Automata Theory - Quiz II (Solutions)

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1 Problems

1. Let $L_1$ and $L_2$ be 2 regular languages, over the same alphabet $\Sigma$ and let them be respectively represented by DFAs $A_1$ and $A_2$. Discuss a strategy by which you could check whether there is a string in $\Sigma^*$, that is neither in $L_1$ nor in $L_2$. (3 points)

Solution: Observe that we want to test whether $M$ is empty, where $M = L_1' \cap L_2'$. This is equivalent to testing whether $M = (L_1 \cup L_2)'$ is empty. Thus we can combine DFA’s $A_1$ and $A_2$ to form an $\epsilon$-NFA (say $B$), which contains a start state that on transition $\epsilon$ goes to both $A_1$’s and $A_2$’s old start states. Similarly, we have a new final state to which all the original final states of $A_1$ and $A_2$ are connected. We now must convert $B$ to a DFA (say $C$). We will then switch the accepting and non-accepting states of $C$ and use the graph reachability algorithm to check whether any accepting state is reachable from the start state, if so, then there is at least one string in $M$. Otherwise, $M$ is empty. \(\square\)

2. Consider the grammar $G = (\{S\}, \{a, b\}, P, S)$, where $P$ is defined as the following set of rules:

$S \rightarrow aSbS \mid bSaS \mid \epsilon$

Argue using mathematical induction, that $L(G)$ is the set of all strings with an equal number of $a$’s and $b$’s. (4 points)

Proof: Using mathematical induction on the length of the string (say $w$), we have the following.

Base Case:
Let $|w| = 0$, then the production $S \rightarrow \epsilon$ derives this string, which has 0 $a$’s and 0 $b$’s.

Let us assume that $w \in L(G)$ for all $|w| \leq 2^{n-1}$.

Inductive Step:
Let $|w| = 2^n$, then $w$ must be of the form $w = aw' b$ or $w = bw' a$. Using the inductive hypothesis, we have $w' \in L(G)$. Now, in order to produce $w$, we must use either production $S \rightarrow aSbS$ or $S \rightarrow bSaS$. However, both productions give us an equal number of $a$’s and $b$’s. Thus $|w| = 2^n \in L(G)$.

Therefore, $L(G)$ is the set of all strings with an equal number of $a$’s and $b$’s. \(\square\)

3. Write a CFG for the following language: $L = \{a^ib^j \mid i, j \geq 1, \ j = i + 1\}$. (3 points)

Solution: A Context-Free Grammar for the language $L$ is:

$S \rightarrow abb$
$S \rightarrow aSb$

\(\square\)