C^4 BAND-PASS DELAY FILTER FOR CONTINUOUS-TIME SUBBAND ADAPTIVE TAPPED-DELAY FILTER

Heejong Yoo, David Graham, David V. Anderson, and Paul Hasler

School of Electrical and Computer Engineering
Georgia Institute of Technology, Atlanta, GA, 30332

ABSTRACT

This paper describes the implementation of delay element using C^4 band-pass filter for subband analog tapped-delay adaptive filter, where implementation of larger group delay is required. Most analog delay elements have been implemented with low-pass or all-pass filters. While they can easily achieve constant group delay within pass band, maximum group delay is severely restricted by the corner frequency because group delay is inversely proportional to the corner frequency.

In this paper, we implemented a delay element with a Capacitively-Coupled Current Conveyer (C^4) band-pass filter to produce larger group delay required for analog subband adaptive filter. Experimental results from circuits fabricated in $0.5\mu m$ CMOS technology through MOSIS are also presented.

1. CONTINUOUS-TIME ADAPTIVE FILTER

Adaptive tapped-delay filter like LMS (Least Mean Square) has been widely implemented in discrete-time domain as well as in continuous-time domain for it's simple structure and stability [1]. Figure 1 shows the configuration of tapped-delay adaptive FIR filter.

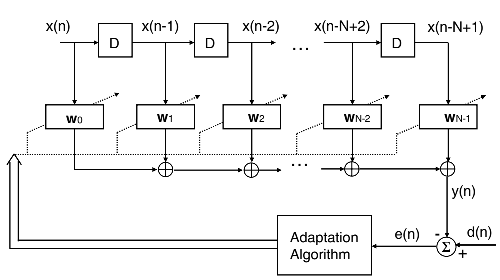


Fig. 1. Tapped-delay adaptive FIR filter structure. It consists of tapped-delay part, weight adaptation part, and multiplication/add part. Most analog delay elements in continuous-time mode have been implemented with first-order low-pass (Gamma filter), first-order all-pass filters, or the combination of first-order low-pass and all-pass filters (Laguerre filter).

This Research is sponsored by the Hewlett-Packard Corporation.

With all the development of adaptive algorithms for the last 30 years in discrete time domain, real-time implementation still remains as a hard problem because of the high computational complexity. The combination of subband processing with fast adaptive algorithms can be used for a real-time implementation. However, even with a high performance DSP hardware combining subband processing and fast adaptive algorithm, high power consumption is still unavoidable because of the DSP system's fast clock rate.

Many efforts have been made among analog circuit engineers to resolve real-time and low-power constraint by implementing adaptive LMS filters with analog circuitry. Sample mode delay elements using a CCD or a switched-capacitor were also investigated [2]. However, the necessity of clocking and the occurrence of aliasing effects still remain as drawbacks. Continuous-time LMS adaptive filter consists of tapped-delay part, weight adaptation part, and multiplication/add part. One of the difficulties we face in continuous-time implementation of tapped-delayed filter is that it requires ideal delay elements, which are impossible to implement with non-ideal circuitry.

2. LOW-PASS OR ALL-PASS DELAY ELEMENT

2.1. Constant Group Delay

Most analog delay elements in continuous-time mode have been implemented with first-order low-pass (Gamma filter) [3], first-order all-pass filters [4], or the combination of first-order low-pass and all-pass filters (Laguerre filter) [3]. While their group delay is generally constant within pass band, maximum group delay is severely restricted by the corner frequency because group delay is inversely proportional to the corner frequency.

The first-order low-pass filter transfer function and group delay become

$$H_l(s) = \frac{1}{\tau s + 1}, \quad D_l(w) = \frac{\tau}{\tau^2 w^2 + 1}, \tag{1}$$

and for the first-order all-pass filter,

$$H_a(s) = \frac{\tau s - 1}{\tau s + 1}, \quad D_a(w) = \frac{2\tau}{\tau^2 w^2 + 1}. \tag{2}$$

It is obvious from Eq. (1) and Eq. (2) that, when $w \ll \tau$, $D_l(w) \approx \tau$ and $D_a(w) \approx 2\tau$.

