Stochastic Programming Models and application

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Outline

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Brief Introduction
Uncertainty is Everywhere

Almost Everything in a real world has some amount of uncertainty let alone Financial activities. Where to send the plane to accomplish a goal? Considering uncertain factors such as noise, demands on the system, equipment failures, wind. How much is its effect on Control variables like angle, velocity, acceleration?
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- Another most popular and traditional measure of risk is volatility. The main problem with volatility, however, is that it does not care about the direction of an investment's movement:
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VaR and the Idea behind that

Value at Risk (VaR)

Value at Risk (VaR) is a measure related to percentiles of loss distributions and represents the predicted maximum loss with a specified probability level (e.g., 95%) over a certain period of time (e.g., one day).

What would be the first priority of an investor to invest in a special market? The amount of benefit that he/she can gain?

Yes, but how about if the failure case happens?

For investors, risk is about the odds of losing money, and VaR is based on that common-sense fact. By assuming investors care about the odds of a really big loss, VaR answers the question, "What is my worst-case scenario?" or "How much could I lose in a really bad month?"
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Methods of Calculating VaR

We are covering three methods in order to calculate the VaR which is the answer of this question: "What is the most (let’s say with 95 % confidence ) I can expect to lose in dollars over the specific time period (let’s say next month) ?"
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- First Method based on historical data
- Second method based on Variance and CoVariance method
- Third Method based on Monte-Carlo Simulation
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Graph Explanation

- At the highest point of the histogram (the highest bar), there were more than 250 days when the daily return was between 0 % and 1 %.

Figure: daily return for the QQQ graph
At the highest point of the histogram (the highest bar), there were more than 250 days when the daily return was between 0% and 1%.

At the far right, you can barely see a tiny bar at 13%; it represents the one single day (in Jan 2000) within a period of five-plus years when the daily return for the QQQ was a stunning 12.4%!
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With 95% confidence, we expect that our worst daily loss will not exceed 4% or we expect that with 95% confidence our gain will exceed -4%.
If we invest 100$, what would be our worst daily loss by 95 confidence level?

By 95% confident our worst daily loss will not exceed 4$ (100 $ * -4% = 4 $).

We know that this answer does not express absolute certainty but instead makes a probabilistic estimate.
Exercise

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Increasing the Confidence level

By moving to the left side of the above histogram, the confidence level will be increased. Let’s go to the left side of the above histogram where the first two red bars, at -8% and -7% represent the worst 1% of daily returns. With 99% confidence, we expect that the worst daily loss will not exceed 7%. Or, if we invest $100, we are 99% confident that our worst daily loss will not exceed $7.
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The Variance-Covariance Method

This method assumes that stock returns are normally distributed. In other words, it requires that we estimate only by having two following factors one can plot the normal distribution curve for that:

- Expected/Average return
- Standard deviation

Here we plot the normal curve against the same actual return data:

In comparison with historical data we use the familiar curve instead of actual data.
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Distribution of Daily Returns
NASDAQ 100 - Ticker: QQQ

Instead of actual returns, here we look at the "worst" 5% (or worst 1%) of the normal curve.

What is the value of VaR for both 95% and 99% confidence level if we know that the standard deviation of the QQQ is equal to 2.64%?

Since this is a Normal distribution curve we know that the confidence of 95% is $-1.96 \sigma$ and confidence of 99% is $-2.58 \sigma$.

For 95% confidence the VaR is $-1.96 \cdot 2.64 = -5.17\%$.

