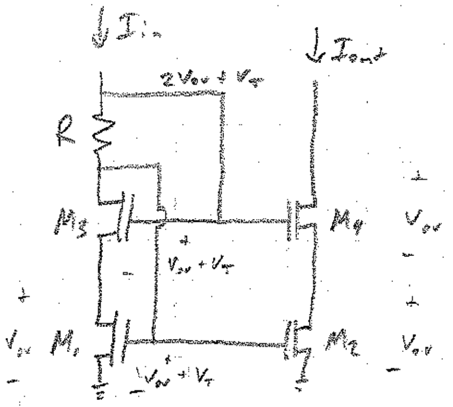


For identically sized transistors, design the current mirror ($\frac{W}{L}$ values and R) for a current of $I_{in} = I_{out} = 100\mu A$ and a $V_{out,min} = 0.4\ V$

> for edge of ohmic/saturation

$V_{out,min} = 2\ V_{ov} \Rightarrow V_{ov} = 0.2V$
for all transistors

Assume operation at the edge of ohmic / Saturation

$$\therefore I = \frac{K}{2}\left(V_{gs} - V_T\right)^2 = \frac{K'}{2}\frac{W}{L}V_{ov}^2$$

$$\frac{W}{L} = \frac{2I}{K' V_{ov}^2} = \frac{(2)(100\mu A)}{(100\mu A/V^2)(0.2)^2} = \boxed{50} \text{ for each transistor}$$

Find the effective threshold voltage for M_3 + M_4 (Body Effect)

$$V_{T3} = V_{T4} = V_{T0} + \gamma\left(\sqrt{V_{SB} + |2\phi_F|} - \sqrt{|2\phi_F|}\right)$$

$$= 0.7V + (0.4)\left(\sqrt{0.2V + 0.7V} - \sqrt{0.7V}\right) = 0.7448V$$

$V_{gs1} = V_{ov} + V_T = 0.2V + 0.7V = 0.9V \leftarrow$ Bottom of R

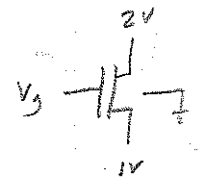
$V_{gs3} = V_{ov} + V_{T3} = 0.2V + 0.7448V = 0.9448V$

Top of R $= 2V_{ov} + V_{T3} = 0.4V + 0.7448V = 1.1448V$

Find the value of R

$$R = \frac{V}{I} = \frac{1.1448 - 0.9V}{100\mu A} = \boxed{2448\ \Omega}$$

Determine the complete small-signal equivalent model for the following transistor



$W = 100\mu m$
$L = 1\mu m$
$C_{ov} = 0.1 fF/\mu m$
$C_{ox} = 3.5 fF/\mu m^2$

$C_{sbo} = C_{dbo} = 100 \leftarrow$ includes area and sidewall
$m = 1$
$V_{bi} = 0.7V$

$500\mu A > I_{th} \Rightarrow$ Above V_T, ohmic or saturation?

V_A is relatively large so, $I \approx \frac{K}{2} V_{ov}^2$ (edge of ohmic / saturation)

$$V_{ov} \approx \sqrt{\frac{2I}{K}} = \sqrt{\frac{2(500\mu A)}{(100\mu A/V^2)(100)}} = 0.3162V < V_{DS} = 1V$$

$\Rightarrow$ saturation

Low-frequency parameters

$$g_m = \sqrt{2KI} = \sqrt{2K'\tfrac{W}{L}I}$$
$$= \sqrt{2(100\mu A/V^2)(100)(500\mu A)} = 3.16 mA/V$$

$$g_{mb} = \frac{\gamma g_m}{2\sqrt{V_{SB}+|2\phi_F|}} = \frac{(0.4)(3.16mA/V)}{(2)\sqrt{1V+0.7V}} = 4.8507\times10^{-4} A/V$$

$$r_o = \frac{50V}{500\mu A} = 100k\Omega$$

Need to find capacitance values $\rightarrow C_{gs}, C_{gd}, C_{gb}, C_{sb}, C_{db}, C_{ds}$

