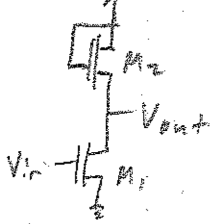
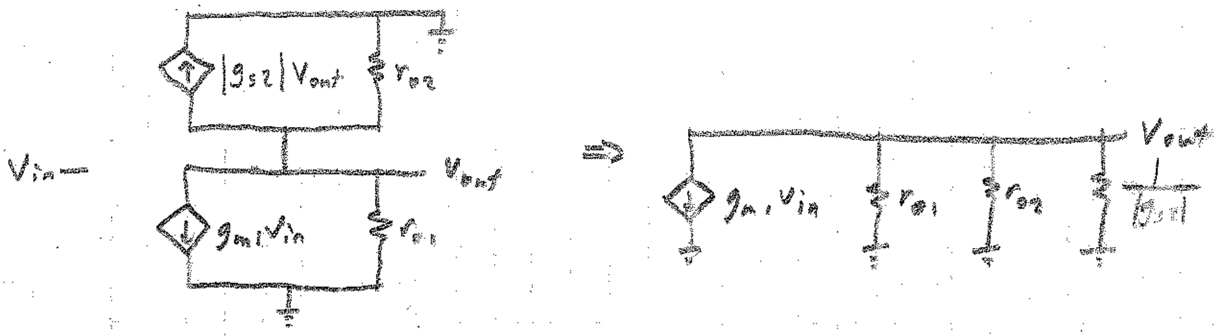
Find the voltage gain and the maximum signal swing of the following amplifier



$I_{D1} = I_{D2} = 10nA \rightarrow$ Sub V_T operation

$$\left(\frac{W}{L}\right)_1 = \frac{50}{0.5} \qquad \left(\frac{W}{L}\right)_2 = \frac{10}{0.5}$$

Small-Signal Model



$$\frac{V_{out}}{V_{in}} = -g_{m1}\, r_{o1} \,//\, r_{o2} \,//\, \frac{1}{|g_{s2}|} \approx -\frac{g_{m1}}{|g_{s2}|}$$ ← Is this approximation valid ?

$$r_{o1} = r_{o2} = \frac{V_A}{I} = \frac{50V}{10nA} = 5G\Omega$$

$$|g_{s2}| = \frac{I}{U_T} = \frac{10nA}{25.9mV} = 3.86 \times 10^{-7}\ S \Rightarrow \frac{1}{|g_{s2}|} = 2.59\,M\Omega$$

Yes, this approximation is valid

$$\therefore \frac{V_{out}}{V_{in}} \approx -\frac{g_{m1}}{|g_{s2}|} = \frac{-\frac{KI}{U_T}}{\frac{I}{U_T}} = -K = -0.65$$

Maximum signal swing

→ both transistors need to stay in saturation → ~100mV for each transistor

$$V_{swing} = V_{dd} - 2V_{sat} \approx V_{dd} - 200mV$$

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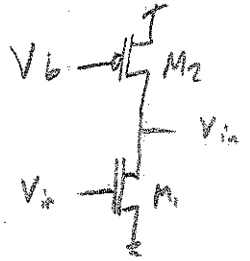
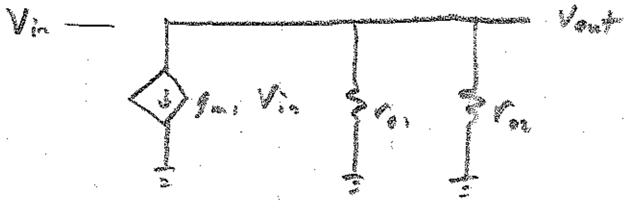


Biased such that $I_{D1} = I_{D2} = 500\mu A$ → Above V_T

$$\left(\frac{W}{L}\right)_1 = \frac{50\mu m}{0.5\mu m} \quad \left(\frac{W}{L}\right)_2 = \frac{50\mu m}{2\mu m}$$



$$\frac{V_{out}}{V_{in}} = -g_{m1}\, r_{o1} // r_{o2}$$

$$g_{m1} = \sqrt{2KI} = \sqrt{2K_n'\left(\frac{W}{L}\right)_1 I} =$$

$$= \sqrt{(2)(1.34\times10^{-4}\ A/V^2)(100)(500\mu A)} = 3.66\times10^{-3}\ S$$

$$r_{o1} = \frac{V_{A1}}{I} = \frac{50V}{500\mu A} = 100k\Omega$$

We do not know V_{A2} because it has a different length. However, we know that

$$V_A \propto L \quad \Rightarrow \quad L_2 = 4L_1$$

$$\therefore V_{A2} = 4V_{A1} = 200V$$

$$r_{o2} = \frac{V_{A2}}{I} = \frac{200V}{500\mu A} = 400k\Omega$$

$$a_v = \frac{V_{out}}{V_{in}} = -g_{m1}\, r_{o1} // r_{o2} = -(3.66\times10^{-3}\ S)(80k\Omega) =$$