For 99% confidence the VaR is $-2.58 \cdot 2.64 = -6.81\%$.
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Definition

A Monte Carlo simulation refers to any method that randomly generates trials. This method involves developing a model for future stock price returns and running multiple hypothetical trials through the model but by itself does not tell us anything about the underlying methodology. Monte Carlo simulation amounts to a black box generator of random outcomes. Without going into further details, we ran a Monte Carlo simulation on the QQQ based on its historical trading pattern. In our simulation, 100 trials were conducted. If we ran it again, we would get a different result, although it is highly likely that the differences would be narrow.
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Please note that while the previous graphs have shown daily returns, this graph displays monthly returns.
There is no correct time period so different calculations may specify different time periods. For example, commercial banks typically calculate a daily VaR, asking themselves how much they can lose in a day; while pension funds often calculate a monthly VaR.

Classic idea in finance: the standard deviation of stock returns tends to increase with the square root of time:

$$\sigma_{\text{monthly}} = \sigma_{\text{daily}} \times \sqrt{T}$$

Having the above assumption, we can use this formula to convert the VaR from daily to monthly index.

$T$ in the above equation is equal to 20 (days of the month excluding the weekends).
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Find the VaR for the Monthly period from given daily Var which is given in the above figure,
Example

<table>
<thead>
<tr>
<th>Investment</th>
<th>VAR Method</th>
<th>Standard Deviation</th>
<th>Time Period</th>
<th>Calculated VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>QQQ</td>
<td>N/A *</td>
<td>Daily</td>
<td>~ - 4.0%</td>
</tr>
<tr>
<td>2</td>
<td>QQQ</td>
<td>Variance-Covariance</td>
<td>Daily</td>
<td>- 6.16%</td>
</tr>
<tr>
<td>3</td>
<td>QQQ</td>
<td>Monta Carlo simulation</td>
<td>Monthly</td>
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\]

\[
\sigma_{\text{monthly}} = 2.64\% \cdot \sqrt{20} = 11.80 \Rightarrow \text{VaR} = -1.65 \cdot \sigma_{\text{monthly}} = -1.65 \cdot 11.80
\]
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<td>2</td>
<td>QQQ</td>
<td>Variance-Covariance</td>
<td>2.64%</td>
<td>Daily</td>
</tr>
<tr>
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Example

<table>
<thead>
<tr>
<th>Investment</th>
<th>VAR Method</th>
<th>Standard Deviation</th>
<th>Time Period</th>
<th>Calculated VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 QQQ</td>
<td>Historical</td>
<td>N/A *</td>
<td>Daily</td>
<td>~ -4.6%</td>
</tr>
<tr>
<td>2 QQQ</td>
<td>Variance-Covariance</td>
<td>2.64%</td>
<td>Daily</td>
<td>- 6.16%</td>
</tr>
<tr>
<td>3 QQQ</td>
<td>Monte Carlo simulation</td>
<td>N/A *</td>
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Optimization problems arise naturally within the context of risk management. Think of a risk manager who wants to optimize a performance measure (e.g. expected return) while making sure that certain risk measures do not exceed a threshold value.

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\max_{x} \mu^T x \quad \text{s.t.} \quad \text{VaR}_\alpha \leq U, \quad j = 1, \ldots, J, \quad x \in X
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2. The historical simulation improves on the accuracy of the VAR calculation, but requires more computational data; it also assumes that "past is prologue".

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Asset/Liability management
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Stochastic program on maximizing the expected wealth at the end of the planning horizon

Let $L_t$ be the liability of the company in year $t$ for $t = 1, \ldots, T$. Let $R_{it}$ denote the return on asset $i$ in year $t$.

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Definition of Pension Funds

A fund established by an employer to facilitate and organize the investment of employees' retirement funds contributed by the employer and employees. The pension fund is a common asset pool meant to generate stable growth over the long term, and provide pensions for employees when they reach the end of their working years and commence retirement. Pension funds and insurance companies are the most typical areas for the integrated management of assets and insurance through ALM.

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Random variables appearing in the stochastic linear program:
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- $F_t =$ deposit inflow from $t - 1$ to $t$,
- $P_t =$ principal payout from $t - 1$ to $t$,
- $I_t =$ interest payout from $t - 1$ to $t$,
- $g_t =$ rate at which interest is credited to policies from $t - 1$ to $t$,
- $L_t =$ liability valuation at $t$.

