CS 491I Approximation algorithms
Solutions of the Quiz Problems
Dejan Desovski

1. Show that the set $S = \{A \cdot \vec{x} \{\leq, =, \geq\} \vec{b}, \vec{x} \geq \vec{0}\}$ is convex.

Answer:
We will show the convexity of the $\{\leq\}$ form of the set. Any other form can be converted to this form. Let $\overrightarrow{x_1}, \overrightarrow{x_2} \in S$ and $\lambda \in [0,1]$. We should prove that any linear combination of these vectors also belongs to the set *S*.

$$\vec{z} = \lambda \overrightarrow{x_1} + (1-\lambda)\overrightarrow{x_2} \geq \vec{0} \;\Rightarrow\; \vec{z} \geq \vec{0}$$

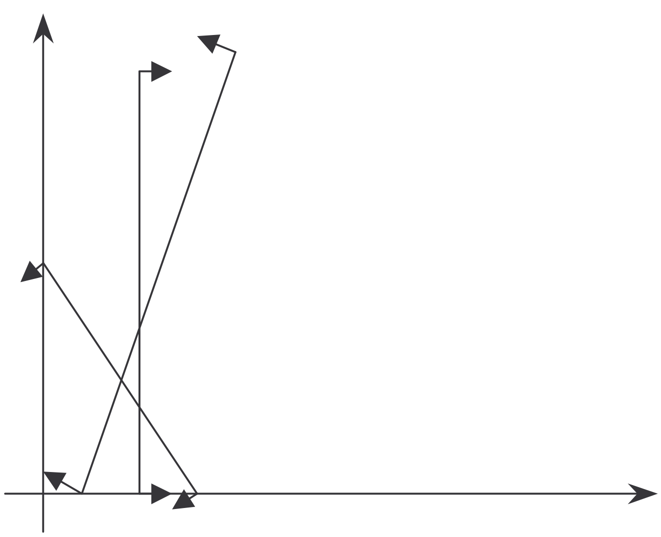
$$A \cdot \vec{z} = \lambda A \cdot \overrightarrow{x_1} + (1-\lambda) A \overrightarrow{x_2} \leq \lambda \vec{b} + (1-\lambda)\vec{b} = \vec{b} \quad \Rightarrow \quad A \cdot \vec{z} \leq \vec{b}$$

$\Rightarrow \vec{z} \in S$**,** and thus *S* is convex.

2. Solve graphically:

$$\min z = 4x_1 + 5x_2$$
$$3x_1 + 2x_2 \leq 24$$
$$x_1 \geq 5$$
$$3x_1 - x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

Answer:



The system is infeasible.

3. Show that the halfspace $H^- = \{\vec{x} : \vec{a} \cdot \vec{x} \le \alpha\}$ is convex.

Answer:

Let $\overrightarrow{x_1}, \overrightarrow{x_2} \in H^-$ and $\lambda \in [0,1]$.

$\vec{z} = \lambda \overrightarrow{x_1} + (1-\lambda)\overrightarrow{x_2}$

$\vec{a} \cdot \vec{z} = \lambda \vec{a} \cdot \overrightarrow{x_1} + (1-\lambda)\vec{a} \cdot \overrightarrow{x_2} \le \lambda\alpha + (1-\lambda)\alpha = \alpha$

$\Rightarrow \vec{z} \in H^-$, and thus H^- is convex.

4. Given three vectors, $\vec{a} = [4,2]^T, \vec{b} = [-2,6]^T, \vec{c} = [2,5]^T$, illustrate graphically

(a) The set of all linear combinations of $\vec{a}, \vec{b}, \vec{c}$,

(b) The set of all non-negative linear combinations of $\vec{a}, \vec{b}, \vec{c}$,

(c) The set of all convex combinations of $\vec{a}, \vec{b}, \vec{c}$.

Answer:

(a) The set of all linear conbinations covers the whole plane.
(b) Represents a part of the plane between the vectors.
(c) A triangle between the vectors.