Constant group delay property of low-pass and all-pass filter is desirable in many applications. However, since their group delay is proportional to τ, when an input signal has a wide bandwidth ($\tau \ll 1$), maximum group delay that each delay element can produce is severely reduced as corner frequency increases. For example, when input signal is band limited by 8 KHz, τ should be less than 1/8000 and the maximum group delay of low-pass and all-pass delay filter is limited to 0.125 msec and 0.25 msec, respectively.

2.2. Delay of Subband Analog Adaptive Filter

Subband adaptive filter in discrete-time domain has gained much attentions these days because it enables faster convergence by reducing eigenvalue spread of input signal and reduces processing time when implemented with hardware [1]. From multi-rate signal processing theory, we know that signal can be decimated by either equal to M or smaller than M without having aliasing for M-ary filter bank. For example, when Fs = 8 KHz and the length of room impulse response is 200 msec, adaptive filter for AEC should have 1600 filter taps. In subband AEC, when number of subband, M, is 64 we only need 25 filter taps for each subband.

While decimation is advantageous in terms of decreasing computational complexity and number of cascade of delay element in discrete-time domain, if it could be directly implementable in continuous-time domain, M times bigger delay should also be implementable from one delay element. For above example, delay that each element generates should be 8 msec for each subband while it is 0.125 msec for full-band AEC. It is apparent from Eq. (1) and Eq. (2) that 8 msec group delay out of all-pass delay elements can be achieved at very lower subbands where $\tau > 1/250$.

3. BAND-PASS DELAY ELEMENT

3.1. Capacitively-Coupled Current Conveyer (C^4)

The C^4 was originally developed for separating corner frequencies, as its model system was the auto-zeroing floating-gate amplifier [5]. The frequency response of the C^4 is characterized by [5]

$$\frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2}\frac{s\tau_l(1-s\tau_f)}{s^2\tau_h\tau_l + s(\tau_l + \tau_f(\frac{C_0}{\kappa C_2}-1))+1}, \quad (3)$$

where the time constants are given by

$$\tau_l = \frac{C_2 U_T}{\kappa I_{\tau_l}},\ \ \tau_f = \frac{C_2 U_T}{\kappa I_{\tau_h}},\ \ \tau_h = \frac{C_T C_O - C_2^2}{C_2}\frac{U_T}{\kappa I_{\tau_h}},$$

where the total capacitance, C_T, and the output capacitance, C_O, are defined as $C_T = C_1 + C_2 + C_W$ and $C_O = C_2 + C_L$. The currents I_{τ_l} and I_{τ_h} are the currents through M_2 and M_3, respectively in Fig. 2. Thus, the C^4 takes on the properties of a band-pass filter with first-order slopes and band-pass gain set by the ratio of the two coupling capacitors as $A_v = -C_1/C_2$. The overall time constant of the filter, which gives the center frequency, is

$$\tau = \sqrt{\tau_l \tau_h}. \quad (4)$$

The corner frequencies can be set independently of each other, and the bandwidth can therefore be tuned at will. If the corners are brought close together, a very tight bandwidth with a single Q peak can be developed and by setting the corners wide apart from each other, all-pass filter or low-pass filter can be implemented by setting lower corner frequency down to a few Hz and higher frequency up to a several hundred KHz.

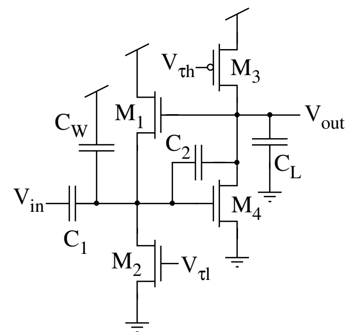


Fig. 2. Circuit diagram of a nFET-based Capacitively-Coupled Current Conveyer (C^4) circuit. C^4 allows both low-frequency and high-frequency cutoffs to be controlled electronically by changing the appropriate bias currents. The ratio of C_2 to C_1 sets the gain of C^4 and overall time constant is $\tau = \sqrt{\tau_l \tau_h}$.