$$= -292.8$$

Maximum signal swing

→ for the low end, assume M_1 is at the edge of ohmic region

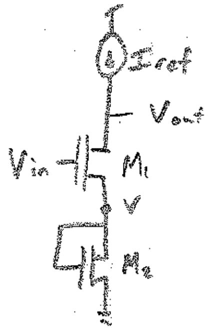
→ for the high end, assume M_2 is at the edge of ohmic region

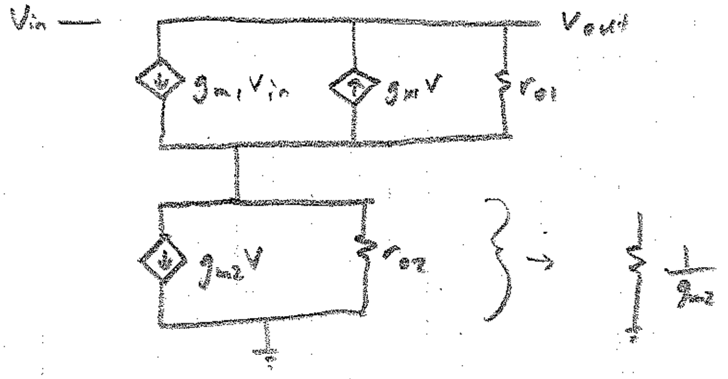
$$\therefore V_{gs} - V_T = V_{ds1} = V_{out} \rightarrow I_{d1} = K\left[(V_{gs1}-V_T)V_{ds1} - \tfrac{1}{2}V_{ds}^2\right] = \tfrac{1}{2}K\,V_{out}^2$$

$$V_{out}\Big|_{low} = \sqrt{\frac{2I_{d1}}{K_1}} = \sqrt{\frac{(2)\ 500\mu A}{(1.34\times10^{-4} A/V^2)(100)}} = 0.2732V \quad \text{(low end)}$$

$$V_{out}\Big|_{high} = \sqrt{\frac{2I_{d2}}{K_2}} = \sqrt{\frac{(2)\ 500\mu A}{(4.0\times10^{-5} A/V^2)(25)}} = 1V \Rightarrow V_{dd} - 1V \quad \text{(high end)}$$

Maximum Signal swing ⇒ $0.2732V < V_{out} < (V_{dd} - 1V)$





$r_{o2} >> \frac{1}{g_{m2}}$

This is simply a source-degenerated CS Amp with $R_S = \frac{1}{g_{m2}}$
$r_{o1} >> \frac{1}{g_{m2}}$

$\therefore R_{in} = \infty\,\Omega$

$$G_m = \frac{g_{m1}}{1 + g_{x1}R_S} = \frac{g_{m1}}{1 + \frac{g_{x1}}{g_{m2}}}$$

$$= \frac{g_{m1}}{1 + \frac{|g_{x1}|}{g_{m2}}} \quad \text{sub } V_T$$

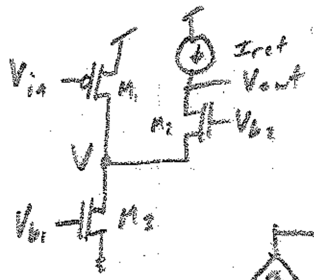
$$= \frac{g_{m1}}{1 + \frac{g_{m1} + g_{mb1}}{g_{m2}}}$$

$$R_{out} = r_{o1} + \frac{1}{g_{m2}} + \frac{g_{x1} r_{o1}}{g_{m2}} \approx r_{o1}\left(1 + \frac{g_{x1}}{g_{m2}}\right)$$

$$a_v = \frac{V_{out}}{V_{in}} = -G_m R_{out} = \frac{-g_{m1}}{\cancel{1 + \frac{g_{x1}}{g_{m2}}}} (r_{o1}) \cancel{\left(1 + \frac{g_{x1}}{g_{m2}}\right)}$$

