Automata Theory - Final

K. Subramani  
LCSEE,  
West Virginia University,  
Morgantown, WV  
{ksmani@csee.wvu.edu}

1 Instructions

1. Attempt as many problems as you can. You will be given partial credit.

2. The alphabet for each language is \( \Sigma = \{0, 1\} \).

2 Problems

1. Design a DFA to accept the language \( L \), that consists of all strings having even length \textit{and not} ending in 1. (5 points)

2. Assume that you are given two DFAs \( A_1 \) and \( A_2 \); let the languages accepted by these DFAs be \( L(A_1) \) and \( L(A_2) \) respectively. Design a strategy to check whether \( L(A_1) \subseteq L(A_2) \). (5 points)

3. Consider the CFG, \( G \), defined by the following productions:

\[
S \rightarrow 0S1S | 1S0S | \epsilon
\]

Show that \( L(G) \) is the set of all strings with an equal number of 0s and 1s. (5 points)  
\textit{Hint: Use Induction.}

4. Consider the CFG \( G_1 \) defined by:

\[
S \rightarrow 0S | 0S1S | \epsilon
\]

Show that

(a) \( G_1 \) is ambiguous. (2 points) \textit{Hint: How many leftmost derivations does \( w = 001 \) have?}

(b) Is \( L(G_1) = \{0^n1^n| n \geq 0\} \)? (3 points)

5. Design a PDA to accept the language of palindromes \( L \), i.e., the language \( L \) consists of all strings \( w \), such that \( w = w^R \). (5 points)

6. Design a Turing Machine to accept the regular language described by the expression \( 1\cdot 0^* + 0\cdot 1^* \). (5 points)

3 Abbreviations

1. CFG - Context Free Grammar

2. PDA - Pushdown Automaton