The objective of the model is to allocate funds among available assets to maximize expected wealth at the end of the planning horizon $T$ less expected penalized shortfalls accumulated through the planning horizon.
• Stages are indexed by $t = 0, 1, \ldots, T$.

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- $RP_{it}$ = price return of asset $i$ from $t - 1$ to $t$,
- $RI_{it}$ = interest income of asset $i$ from $t - 1$ to $t$,
- $F_t$ = deposit inflow from $t - 1$ to $t$,
- $P_t$ = principal payout from $t - 1$ to $t$,
- $I_t$ = interest payout from $t - 1$ to $t$,
- $g_t$ = rate at which interest is credited to policies from $t - 1$ to $t$,
- $L_t$ = liability valuation at $t$. 

The objective of the model is to allocate funds among available assets to maximize expected wealth at the end of the planning horizon $T$ less expected penalized shortfalls accumulated through the planning horizon:

$\gamma_t = $ Piecewise linear convex cost function.
Stages are indexed by \( t = 0, 1, \ldots, T \).

Decision variables of the stochastic program:
- \( x_{it} \) = market value in asset \( i \) at \( t \),
- \( w_t \) = interest income shortfall at \( t \geq 1 \),
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The objective of the model is to allocate funds among available assets to maximize expected wealth at the end of the planning horizon \( T \) less expected penalized shortfalls accumulated through the planning horizon:
- \( c_t \) = Piecewise linear convex cost function.
Liability balances and cash flows are computed so as to satisfy the liability accumulation relations:
\[ L_t = (1 + g_t) L_{t-1} + F_t - P_t - I_t \text{ for } t \geq 1. \]

**Method of solving**
This stochastic linear program is converted into a large linear program using a finite number of scenarios to deal with the random elements in the data. Creation of scenario inputs is made in stages using a tree. The tree structure can be described by the number of branches at each stage.

Solving this model yielded extra income estimated to be about US $80 million per year for the company.

**Solution**

\[
\begin{align*}
\max \quad & E \left[ \sum_i x_{iT} - \sum_{t=1}^T c_t(w_t) \right] \\
\text{subject to} \quad & \sum_i x_{it} - \sum_i (1 + RP_{it} + RI_{it}) x_{i,t-1} \\
\text{asset accumulation:} \quad & = F_t - P_t - L_t \\
\text{interest income shortfall:} \quad & \sum_i RI_{it} x_{i,t-1} + w_t - v_t = g_t L_{t-1} \\
& \text{for } t = 1, \ldots, T, \\
& x_{it} \geq 0, \quad w_t \geq 0, \quad v_t \geq 0.
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& \quad = F_t - P_t - l_t \\
& \quad \text{for } t = 1, \ldots, T, \\
& \quad \sum_i R I_{it} x_{i,t-1} + w_t - v_t = g_t L_{t-1} \\
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- Liability balances and cash flows are computed so as to satisfy the liability accumulation relations: \(L_t = (1 + g_t) L_{t-1} + F_t - P_t - l_t\) for \(t \geq 1\).

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Synthetic Options
Brief Introduction
Value at Risk (VaR)
Asset/Liability management
**Synthetic Options**
References
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- While one may be able to construct a diversified portfolio well suited for a corporate investor, there may be no option market available on this portfolio.
- One solution could be using the synthetic options: A financial instrument that is created artificially by simulating another instrument with the combined features of a collection of other assets.
- Example: you can create a synthetic stock by purchasing a call option and simultaneously selling a put option on the same stock.
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- \( A_{it} \) = amount of asset i bought at time t,
- \( D_{it} \) = amount of asset i sold at time t,
- \( \alpha_t \) = amount allocated to riskless asset at time t.
The initial portfolio is:

\[ \alpha_0 + x_1 + \ldots + x_n = W_0 \]

