Midterm - Practice Questions

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1. Suppose my linear program $\mathbf{A}.\vec{\mathbf{x}} \leq \vec{\mathbf{b}}$ has the specification that all variables except x_1 are ≥ 0; x_1 is an unrestricted variable. How would you model it in the standard form with all variables ≥ 0?

2. Suppose that all n variables are unrestricted variables. Show that you can replace this set with a set of $n+1$ variables that are constrained to be non-negative?

3. ***Definition: 0.1*** *Polytope - A bounded polyhedron*

   Definition: 0.2 *$\epsilon-$neighbourhood - The ϵ-neighbourhood of a point $\vec{\mathbf{x}}$, denoted by $N_\epsilon(\vec{\mathbf{x}})$ is defined as a ball of radius ϵ centred around $\vec{\mathbf{x}}$.*

   Definition: 0.3 *A point $\vec{\mathbf{x}}$ is said to be in the closure of a set S, if for all $\epsilon > 0$, $N_\epsilon(\vec{\mathbf{x}}) \cap S \neq \phi$. The set of all points in the closure of S are denoted by cl S. S is said to be closed if $S =$ cl S.*

   Show that a polytope defined in the usual way is closed.

4. Show that the set of optimal points of a Linear Program is a convex set.

5. Can a pivot of the Simplex Algorithm move the feasible point in the basis, while leaving the cost unchanged ?

6. Prove or disprove: If an LP is unbounded then there exists a vector $\vec{\alpha}$ such that for any feasible $\vec{\mathbf{x}}$, $\vec{\mathbf{x}} + \mathbf{k}.\vec{\alpha}$ is also feasible, for all $k > 0$.

7. Use the Simplex Method to solve

$$\begin{aligned} \max z = 3.x_1 + 10.x_2 + 5.x_3 + 11.x_4 + 7.x_5 + 14.x_6 \\ s.t. \\ x_1 + 7.x_2 + 3.x_3 + 4.x_4 + 2.x_5 + 5.x_6 = 42 \\ x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{aligned}$$

8. Show that the point $\vec{\mathbf{x}} = [10, 0, 16, 6]$ is an optimal solution to the problem:

$$\begin{aligned} \max z = x_1 + 2.x_2 + 5.x_3 + x_4 \\ s.t. \\ x_1 + 2.x_2 + x_3 - x_4 \leq 20 \\ -x_1 + x_2 + x_3 + x_4 \leq 12 \\ 2.x_1 + x_2 + x_3 - x_4 \leq 30 \\ x_i \geq 0, i = 1, 2, 3, 4 \end{aligned}$$

9. Consider the following linear program:

$$\begin{aligned} \min z = \vec{\mathbf{b}}.\vec{\mathbf{w}} - \vec{\mathbf{c}}\mathbf{x} \\ s.t. \\ \mathbf{A}.\vec{\mathbf{x}} \leq \vec{\mathbf{b}} \\ \mathbf{A}^{\mathbf{T}}.\vec{\mathbf{w}} \leq \vec{\mathbf{c}}^{\mathbf{T}} \\ \vec{\mathbf{x}} \geq \vec{\mathbf{0}} \\ \vec{\mathbf{w}} \geq \vec{\mathbf{0}} \end{aligned}$$

where $\mathbf{A}$ is $m \times n$, $\vec{\mathbf{b}}$ is $m \times 1$, $\vec{\mathbf{c}}$ is $n \times 1$. Show that the optimal objective value is 0 or the problem is infeasible.