Homework II - Solutions

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1. There are 2 key observations to make:

(a) Any optimal solution on \( k \) lines consists of \( p \) (say) words on the first line and the remaining \( n - p \) words on the remaining \( k - 1 \) lines.

(b) If all the words fit on one line, it is sub-optimal to break up the words into two or more lines.

These two simple observations are enough to provide you with the recurrence relation. Let \( s[i,j] \) denote the packing cost of packing words \( w_i \) through \( w_j \) in one line. The following recurrences are immediate

\[
s[i,j] = (M - j + i - \sum_{t=i}^{j} l_t)^{3}, \quad \text{if } (M - j + i - \sum_{t=i}^{j} l_t) \geq 0 \quad \text{AND } j \neq n \quad (1)
\]
\[
= 0, \quad \text{if } (M - j + i - \sum_{t=i}^{j} l_t) \geq 0 \quad \text{AND } j = n \quad (2)
\]
\[
= \infty, \quad \text{if } (M - j + i - \sum_{t=i}^{j} l_t) < 0 \quad (3)
\]

Equation (2) captures the fact that the last line should not be charged a cost, if the words fit in one line.

Let us now define \( m[i,j] \) to be the optimal cost of packing words \( w_i \) through \( w_j \) with word \( w_i \) starting on a fresh line. According to this definition, the solution to our problem is given by \( m[1,n] \). The following recurrence is immediate:

\[
m[i,j] = s[i,j] \text{ if } (M - j + i - \sum_{t=i}^{j} l_t) \geq 0 \quad (4)
\]
\[
= \min_{i \leq k \leq j} s[i,k] + m[k+1,j], \quad \text{otherwise} \quad (5)
\]