The portfolio at time \( t \) is

\[ x_{i,t} = R_{i,t} x_{i,t} - x_{i,t-1} + A_{i,t} - D_{i,t} \]

for \( t = 1, \ldots, T \)

\[ \alpha_t = R_\alpha_t - \alpha_{t-1} - \sum_{i=1}^n \left( 1 + \Theta_{i,t} \right) A_{i,t} + \sum_{i=1}^n \left( 1 - \Theta_{i,t} \right) D_{i,t} \]

for \( t = 1, \ldots, T \)

How much is the value of the portfolio at the end of the planning horizon? It is the summation of the value of the riskless and risky assets minus the transaction costs at the end of the time period:

\[ \nu = R_\alpha_T - \nu_{t-1} + \sum_{i=1}^n \left( 1 - \Theta_{i,T} \right) R_{i,T} x_{i,T} - 1 \]

To construct the desired synthetic option, we split \( \nu \) into the riskless value of portfolio \( Z \) and a surplus \( z \). Using the scenario approach to the stochastic program, \( Z \) is the worst-case payoff over all the scenarios and the surplus \( z \) is a random variable that depends on the scenario.

\[ \nu = Z + z \]

Now the objective function of the stochastic program is

\[ \max E(z) + \mu Z, \] where \( \mu \geq 1 \) is the risk aversion of the investor. The risk aversion \( \mu \) is given data.

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When \( \mu = 1 \), the objective is to maximize the expected return.
Example

- Consider an investor with initial wealth $W_0 = 1$ who wants to construct a portfolio comprising one risky asset and one riskless asset using the ”synthetic option” model described above.

- We write the model for a two-period planning horizon, i.e., $T = 2$. The return on the riskless asset is $R$ per period.

- For the risky asset, the return is $R_1^+$ with probability 0.5 and $R_1^-$ with the same probability at time $t = 1$.

- Similarly, the return of the risky asset is $R_2^+$ with probability 0.5 and $R_2^-$ with the same probability at time $t = 2$.

- The transaction cost for purchases and sales of the risky asset is $\Theta$. 
Solution

- There are four scenarios in this example, each occurring with probability 0.25, which we can represent by a binary tree.
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- $x_i$ denote the amount of risky asset at node $i$
- $\alpha_i$ denote the amount of riskless asset at node $i$
- $Z$ is the riskless value of the portfolio
Solution

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- $x_i$ denote the amount of risky asset at node $i$
- $\alpha_i$ denote the amount of riskless asset at node $i$
- $Z$ is the riskless value of the portfolio
- $z_i$ is the surplus at node $i$.
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- \( \alpha_i \) denote the amount of riskless asset at node \( i \)
- \( Z \) is the riskless value of the portfolio
- \( z_i \) is the surplus at node \( i \).

The linear program for this problem is:

\[
\begin{align*}
\text{max} & \quad 0.25z_3 + 0.25z_4 + 0.25z_5 + 0.25z_6 + \mu Z \\
\text{subject to} & \quad \alpha_0 + x_0 = 1 \\
& \quad \alpha_i = R\alpha_i - (1 + \theta)A_i + (1 - \theta)D_i \\
& \quad x_1 = R^+x_0 + A_1 - D_1 \\
& \quad x_2 = R^-x_0 + A_2 - D_2 \\
& \quad z_3 + Z = R\alpha_1 + (1 - \theta)R^+x_1 \\
& \quad z_4 + Z = R\alpha_1 + (1 - \theta)R^-x_1 \\
& \quad z_5 + Z = R\alpha_2 + (1 - \theta)R^+x_2 \\
& \quad z_6 + Z = R\alpha_2 + (1 - \theta)R^-x_2 \\
\text{nonnegativity:} & \quad \alpha_i, x_i, z_i, A_i, D_i \geq 0.
\end{align*}
\]
References
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