Final exam

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1. Consider the following greedy strategy to solve the vertex cover problem:

Function VERTEX-COVER ($G =< V, E >$)
$Cover = \phi$
while (there exist uncovered edges in E) **do**
  Select the vertex v' with the largest degree
  $Cover = Cover \cup v'$
  Delete from the edge set E, all edges that are incident to v'
end while
return($Cover$)

Algorithm 0.1: A greedy algorithm for Vertex Cover

   Show that Algorithm (0.1) is not optimal by providing a counter-example? How good is the quality of the approximation? (*Hint: Set Cover*) (10 pts.)

2. Formulate a Dynamic Programming algorithm to solve the minimum partition problem. Argue correctness and provide an analysis of the running time. (10 pts.)

3. Show that the β in the semidefinite programming approximation algorithm for the MAX-CUT problem is at least 0.87. You may use analytical or software techniques. (10 pts.)

4. In class we analyzed the *First-Fit Decreasing* heuristic for the bin-packing problem. Show that the bound of $\frac{3}{2}.OPT + 1$ is tight by providing an example. (10 pts.)

5. There are m people and n jobs. Let $w(i, j)$ denote the affinity of person i for job j. We want to assign people to jobs, such that

   (a) Each person is assgned to at most 1 job,
   (b) Each job gets exactly one person,
   (c) The overall affinity is maximized.

   You can assume that $m \geq n$.

   (a) Formulate the above problem as an Integer Program
   (b) Formulate the above problem as a Linear Program
   (c) Write the dual of the Linear Program.

   (10 pts.)