1 Instructions

1. The homework is due on September 14, in class.
2. Each question is worth 3 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Write a recursive algorithm to compute the maximum element in an array of integers. You may assume the existence of a function “\( \text{max}(a, b) \)” that returns the maximum of two integers \( a \) and \( b \).

2. Argue that your algorithm is correct. \textit{Hint: First Principle of Mathematical Induction}.

3. What is the \textit{exact} comparison complexity of your algorithm? Derive a recurrence relation and solve it to justify your answer.

4. Argue using induction that the exact solution to the recurrence relation:

\[
\begin{align*}
T(1) &= 0 \\
T(n) &= 2 \cdot T\left(\frac{n}{2}\right) + n, \quad n \geq 2
\end{align*}
\]

is \( T(n) = n \cdot \log n \).

5. Show that \( \log(n!) \in O(n \cdot \log n) \).