$C_{ds} = 0F$

$$C_{gs} = \frac{2}{3}C_{ox}W\cdot L + C_{ov}W = \left(\frac{2}{3}\right)(3.5fF/\mu m^2)(100\mu m)(1\mu m) + (0.1fF/\mu m)(100\mu m) =$$
$$= 243.3 fF$$

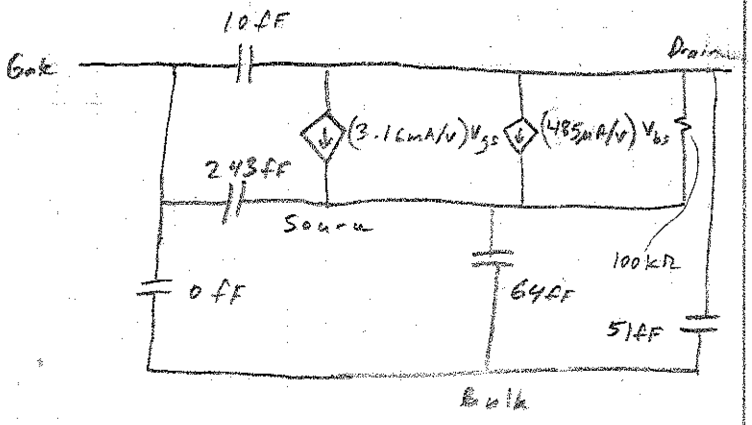
$C_{gd} = C_{ov}W = 10fF$

$C_{gb} \approx 0F$

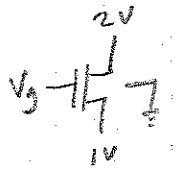
$C_{sbo} = C_{dbo} = 10fF$

$$C_{sb} = \frac{C_{sbo}}{\sqrt{1+\frac{1V}{0.7V}}} = 6.42 fF$$

$$C_{dbo} = \frac{C_{dbo}}{\sqrt{1+\frac{2V}{0.7V}}} = 50.9 fF$$



$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gb}+C_{gd})} = \frac{3.16mA/V}{(2\pi)(243fF+10fF)}$$
$$= 1.99 GHz$$



$\downarrow$ 10nA $\rightarrow$ subthreshold operation, saturation ($V_{ds} > 100\,mV$)

C_{sb}, C_{db} $\rightarrow$ unchanged from above V_T case

$C_{sb} = 64.2\,fF$, $C_{db} = 50.9\,fF$

$$g_m = \frac{\kappa I}{U_T} = \frac{(0.65)(10\,nA)}{0.0259\,mV} = 251\,nA/V$$

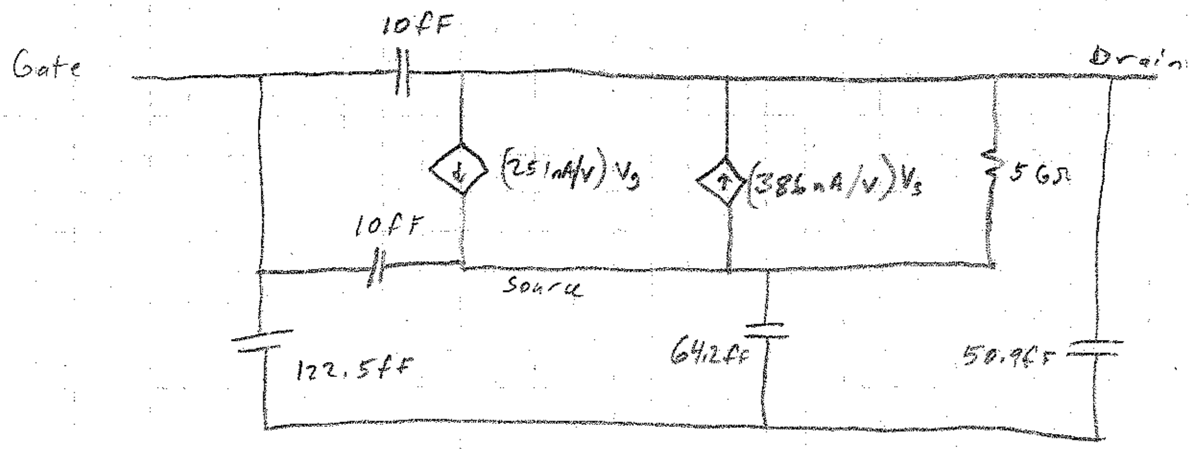
$$g_s = \frac{-I}{U_T} = \frac{-10\,nA}{0.0259\,mV} = -386\,nA/V$$

