Linear Programming
Class Notes
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Karl Stump

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1 Introduction

Mathematical programming models use *decision variables* to minimize or maximize an *objective function*, subject to a set of *constraints*. Not all problems can be solved using this approach, but many can be.

2 LP Models

Constructing an LP model requires three steps.

1. Determine the decision variables
2. Write the objective function
3. Formulate the constraints

The general form is as follows:

$z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Subject to:

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n\{\leq, =, \geq\}b_1$
$\ldots$
$\ldots$
$\ldots$
$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n\{\leq, =, \geq\}b_m$
$x_1, x_2, \ldots, x_n \geq 0$

3 Assumptions

There are four basic assumptions that must hold for LP problems:

1. Certainity
2. Proportionality
3. Additivity
4. Divisibility

4 Example

The first problem in the class handout is solved as follows:

Variables

$x_1 = \#$ of A switches
$x_2 = \#$ of B switches

Objective Function

$$z = \max\ 20x_1 + 30x_2$$

Constraints

$4x_1 + 3x_2 \leq 240$
$x_1 + 2x_2 \leq 140$
$x_1 \geq 25$
$x_1, x_2 \geq 0$

Objective Function

$$z = \begin{vmatrix} 20 & 30 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix}$$

Constraints

$$\begin{vmatrix} 4 & 3 \\ 1 & 2 \\ -1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 3 \\ 1 & 2 \\ -1 & 0 \end{vmatrix} \leq \begin{vmatrix} 240 \\ 140 \\ -25 \end{vmatrix}$$

$$\begin{vmatrix} x_1 \\ x_2 \end{vmatrix} \geq \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

5 Miscellaneous

For those interested, you can install an LP utility called lp_solve; go to

`ftp://ftp.es.ele.tue.nl/pub/lp_solve`

Here you'll find both Unix and DOS versions. I installed the Unix version on my Linux box, and it seems to work fine. As I understand it, MS-Excel also has an LP solver.