1 Problems

1. Minimize the states in the DFA depicted in the following diagram.

![DFA Diagram]

Figure 1: DFA
2. Convert the following regular expression to an NFA $a + b^n$ where $n \geq 1$.
   **Solution:** □

3. Convert the following $\epsilon$-NFA into an equivalent DFA.
4. Give a grammar for the following language \( L = \{ a^n b^m | n + m \text{ is even} \} \).

**Solution:**

\[
\begin{align*}
S & \rightarrow a a S | \lambda \\
A & \rightarrow b b A | \epsilon
\end{align*}
\]

5. Give a PDA that accepts the following language \( L = \{ a^n b^m | n \geq 0 \} \).

**Solution:** \( P = \langle Q, \Sigma, \Gamma, \delta, q_0, Z_0, F \rangle \), where

- \( Q = \{ q_0, q_1, q_2 \} \),
- \( \Sigma = \{ a, b \} \),
- \( \Gamma = \{ 1, Z_0 \} \),
- \( F = \{ q_0 \} \), and
- \( \delta =
\begin{align*}
\delta(q_0, a, Z_0) & = \{(q_1, 1Z_0)\} \\
\delta(q_1, a, 1) & = \{(q_1, 11)\} \\
\delta(q_1, b, 1) & = \{(q_2, \epsilon)\} \\
\delta(q_2, b, 1) & = \{(q_2, \epsilon)\} \\
\delta(q_2, \epsilon, Z_0) & = \{(q_0, \epsilon)\}
\end{align*}
\]

6. Design a Turing Machine that accepts the language defined by the regular expression \( a(a + b)^* \). Assume that \( \Sigma = \{ a, b \} \).
Solution: $M = \langle Q, \Sigma, \Gamma, \delta, q_0, B, F \rangle$, where

- $Q = \{q_0, q_1, q_2\}$,
- $\Sigma = \{a, b\}$,
- $\Gamma = \{a, b, B\}$,
- $B$ is the blank symbol,
- $F = \{q_2\}$, and
- $\delta = $

\[
\begin{align*}
\delta(q_0, a) &= (q_1, a, R) \\
\delta(q_1, a) &= (q_1, a, R) \\
\delta(q_1, b) &= (q_1, b, R) \\
\delta(q_1, B) &= (q_2, B, R)
\end{align*}
\]