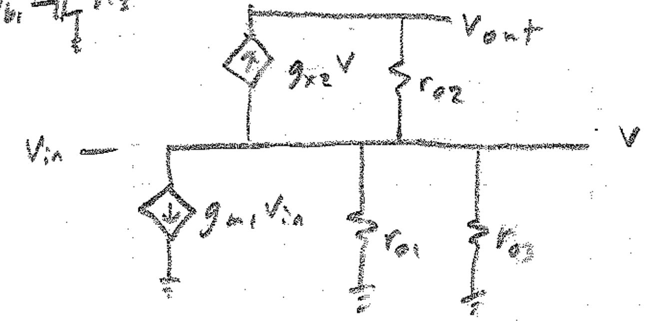
$$= -g_{m1} r_{o1}$$

With an infinite load impedance, R_S has no effect on the small signal gain



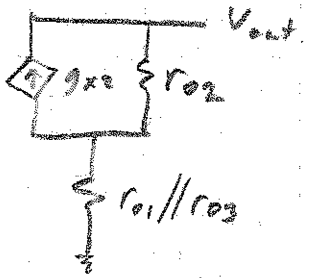
Folded cascode structure with an ideal current-source load

$R_{in} = \infty\ \Omega$ MOSFET gate



$$R_{out} = \left.\frac{V_{out}}{I_{out}}\right|_{V_{in}=0}$$

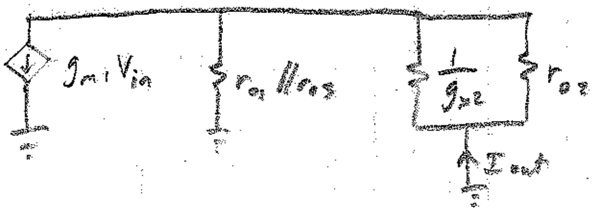
→ This becomes the same as the output impedance of a source-degenerated CS Amp



$$R_{out} = r_{o2} + r_{o1} \| r_{o3} + g_{x2}(r_{o1} \| r_{o3})\, r_{o2}$$

→ less R_{out} than for a standard telescopic cascode structure

$$G_m = \left.\frac{I_{out}}{V_{in}}\right|_{V_{out}=0}$$

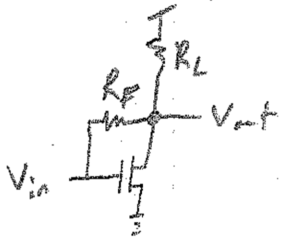


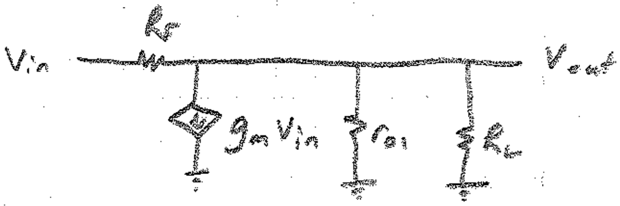
$$I_{out} = \frac{g_{m1} V_{in} (r_{o1} \| r_{o3})}{r_{o1} \| r_{o3} + \frac{1}{g_{x2}} \| r_{o2}} \Rightarrow G_m = \frac{g_{m1}\, r_{o1} \| r_{o3}}{r_{o1} \| r_{o3} + \frac{1}{g_{x2}} \| r_{o2}}$$

$$\approx g_{m1}$$

$$a_v = -G_m R_{out} \approx -g_{m1}\left(r_{o2} + r_{o1} \| r_{o3} + g_{x2}(r_{o1} \| r_{o3})\, r_{o2}\right)$$

$$\approx \underbrace{-g_{m1}\, r_{o1} \| r_{o3}}_{\text{gain of first stage}}\ \underbrace{g_{x2}\, r_{o2}}_{\text{gain of second stage}}$$



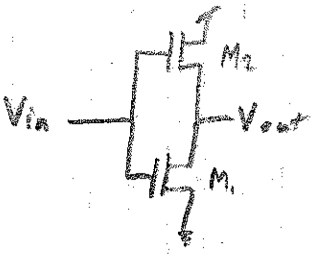


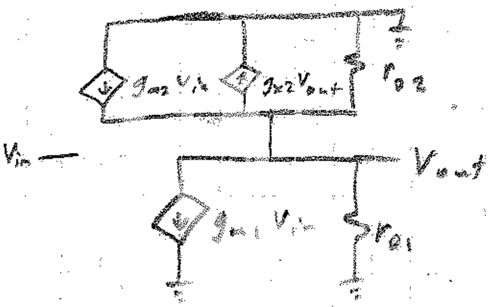
KCL at V_{out}

$$\frac{V_{in} - V_{out}}{R_F} = g_m V_{in} + \frac{V_{out}}{r_o} + \frac{V_{out}}{R_L}$$