$$r_o = \frac{50V}{I} = \frac{50V}{10nA} = 5\,G\Omega$$

$$C_{gs} = C_{gd} = C_{ov}W = 10\,fF$$

$$C_{gb} = (1-\kappa)\,C_{ox}W \cdot L = (0.35)(3.5\,fF/\mu m^2)(100\,\mu m)(1\,\mu m) =$$

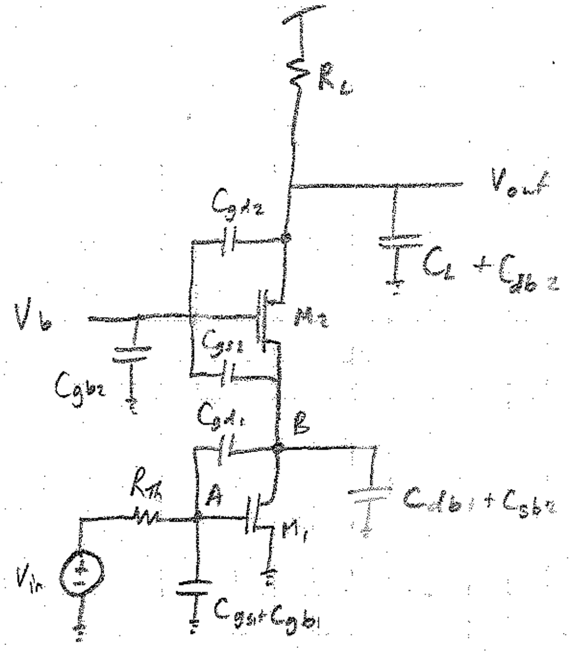
$$= 122.5\,fF$$



$$f_T = \frac{g_m}{(2\pi)(C_{gs}+C_{gd}+C_{gb})} = \frac{251\,nA/V}{(2\pi)(10fF + 10fF + 122.5fF)}$$

$$= 280\,kHz$$

Use the Miller Effect to estimate expressions for all three poles for a cascoded CS Amp with a resistive load.
Explain why a cascode improve the frequency of operation.



What is the gain from A to B?

$$\underbrace{\frac{r_{o2} + R_L}{1 + g_{x2} r_{o2}}}_{\approx \frac{1}{g_{x2}}\left(1 + \frac{R_L}{r_{o2}}\right)}$$

$$\frac{V_b}{V_a} = -g_{m1}\, r_{o1} \,\Big\|\, \left(\frac{1}{g_{x2}}\left(1 + \frac{R_L}{r_{o2}}\right)\right)$$

if $R_L < r_{o2}$

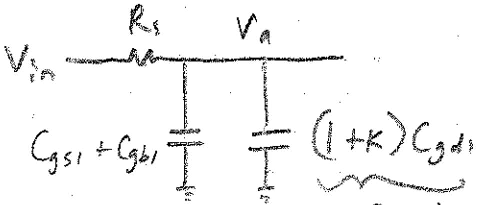
$$\frac{V_b}{V_a} \simeq -\frac{g_{m1}}{g_{x2}}$$

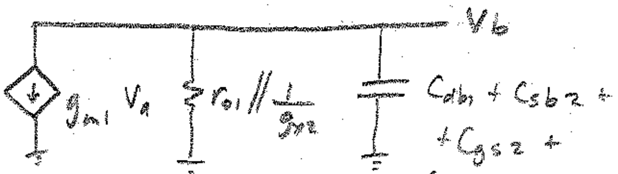
if $R_L \approx r_{o2}$ (e.g. r_o from a transistor)

$$\frac{V_b}{V_a} \approx -\frac{2 g_{m1}}{g_{x2}}$$

Assume R_L is relatively small. If not, there will only be a small error

Use Miller approximation





$$\underbrace{(1+K)C_{gd1}}_{\left(1 + \frac{g_{m1}}{g_{x2}}\right) C_{gd1} \approx 2 C_{gd1}}$$

