Advanced Analysis of Algorithms - Practice Midterm

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Please attempt as many problems as you can in class. The practice midterm will not be graded, i.e. there are no points. The solutions are posted at:
http://www.csee.wvu.edu/~ksmani/courses/fa03/algos/algos.html

1 Problems

1. Show that

\[ \sum_{i=1}^{n} i^3 = \left[ \frac{n \cdot (n+1)}{2} \right]^2. \]

2. Consider the recurrence relation:

\[ T(n) = \begin{cases} 1, & \text{if } n = 1 \\ T(n-1) + 2^n, & \text{otherwise} \end{cases} \]

Show that \( T(n) = 2^{n+1} - 3. \)

3. Give an algorithm which will return the third largest element of a heap with heap-size \( \geq 3, \) in \( O(1) \) time. Assume that all elements are unique.

4. Describe an algorithm that, given \( n \) integers in the range from 1 to \( k, \) preprocesses its input and then answers any query about how many of the \( n \) integers fall into a range from \([a \ldots b]\) in \( O(1) \) time. Your algorithm should use \( O(n + k) \) preprocessing time.

5. Suppose a biker is about to go on a ride on a bike trail carrying a single knapsack. Suppose further that she knows the maximum total weight \( W \) that she can carry, and she has a set \( S \) of \( n \) different useful items that she can potentially take with her. Let us assume that each item \( j \) has an integer weight \( w_j \) and a benefit value \( b_j, \) which is a value that the biker assigns to item \( j. \) Her problem is to optimize the value of the items that she places into her knapsack, without going over the weight limit \( W \) (i.e., maximize \( \sum_{j \in T} b_j \) subject to \( \sum_{j \in T} w_j \leq W \)). Give a recursive definition for the profit value of the optimal solution to this problem.