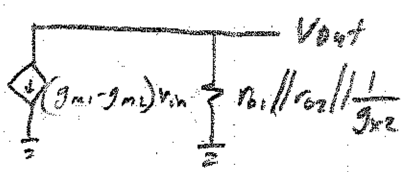
$$V_{in}\left(\frac{1}{R_F} - g_m\right) = V_{out}\left(\frac{1}{r_o} + \frac{1}{R_L} + \frac{1}{R_F}\right)$$

$$a_v = \frac{V_{out}}{V_{in}} = -\left(g_m - \frac{1}{R_F}\right) r_o /\!/ R_L /\!/ R_F$$





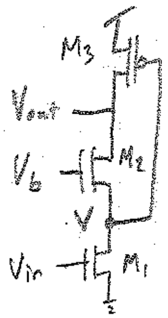
$\Rightarrow$

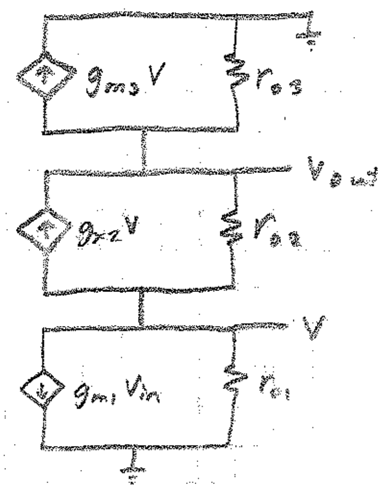


$$a_v = \frac{V_{out}}{V_{in}} = -(g_{m1} - g_{m2})\, r_{o1} /\!/ r_{o2} /\!/ \frac{1}{g_{x2}}$$

$$= \frac{-(g_{m1} - g_{m2})\, r_{o1} /\!/ r_{o2}}{1 + g_{x2}\, r_{o1} /\!/ r_{o2}} \approx \frac{-(g_{m1} - g_{m2})}{g_{x2}}$$

Find the small-signal voltage gain of the following amplifier





The current flowing through each transistor is equal to each other.

$$g_{m1}V_{in} + \frac{V}{r_{o1}} = -g_{x2}V + \frac{V_{out}-V}{r_{o2}} = -\left(g_{m3}V + \frac{V_{out}}{r_{o3}}\right)$$

Transitor → ① ② ③

② = ③ ⇒ $$-g_{x2}V + \frac{V_{out}-V}{r_{o2}} = -\left(g_{m3}V + \frac{V_{out}}{r_{o3}}\right)$$

$$V_{out}\left(\frac{1}{r_{o2}} + \frac{1}{r_{o3}}\right) = V\left(\frac{1}{r_{o2}} + g_{x2} - g_{m3}\right)$$

$$V = \frac{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}}{\frac{1}{r_{o2}} + g_{x2} - g_{m3}} V_{out}$$

① = ③ ⇒ $$g_{m1}V_{in} + \frac{V}{r_{o1}} = -\left(g_{m3}V + \frac{V_{out}}{r_{o3}}\right)$$

$$g_{m1}V_{in} + V\left(\frac{1}{r_{o1}} + g_{m3}\right) = -\frac{V_{out}}{r_{o3}}$$

$$g_{m1}V_{in} + V_{out}\frac{\left(\frac{1}{r_{o2}} + \frac{1}{r_{o3}}\right)\left(\frac{1}{r_{o1}} + g_{m3}\right)}{\frac{1}{r_{o2}} + g_{x2} - g_{m3}} = \frac{-V_{out}}{r_{o3}}$$

$$g_{m1}V_{in} = -V_{out}\left[\frac{1}{r_{o3}} + \frac{\left(\frac{1}{r_{o2}} + \frac{1}{r_{o3}}\right)\left(\frac{1}{r_{o1}} + g_{m3}\right)}{\frac{1}{r_{o2}} + g_{x2} - g_{m3}}\right]$$

