Project Problems

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1. Given an arbitrary graph $\mathbf{G} =< \mathbf{V}, \mathbf{E} >$ with positive and negative edge-weights, what is the minimum number of edges to delete so that the resultant graph does not contain a negative weight cycle ?

   References and Motivation - It is well known that graphs can be used to represent systems of difference constraints [CLR92]. We are thus trying to minimize the number of constraints to be eliminated to make the system feasible. It is known that the problem is `NP-complete`.

   Project credit - Find a constant bound approximation (or better!)

2. ***Definition: 0.1*** *A graph is said to be an interval graph if its vertices can be put into* $1-1$ *correspondence with a set of intervals on the real-line, so that two intervals intersect if and only if their corresponding vertices in the graph have an edge between them.*

   Given an arbitrary graph $\mathbf{G} =< \mathbf{V}, \mathbf{E} >$ (unweighted) what is the minimum number of edges to delete so that the resultant graph is an interval graph ?

   References and Motivation - Interval graphs are used to model a number of problems in diverse domains [Spi]. The interval property is crucial to the solution of these problems.

   Project Credit - Show that the problem is `NP-complete` and find a constant factor approximation algorithm.

3. ***Definition: 0.2*** *A graph* $\mathbf{G}$ *is said to be a split graph, if its vertex set can be partitioned into two parts* $\mathbf{V_1}$ *and* $\mathbf{V_2}$ *such that the subgraph induced by* $\mathbf{V_1}$ *is an interval graph and the that induced by* $\mathbf{V_2}$ *is an independent set.*

   Given an arbitrary graph $\mathbf{G} =< \mathbf{V}, \mathbf{E} >$ (unweighted) is it a *split graph*?

   References and Motivation - Split interval graphs are used in modeling problems in Computational Biology [MWZ98]. The hardness of this problem is not known.

   Project credit - Show that the problem is `NP-complete`.

4. ***Definition: 0.3*** *A graph* $\mathbf{G}$ *is said to be a probe-interval graph if there exists a probe/non-probe assignment to its vertices i.e. a partition of the vertex set into* $\mathbf{P}, \mathbf{N}$*, such that*

   (a) The graph $\mathbf{G}$ *is an interval graph, and*

   (b) For every edge, one of the vertices is is in the set $\mathbf{P}$*?*

   References and Motivation - Probe interval graphs are used to model gene sequencing problems in Computational Biology [JS00]. It is known that the problem can be solved in polynomial time *if the probe/non-probe assignment is given.*

   Project Credit - Show that the problem is `NP-complete`.

5. ***Definition: 0.4*** *A graph* $\mathbf{G} =< \mathbf{V}, \mathbf{E} >$ *is said to be collapsible, if for every subset* $\mathbf{S}$ *of* $\mathbf{V}$*, with* $|\mathbf{S}|$ *even (i.e. all even subsets of the vertex set),* $\mathbf{G}$ *has a spanning connected subgraph* $\mathbf{H}$ *such that the set of odd degree vertices in* $\mathbf{H}$ *is precisely* $\mathbf{S}$*.*

   Given an arbitrary graph $\mathbf{G}$, is it collapsible ?

   References and Motivation - This is a pure Discrete Structure problem, with absolutely no practical relevance! However, establishing the hardness of the problem will settle a 10 year old conjecture [CL93]

   Project Credit - Show that the problem is `coNP-complete`.

References

[CL93] Chen and Lai. Collapsible graphs and matchings. *JGT: Journal of Graph Theory*, 17, 1993.

[CLR92] T. H. Cormen, C. E. Leiserson, and R. L. Rivest. *Introduction to Algorithms*. MIT Press and McGraw-Hill Book Company, 6th edition, 1992.

[JS00] Julie Johnson and J. Spinrad. A polynomial time recognition algorithm for probe interval graphs. In *Symposium on Discrete Algorithms*, 2000.

[MWZ98] McMorris, Wang, and Zhang. On probe interval graphs. *DAMATH: Discrete Applied Mathematics and Combinatorial Operations Research and Computer Science*, 88, 1998.

[Spi] J. Spinrad. *Graph Theory*. http://www.vuse.vanderbilt.edu/ spin/research.html.