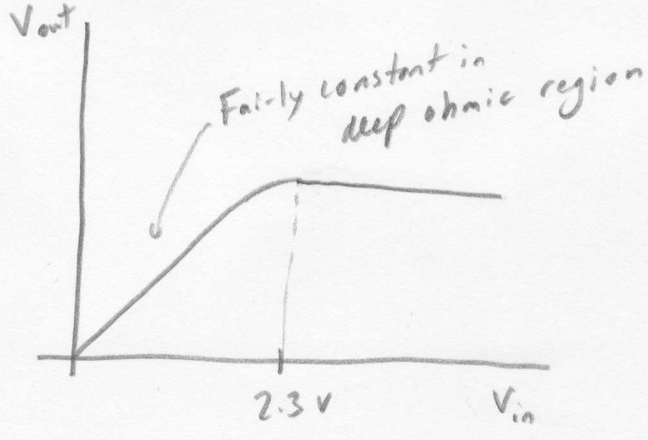
$$V_{out} = I_D R = R K\left[\left(V_{gs} - V_T\right) V_{ds} - \frac{1}{2}\left(V_{gs} - V_T\right)^2\right] =$$

$$= R K\left[\left(2.3V - V_{out}\right)\left(V_{in} - V_{out}\right) - \frac{1}{2}\left(2.3V - V_{out}\right)^2\right]$$

For $V_{in} > 2.3V \rightarrow$ Saturation

$$V_{out} = I_D R = R \frac{K}{2}\left(V_{gs} - V_T\right)^2 = \frac{RK}{2}\left(2.3V - V_{out}\right)^2$$

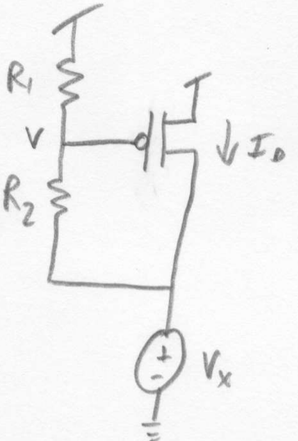
$\rightarrow$ constant, does not depend on V_{in}



Determine I_D as V_x is swept from ground to V_{dd}

Assume $\gamma = \lambda = 0$

$I_D = 0$ (no sub V_T current)



$$V = V_{dd}\frac{R_2}{R_1+R_2} + V_x\frac{R_1}{R_1+R_2}$$

$$V_{sg} = V_{dd} - V$$

$$= V_{dd} - V_{dd}\frac{R_2}{R_1+R_2} - V_x\frac{R_1}{R_1+R_2}$$

$$= \frac{R_1}{R_1+R_2}(V_{dd} - V_x)$$

$$V_{sd} = V_{dd} - V_x$$

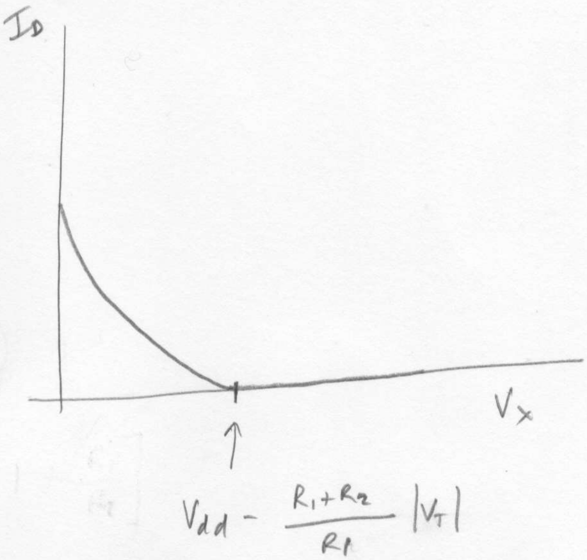
$\therefore V_{sd} \geq V_{sg}$ $\Rightarrow$ If the transistor is turned on, it will be in saturation

Turned on when

$$V_{sg} > |V_T|$$

$$\frac{R_1}{R_1+R_2}(V_{dd} - V_x) > |V_T|$$

$$V_x < V_{dd} - \left(\frac{R_1+R_2}{R_1}\right)|V_T|$$



Then

$$I_D = \frac{K}{2}\left(V_{sg} - |V_T|\right)^2 =$$

$$= \frac{K}{2}\left(\frac{R_1}{R_1+R_2}(V_{dd} - V_x) - |V_T|\right)^2$$