The value of Q peak is

$$Q = \sqrt{\frac{\tau_h}{\tau_l}}\frac{1}{1+\frac{I_{\tau_l}}{I_{\tau_h}}(\frac{C_O}{\kappa C_2}-1)} \quad (5)$$

and the maximum value of the Q peak can be written as

$$Q_{max} = \frac{1}{2}\sqrt{\frac{\kappa(C_T C_O - C_2^2)}{C_2(C_O - \kappa C_2)}}. \quad (6)$$

3.2. Group Delay of Band-pass Filter

The general second-order band-pass filter transfer function is

$$H_b(s) = \frac{K(\frac{w_o}{Q})s}{s^2 + (\frac{w_o}{Q})s + w_o^2}, \quad (7)$$

where w_o is a center frequency, K is a gain, Q is a quality factor defined as w_o/BW, and BW is the bandwidth of the filter. Group delay can be written as

$$D_b(w) = \frac{(\frac{w_o}{Q})({w_o}^2 + w^2)}{({w_o}^2 - w^2)^2 + (\frac{w_o}{Q})^2 w^2}. \quad (8)$$

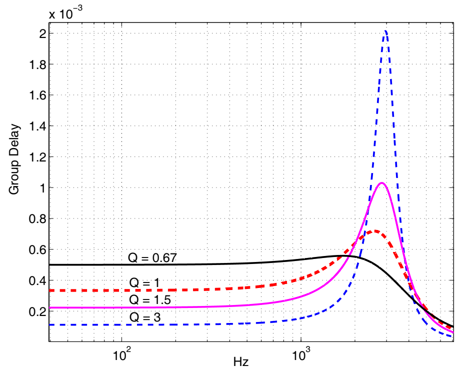


Fig. 3. Theoretical group delay of 2nd order band-pass filter for different Q values with $f_o = 3/2\pi$ KHz. When Q is low, group delay of band-pass filter has a constant group delay for the frequency lower than lower cut-off frequency. As Q increases, maximum group delay of the center frequency approaches to $2/BW$ and group delay at constant group delay region, where frequency is smaller than lower cut-off frequency, decreases.

Group delays of 2nd order band-pass filter for different Q values are shown at Fig. 3. When Q is low, group delay of band-pass filter is very similar to the group delay of 1st order low-pass filter having constant group delay for the frequency lower than w_o. When $w \approx w_o$, it is obvious from Eq. (8) that $D_b(w)$ is close to its maximum delay, $2/BW$, and is independent of center frequency. The property of band-pass delay filter that maximum delay is independent of the center frequency allows us to use band-pass delay element for the other subbands without increasing filter order to achieve same delay. As Q increases, maximum group delay at the center frequency approaches to $2/BW$ and group delay at constant group delay region, where frequency is smaller than lower cut-off frequency, decreases.

3.3. Group Delay of C^4 Band-pass Filter

High-Q and low-Q C^4 delay elements and C^4 SOS delay element have been implemented using MOSIS $0.5\mu m$ process technology. The C^4 is operated at the subthreshold region, where current and voltage has a exponential relationship, and consequently extremely low-power computation is achievable.