↳ <u>much smaller than $g_m r_o C_{gd1}$</u>

$$\omega_a = \frac{1}{R_{a,eff}\, C_{a,eff}} = \frac{1}{(R_s)\left(C_{gs1} + C_{gb1} + \left(1 + \frac{g_{m1}}{g_{x2}}\right) C_{gd1}\right)}$$

$$\omega_b = \frac{1}{R_{b,eff}\, C_{b,eff}} =$$

$$= \frac{1}{\left[r_{o1} \,//\, \left[\frac{1}{g_{x2}}\left(1 + \frac{R_L}{r_{o2}}\right)\right]\right]\left[C_{db1} + C_{sb2} + C_{gs2} + \left(1 + \frac{g_{x2}}{g_{m1}}\right) C_{gd1}\right]}$$

$$\approx \frac{g_{x2}}{C_{db1} + C_{sb2} + C_{gs2} + \left(1 + \frac{g_{x2}}{g_{m1}}\right) C_{gd1}}$$

$$\omega_{out} = \frac{1}{R_{out,eff}\, C_{out,eff}} =$$

$$= \frac{1}{\left[R_L \,//\, \left[r_{o1} + r_{o2} + g_{x2} r_{o1} r_{o2}\right]\right]\left[C_L + C_{db2} + C_{gd2}\right]}$$

$$\approx \frac{1}{(R_L)(C_L + C_{db2} + C_{gd2})}$$

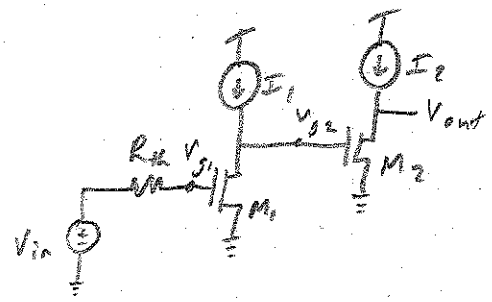
Speed increases

For a non-cascoded CS Amp,

$$\omega_{in} = \frac{1}{(R_S)\left(C_{gs} + C_{gb} + \underbrace{\left(1 + \underbrace{g_m r_o}_{a_v \to \text{large}}\right) C_{gd}}_{\text{large effective capacitance}}\right)}$$

The cascode greatly reduces this associated time constant, moving the pole to a much higher frequency

Use the Miller Theorem to determine all poles in the following circuit



There is one pole for each node
$\therefore$ 3 poles

pole $= -\omega_x$

Pole frequency at V_{g1}

$$\omega_{Vg1} = \frac{1}{R_{Vg1,eff}\ C_{Vg1,eff}} =$$

$$= \frac{1}{(R_{th})(C_{gs1} + C_{gb1} + (1+g_{m1}r_{o1})C_{gd1})}$$

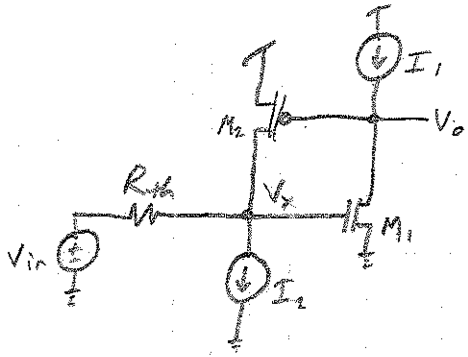
$$\omega_{Vg2} = \frac{1}{(r_{o1})\left(C_{db1} + \left(1+\frac{1}{g_{m1}r_{o1}}\right)C_{gd1} + C_{gs2} + C_{gb2} + (1+g_{m2}r_{o2})C_{gd2}\right)}$$

$$\approx \frac{1}{(r_{o1})(C_{db1} + C_{gd1} + C_{gs2} + C_{gb2} + (1+g_{m2}r_{o2})C_{gd2})}$$

$$\omega_{Vout} = \frac{1}{(r_{o2})\left(C_{db2} + \left(1+\frac{1}{g_{m2}r_{o2}}\right)C_{gd2}\right)}$$

$$\approx \frac{1}{(r_{o2})(C_{db2} + C_{gd2})}$$

Determine the transfer function of the following circuit including all parasitic capacitances