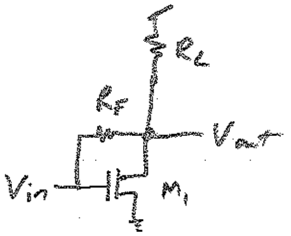
$$g_{m1}V_{in} = -V_{out}\left[\frac{1}{r_{o3}} + \frac{(r_{o2}+r_{o3})(1+g_{m3}r_{o1})}{r_{o1}r_{o3}\left(1+(g_{x2}-g_{m3})r_{o2}\right)}\right]$$

$$g_{m1} v_{in} = -v_{out}\left[\frac{r_{o3}\left(1+(g_{x2}-g_{m3})r_{o2}\right)+(r_{o2}+r_{o3})(1+g_{m3}r_{o1})}{r_{o1}r_{o3}\left(1+(g_{x2}-g_{m3})r_{o2}\right)}\right]$$

$$\frac{v_{out}}{v_{in}} = \frac{-g_{m1}r_{o1}r_{o3}\left[1+(g_{x2}-g_{m3})r_{o2}\right]}{r_{o3}\left[1+(g_{x2}-g_{m3})r_{o2}\right]+(r_{o2}+r_{o3})(1+g_{m3}r_{o1})}$$

$$= \frac{-g_{m1}r_{o1}r_{o3}\left[1+(|g_{s2}|-g_{m3})r_{o2}\right]}{r_{o3}\left[1+(|g_{s2}|-g_{m3})r_{o2}\right]+(r_{o2}+r_{o3})(1+g_{m3}r_{o1})} \quad (\text{sub } V_T)$$

$$= \frac{-g_{m1}r_{o1}r_{o3}\left[1+(g_{m2}+g_{mb2}-g_{m3})r_{o2}\right]}{r_{o3}\left[1+(g_{m2}+g_{mb2}-g_{m3})r_{o2}\right]+(r_{o2}+r_{o3})(1+g_{m3}r_{o1})} \quad (\text{above } V_T)$$



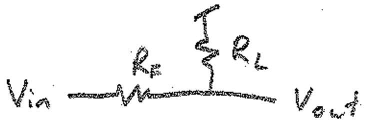
Sketch V_{out} vs. V_{in} as V_{in} is swept from 0 to V_{dd}

Let $I_D = 0$ (no sub V_T current flow)

↳ When $V_{in} < V_T$ ⇒ no current flow in M_1

$R_F = 20k\Omega$ $R_L = 10k\Omega$

When $V_{in} < V_T$



⇒ Resistive Divider with two voltage sources
- V_{in}
- V_{dd}

⇒ use Super-position

$$V_{out} = \underbrace{\frac{R_L}{R_L+R_F}}_{slope} V_{in} + \underbrace{\frac{R_F}{R_L+R_F} V_{dd}}_{constant} = \frac{1}{3} V_{in} + \frac{2}{3} V_{dd}$$
$$= 0.33 V_{in} + 3.33V$$

For $V_{in} > V_T$, the transistor turns on

At $V_{in} = V_T$ → $V_{out} = \frac{1}{3}(0.7V) + 3.33V = 3.57V$

↳ Therefore when M_1 turns on, it will be in saturation

M_1 is in saturation for $V_T < V_{in} < \underbrace{V_{out}}_{V_{in}-V_T}$

Cannot use the small-signal model because we are looking at large-signal parameters

KCL at V_{out}

$$\frac{V_{in} - V_{out}}{R_F} + \frac{V_{dd} - V_{out}}{R_L} = I_D$$

At the edge of saturation

$$V_{out} = V_{in} - V_T \Rightarrow V_{in} = V_{out} + V_T$$

$$I_D = \frac{K}{2}(V_{GS} - V_T)^2 = \frac{K}{2}(V_{in} - V_T)^2 = \frac{K V_{out}^2}{2}$$ assume V_A = large

$$\frac{V_{out} + V_T - V_{out}}{R_F} + \frac{V_{dd} - V_{out}}{R_L} = \frac{K}{2} V_{out}^2$$

$$\frac{K}{2} V_{out}^2 + \frac{1}{R_L} V_{out} - \left(\frac{V_{dd}}{R_L} + \frac{V_T}{R_F}\right) = 0$$

$$(25\,\mu A/V^2)\,V_{out}^2 + (1\times10^{-4}\,S)\,V_{out} - (5.35\times10^{-4}\,A) = 0$$

solve for V_{out} (quadratic equation)

$$V_{out} = \frac{-(1\times10^{-4}) \pm \sqrt{(1\times10^{-4})^2 + (4)(25\times10^{-6})(5.35\times10^{-4})}}{(2)(25\times10^{-6})}$$

$$= -7.0398,\ 3.0398$$

The only valid value

V_{in} at the edge of ohmic

$$V_{in} = V_{out} + V_T = 3.7398\,V$$