For the application to the audio signal processing, τ_f from Eq. (3) is so fast that the zero it produces lies far outside of the operating range and without loss of generality we can omit $s\tau_f$ from the numerator of Eq. (3). Then, the phase response of C^4 band-pass filter can be derived from this reduced transfer function as

$$\arg H(w) = -90^\circ - \arctan\Big(\frac{\frac{\tau_l + \tau_f(\frac{C_0}{\kappa C_2} - 1)}{\tau_l \tau_h} w}{\frac{1}{\tau_l \tau_h} - w^2}\Big), \quad (9)$$

and the constant group delay of C^4 is

$$D_b(w) = \frac{\alpha({w_o}^2 + w^2)}{(w_o^2 - w^2)^2 + \alpha^2 w^2}, \quad (10)$$

where $\alpha = \frac{\tau_l + \tau_f(\frac{C_0}{\kappa C_2} - 1)}{\tau_l \tau_h}$ and $w_o = 1/\tau_l \tau_h$. The maximum group delay at the center frequency, where $w \approx w_o$, is

$$D_b(w_o) \approx 2\frac{\tau_l \tau_h}{\tau_l + \tau_f(\frac{C_0}{\kappa C_2} - 1)}, \quad (11)$$

and group delay, where $w \ll w_o$, is

$$D_b(w) \approx \frac{\tau_l + \tau_f(\frac{C_0}{\kappa C_2} - 1)}{\tau_l \tau_h} \frac{1}{w_o^2}. \quad (12)$$

We designed the drawn capacitance ratio of C_O/C_2 to be 5 and extracted κ is close to 0.65 for the high-Q C^4 delay element. With these parameters Eq. (11) can be rewritten as

$$D_b(w_o) \approx \frac{2\tau_h}{1 + 6.7(\tau_f/\tau_l)}, \quad (13)$$

and Eq. (12) also as

$$D_b(w) \approx \frac{1 + 6.7(\tau_f/\tau_l)}{\tau_h} \frac{1}{w_o^2}. \quad (14)$$

Figure 4 shows group delays of measured data of low-Q C^4 band-pass filter of which magnitude and phase response are shown at Fig. 5. Group delay in Fig. 4 is similar to the group delay of typical low-pass or all-pass filter showing no peak around the center frequency and constant delay at $w \ll w_o$. The low-Q C^4 delay element is fabricated to have a unity gain for further cascade of delay element in delay networks and its Q_{max} defined in Eq. (6) is ≈ 0.828.

Group delays of the high-Q C^4 band-pass filters of Fig. 7 are shown at Fig. 6. As we have seen from Fig. 3, high peak delay around the center frequency is sharper as Q increases. High-Q C4 has a 20 dB magnitude gain and consequently the magnitude of input signal should be limited to a few tenth mV range for linearity. The input signal to the delay element should have a narrower bandwidth than the bandwidth of band-pass delay element in order not to have a spectral loss while exploring larger group delay. The peak of Fig. 6, where is pointed by A, is governed by Eq. (13) and the region B in the figure can be approximated by Eq. (14).

Group delays shown at Fig. 4 and Fig. 6 are reconstructed from measured data by taking numerical derivative of measured phase responses of C^4 and then adding negative sign. Since the measured phase responses, unlike ideal phase response, was not monotonically decreasing function of w, phase response was low-pass filtered for better approximation of the group delay.

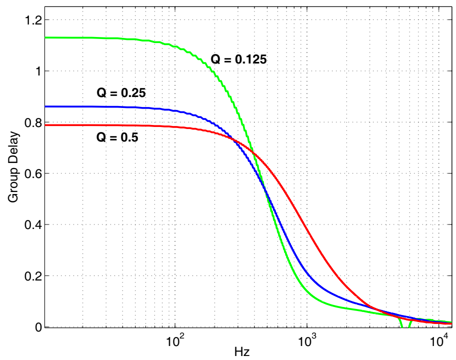
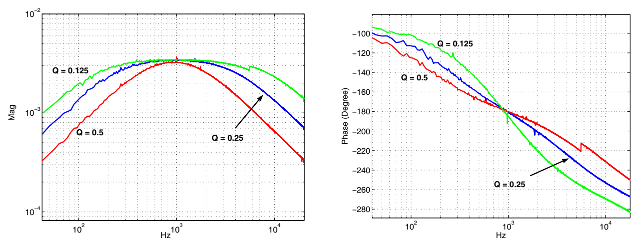


Fig. 4. Group delays of measured phase response of C^4 band-pass filter for low-Q values of Fig. 5. This figure suggests that group delays of C^4 band-pass delay element can be made similar to that of typical low-pass filter by having low Q.