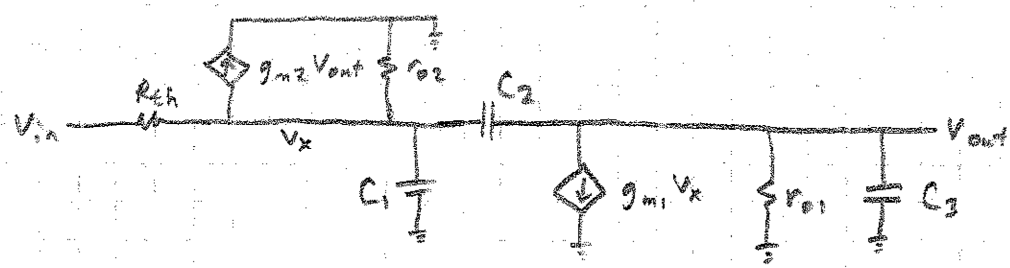


Let $C_1 = C_{gs1} + C_{gb1} + C_{dw2}$

$C_2 = C_{gd1} + C_{gd2}$

$C_3 = C_{db1} + C_{gs2} + C_{gw2}$

Small-signal model



KCL at V_{out}

$$g_{m1} V_x + V_{out}\left(\frac{1}{r_{o1}} + sC_3\right) + (V_{out} - V_x)\,sC_2 = 0$$

$$V_{out}\left(\frac{1}{r_{o1}} + s(C_2 + C_3)\right) = V_x\,(sC_2 - g_{m1})$$

KCL at V_x

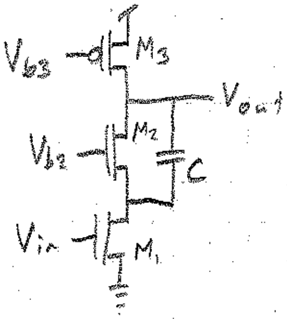
$$\frac{(V_x - V_{in})}{R_{th}} + g_{m2} V_{out} + V_x\left(\frac{1}{r_{o2}} + sC_1\right) + (V_x - V_{out})\,sC_2 = 0$$

$$\frac{V_{in}}{R_{th}} = V_{out}(g_{m2} - sC_2) + V_x\left(\frac{1}{R_{th}} + \frac{1}{r_{o1}} + s(C_1 + C_2)\right)$$

$$\left(\frac{V_{in}}{R_{th}}\right)(sC_2 - g_{m1}) = V_{out}\Big[(sC_2 - g_{m1})(g_{m2} - sC_2) + \left(\frac{1}{r_{o1}} + s(C_2 + C_3)\right)\left(\frac{1}{R_{th}} + \frac{1}{r_{o2}} + s(C_1 + C_2)\right)\Big]$$

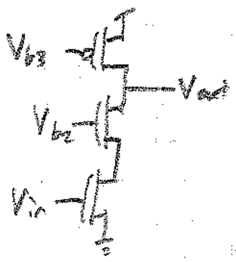
$$\frac{V_{out}}{V_{in}} = \frac{\left(\frac{1}{R_{th}}\right)(sC_2 - g_{m1})}{s^2\left[C_1C_2 + C_1C_3 + C_2C_3\right] + s\left[C_2(g_{m1} + g_{m2}) + \frac{1}{r_{o1}}(C_1 + C_2) + (C_1 + C_2)\left(\frac{1}{R_{th}} + \frac{1}{r_{o2}}\right)\right] + \left(\frac{1}{r_{o1}}\right)\left(\frac{1}{R_{th}} + \frac{1}{r_{o2}}\right) - g_{m1}g_{m2}}$$

