Midterm

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1 Instructions

Each section is worth 10 points for CS 391 students and 8 points for CS 491 students. CS 391 students can pick any 4 out of 5 questions. (Go ahead and answer all 5; I will take the top 4!)

2 Constraints

Let $S = \{\vec{\mathbf{x}} : \mathbf{A}.\vec{\mathbf{x}} \leq \vec{\mathbf{b}}\}$ and let z^* be the optimum value of the objective function $\max \vec{\mathbf{c}}.\vec{\mathbf{x}}$.

1. Let z' be the optimum value obtained after a new constraint has been added. What is the relationship between z' and z^*?
2. Let z' be the optimum value obtained after an existsing constraint is deleted. What is the relationship between z' and z^*?

3 Theorems of the Alternative

Let $\mathbf{A}$ be an $m \times n$ rational matrix. Prove the following theorem

Theorem: 3.1 *Either I:* $\exists \vec{\mathbf{y}} \in R^m, \vec{\mathbf{y}}.\mathbf{A} = \vec{\mathbf{c}}$*, or (exclusively)*
II: $\exists \vec{\mathbf{x}} \in R^n, \mathbf{A}.\vec{\mathbf{x}} = \vec{\mathbf{0}}$*, and* $\vec{\mathbf{c}}.\vec{\mathbf{x}} = 1$.

(Note: This theorem is known as Gale's theorem for equations)

4 Polytopes

Given two bounded polyhedral sets $S = \{\vec{\mathbf{x}} : \mathbf{A}.\vec{\mathbf{x}} \leq \vec{\mathbf{b}}\}$ and $S' = \{\vec{\mathbf{x}} : \mathbf{A}'.\vec{\mathbf{x}} \leq \vec{\mathbf{b}'}\}$, design an efficient procedure to check whether $S \subseteq S'$. You may assume that $\mathbf{A}$ has size $m \times n$, while $\mathbf{A}'$ has size $m' \times n$. (Hint: Assume the existence of an oracle that solves linear programs efficiently. If it helps, assume that the polyhedral sets are bounded.)

5 Integer Modeling

Let $\mathbf{G} =< \mathbf{V}, \mathbf{E} >$ be a directed graph and let $s \in V$ be a source. Associated with the edges is a cost function $c : \mathbf{E} \Rightarrow \mathbf{R}_+$ which associates a positive cost for going from Vertex i to vertex j, using edge (i, j). If edge $(i, j) \notin \mathbf{E}$, we assume that $c(i, j) = \infty$. The Single-Source Shortest Path Value Problem is concerned with finding the *distance* from the source to each vertex in the graph. Formulate a Mathematical Programming Model for this problem. Argue its correctness.

6 Unimodularity

Show that

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

is not totally unimodular, but $\{\vec{\mathbf{x}} : \mathbf{A}.\vec{\mathbf{x}} = \vec{\mathbf{b}}\}$ is integral for all integral vectors $\vec{\mathbf{b}}$. (Are we disproving the Hoffman-Kruskal theorem?!%$)