(a) (b)

Fig. 5. (a) Measured frequency responses for low-Q values. All the frequency response has a 1 KHz center frequency. The slope is ± 20 dB/dec. (b) Measured phase responses. The slope of phase response at the center frequency, which gives maximum group delay, increases as Q increases.

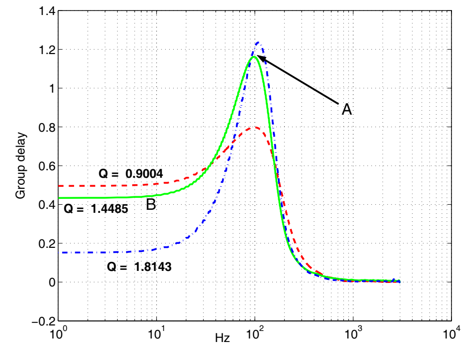
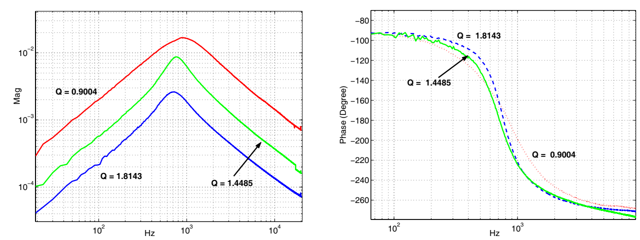


Fig. 6. Group delays of measured phase response of C^4 band-pass filter for high-Q values of Fig. 7. The peak pointed by A is governed by Eq. (13) and the region B can be approximated by Eq. (14). For Q = 1.4485, BW is 500 Hz and τ_l and τ_h are 0.32 msec and 0.16 msec, respectively.



(a) (b)

Fig. 7. (a) Measured frequency responses. The frequency response of Q = 0.9004 has center frequency at 900 Hz and the center frequency of the rest two plots is close to 750 Hz. The slope is ± 20 dB/dec. (b) Measured phase responses.

4. CONCLUSIONS

We have described a C^4 second order band-pass delay element which can be used in continuous-time subband adaptive tapped-delay filter. C^4 band-pass filter has been investigated extensively, however, most of them were focused on the characteristics of magnitude response. In this paper, we consider C^4 as a delay element focusing on phase response and group delay. We showed that all-pass or low-pass delay element, in spite of their constant group delay property where $w \ll w_o$, are not suitable for the application where larger delay is required. Delay elements with high-Q and low-Q C^4 band-pass filter were fabricated and tested to meet the delay constraint of subband adaptive tapped-delay filter. Analysis of C^4 band-pass filter as a delay element was also presented.

5. REFERENCES

[1] S. L. Gay and J. Benesty, *Acoustic Signal Processing for Telecommunication*, Kluwer Academic Publishers, 2000.

[2] K. R. Laker, A. Ganesan, and P. E. Fleischer, "Design and implementation of cascaded switched-capacitor delay equalizers," *IEEE Transactions on Circuits and Systems*, vol. 32, no. 7, pp. 700–711, July 1985.

[3] J. Juan, J. G. Harris, and J. C. Principe, "Analog hardware implementation of adaptive filter structures," in *Proceedings of the International Joint Conference on Neural Networks*, June 1997, vol. 2, pp. 916–921.

[4] K. Bult and H. Wallinga, "A CMOS analog continuous-time delay line with adaptive delay-time control," *IEEE Journal of Solid-State Circuits*, vol. 23, no. 3, pp. 759–766, 1988.

[5] D. Graham and P. Hasler, "Capacitively-coupled current conveyer second-order section for continuous-time bandpass filtering and cochlea modeling," in *Proceedings of the IEEE International Symposium on Circuits and Systems*, Phoenix, AZ, May 2002, vol. 5, pp. 485–488.