1 Problems

1. For $\Sigma = \{a, b\}$, construct a DFA that accepts the set consisting of all strings with no more than 3 $a$’s.

   **Solution:** □

   ![DFA Diagram](image)

   Figure 1: DFA

2. For $\Sigma = \{a, b, c\}$, construct an $\epsilon$-NFA that accepts the language $L = \{ab + abc\}^*$.

   **Solution:** □

   ![\epsilon-NFA Diagram](image)

   Figure 2: $\epsilon$-NFA

3. Give a regular expression for the following languages.

   (a) $L = \{a^n b^m \mid n \geq 4, m \leq 3\}$

   **Solution:** Observe that we can break the solution into the cases $m = 0, 1, 2, 3$. Now, we can write the solution by first generating 4 or more $a$’s followed by the requisite number of $b$’s. Thus, the regular expression for $L$ is $aaaaa^*(\epsilon + b + bb + bbb)$.

   (b) $L'$

   **Solution:** Observe that a string is not in $L$ if it is of the form $a^n b^m$, with either $n < 4$ or $m > 3$;
we must also include strings in which a b is followed by an a. Thus, the regular expression for \( L' \) is
\[
(\epsilon + a + aa + aaa)b^* + a^*bbb^* + (a + b)^*ba(a + b)^*.
\]
\( \square \)

4. Prove that the following language \( L = \{ a^n b^k a^k | k \geq n + 1 \} \) is not regular.

**Proof:**

(a) Player 1 picks the language \( L \) to be proved nonregular, where \( L = \{ a^n b^k a^k | k \geq n + 1 \} \).
(b) Player 2 picks \( n \).
(c) Player 1 picks \( w = a^n b^n a^{2n} \).
(d) Player 2 breaks \( w \) into \( xyz \), in which \( y \neq \epsilon \) and \( |xy| \leq n \).
(e) Player 1 wins. Since \( |xy| \leq n \) and \( xy \) comes at the front of \( w \), we know that \( x \) and \( y \) consist of only a’s. Thus, \( y = a^k \) for \( 0 < k \leq n \), since \( y \neq \epsilon \). The Pumping Lemma tells us that \( xy^2z \) is in \( L \) if \( L \) is regular. If we choose \( k = 2 \), the resulting string is \( w' = a^{n+2}b^n a^{2n} \). Clearly \( w' \) is not in \( L \). Therefore, we have contradicted our assumption that \( L \) is regular.

\( \square \)

5. Design a context-free grammar for the language \( L = \{ a^n b^m | 2n \leq 3 \cdot n \leq m, n \geq 0, m \geq 0 \} \).

**Solution:** The following rules define the context-free grammar.

(a) \( S \to \epsilon \)
(b) \( S \to aSbb \)
(c) \( S \to aSbbb \)

\( \square \)