Find the gain at very low frequencies and at high frequencies



At low frequencies

C = open circuit

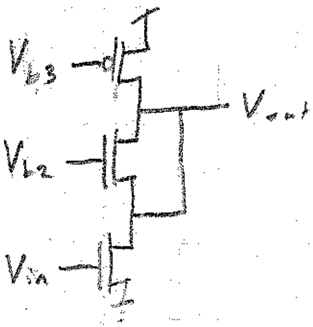


$$a_v = -g_{m1}(g_{m2} r_{o1} r_{o2} \,\|\, r_{o3})$$

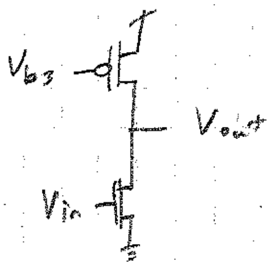
$$\approx -g_{m1} r_{o3}$$

For high frequencies

C = short circuit

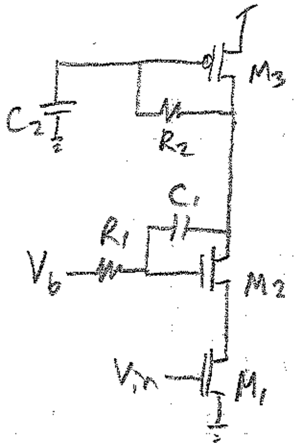


⇒



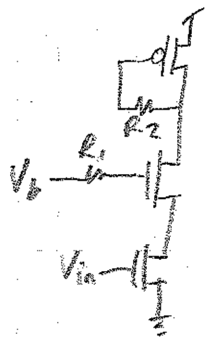
$$a_v = -g_{m1} r_{o1} \,\|\, r_{o3}$$

Find the gain at very low frequencies and at high frequencies



Let $\gamma = 0$

Low frequencies → $C_1 = C_2 =$ open circuit

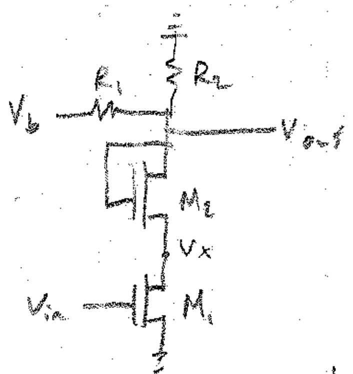


R_1 and R_2 have no effect

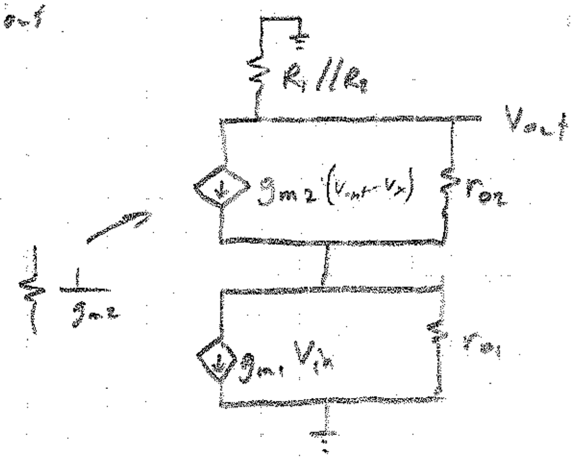
$$a_v = -g_{m1}\left(g_{m2} r_{o1} r_{o2} \,//\, \frac{1}{g_{m3}}\right)$$

$$= -g_{m1}\, g_{m3}$$

High frequencies → $C_1 = C_2 =$ short circuits



Small-signal model

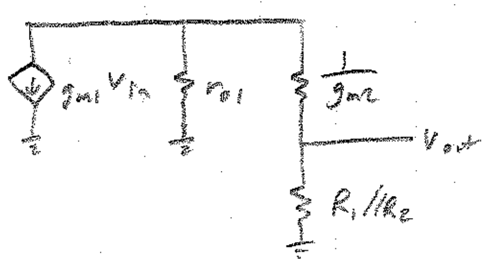


$\gamma = 0$

Use source-referred model

$$\frac{1}{g_{m2}} \,//\, r_{o2} \approx \frac{1}{g_{m2}}$$

Redraw the small-signal model
→ use a current divider



$$V_{out} = -g_{m1} v_{in} \frac{r_{o1}}{r_{o1} + \frac{1}{g_{m2}} + R_1 // R_2} \left(R_1 // R_2\right)$$

Assuming $r_{o1} >> R_1, R_2$

$$a_v \approx -g_{m1} R_1 // R_2$$