Analysis of Algorithms - Midterm

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1 Instructions

1. The Midterm is to be turned in by 9:00 am in class.

2. Each question is worth 4 points.

3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Asymptotics:
   
   (a) Show that \([f(n) \in \Omega(g(n))) \land (g(n) \in \Omega(h(n))) \Rightarrow f(n) \in \Omega(h(n))\).
   
   (b) Does \(\log^3 n \in O(n^{0.5})\)?

2. Algorithm Design for order: Given an integer array of \(n\) elements, design an algorithm to find both the maximum element and the minimum element, using at most \(3n^2\) element to element comparisons. Comparisons for iterators (e.g., for loops) do not count.

3. Binary Search Trees: Enumerate all the binary search trees on the keys 1, 2 and 3.

4. Sorting: Explain briefly how Randomized Quicksort performs \(O(n \cdot \log n)\) comparisons, in the expected case, to sort an array of \(n\) elements. (You may assume the algorithm discussed in class.)

5. Properties of Binary Trees: Let \(T\) be a proper binary tree with \(n\) nodes and height \(h\). Argue that the number of external nodes in \(T\) is at least \(h + 1\) and at most \(2^h\).