Outage Probability of a Multi-User Cooperation Protocol in an Asynchronous CDMA Cellular Uplink

Kanchan G. Vardhe, Daryl Reynolds, and Matthew C. Valenti

Lane Dept. of Comp. Sci and Elec. Eng.
West Virginia University
Morgantown, WV 26505
kvardhe@mix.wvu.edu, {Daryl.Reynolds, Matthew.Valenti }@mail.wvu.edu

Presentation Outline

- Cooperative communications overview
- Previous work
- The proposed multi-user cooperation protocol in a CDMA cellular uplink
- Results
  - The performance comparison between diversity combining and code combining for the proposed protocol
  - Performance under modulation constraints
- Conclusions

Cooperative Communications Overview

- Multi-antenna systems with m transmit and n receive antennas have

$$C(\mathsf{SNR}) = \min\{m,n\}\log(1+\mathsf{SNR}) \quad (1)$$

$$\text{BER} \propto \frac{1}{(\mathsf{SNR})^{mn}} \quad (2)$$

- Difficult to implement multiple antennas on a single mobile unit (e.g. cellular uplink) due to size constraints.
- Spatial diversity can then be achieved through cooperation among single antenna mobile units - cooperative diversity.

Previous Work

- Full duplex, Two-user cooperation for a CDMA cellular uplink. [Sendonaris *et al.* TCOM 2003]
- Half duplex, Multi-user space-time coded cooperative diversity protocol [Laneman *et al.* IT 2003]
- Coded cooperation [Hunter *et al.* IT, TWC 2003]
- Limitations: Allocation of orthogonal channels to users, synchronous communication assumed, multiple-access interference ignored, information-theoretic analysis of user cooperation assumes Gaussian distribution of input symbols
- Asynchronous cooperative diversity [Wei *et al.* IT, TWC 2006]

The Proposed Multi-User Cooperation Scheme

- A multi-source multi-relay cooperation scheme for CDMA cellular uplink [K. Vardhe and D. Reynolds, Milcom 2006, Signal Processing 2007]
  - Space-time relaying takes into account multiple sources, asynchronism, multiple-access and inter-symbol interference
  - Relays (other users) receive messages from multiple sources and upon successful decoding can forward the superposition of multiple re-encoded and re-spread messages
  - Decorrelating MUD is employed at the base station and at the user side
  - Performance metric : Information outage probability

Protocol Design

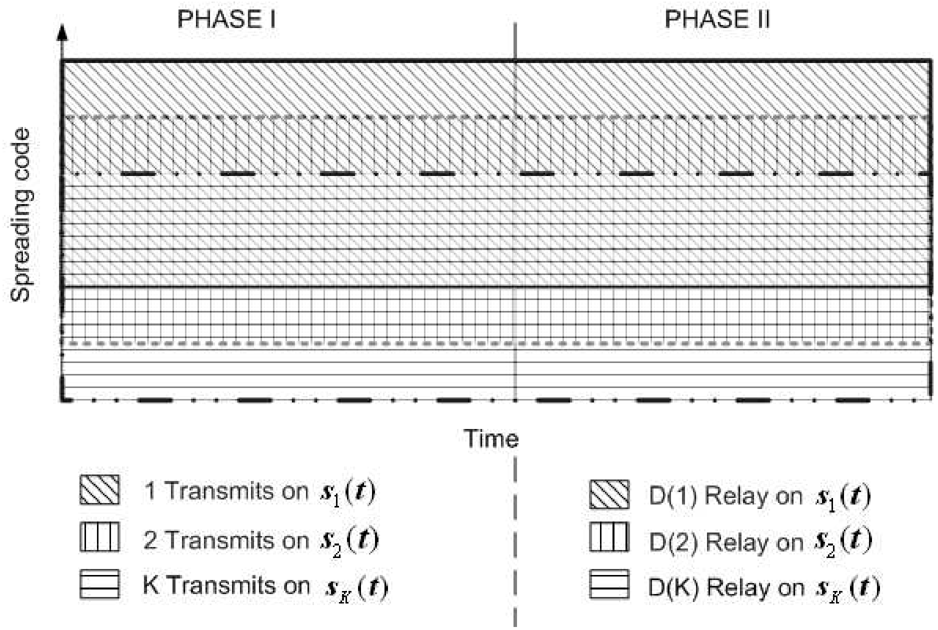


Figure 1: Medium access control for the proposed cooperation scheme.

- Non-orthogonal spreading codes : Inter-user non-orthogonality
- Phase II - Asynchronous relayed transmissions
- Non-orthogonality across the subchannels and also within a subchannel.

Received Signal Model

- The proposed sharing scheme operates in an asynchronous CDMA cellular uplink in the presence of MAI and ISI.
- The received signal at the base-station with total K users and $|\mathcal{D}(k)|$ cooperating users is given by

$$r(t) = \sum_{k=1}^{K} \sum_{l \in \mathcal{D}(k)} \sum_{i=0}^{B-1} x_{l,k}[i] \alpha_l s_k(t - iT_s - \tau_l) + n(t). \qquad (3)$$

- Matched filtering w.r.t received waveform over the channel and stacking all matched filtered outputs: $\boldsymbol{r} = \underbrace{\boldsymbol{A}\boldsymbol{R}\boldsymbol{A}^{\mathsf{H}}}_{\tilde{\boldsymbol{H}}} \boldsymbol{x} + \boldsymbol{n} \qquad \boldsymbol{n} \sim \mathcal{N}_c(\boldsymbol{0}, \sigma^2 \tilde{\boldsymbol{H}})$
  - $\boldsymbol{A}$ is a diagonal matrix and is a function of channel gains only, $\boldsymbol{R}$ is a function of cross-correlation between delayed signature waveforms.
- Decorrelating MUD: $\boldsymbol{y} = (\boldsymbol{A}\boldsymbol{R})^{-1}\boldsymbol{r} + \boldsymbol{v} \qquad \boldsymbol{v} \sim \mathcal{N}_c(\boldsymbol{0}, \sigma^2 \boldsymbol{R}^{-1})$
- Scalar channel model: $y_i \triangleq [\boldsymbol{y}]_i = \alpha_i x_i + v_i, \qquad v_i \sim \mathcal{N}_c(0, \sigma^2 [\boldsymbol{R}]_{i,i}^{-1})$

Performance Under Diversity Combining

- All the relays in the decoding set of a particular user transmit on the same subchannel using a delay diversity. This leads to diversity combining of the relayed information at the base station
- Conditioned on the decoding set $\mathcal{D}(k)$, the mutual information between k-th user and destination under diversity combining and fully loaded CDMA is

$$I_{\text{f-CDMA}} = \frac{1}{2}\log\left(1 + \frac{2\mathsf{SNR}}{K}\frac{|\alpha_{k,d}|^2}{[\boldsymbol{R}^{-1}]_{1,1}}\right) + \frac{1}{2}\log\left(1 + \frac{2\mathsf{SNR}}{K}\sum_{r\in\mathcal{D}(k)}\frac{|\alpha_{r,d}|^2}{[\boldsymbol{R}^{-1}]_{r,r}}\right). \tag{4}$$

The mutual information between the k-th user and the potential relay r:

$$I_{k,r} = \frac{1}{2}\log\left(1 + \frac{2\mathsf{SNR}}{K}\frac{|\alpha_{k,r}|^2}{[\boldsymbol{R}^{-1}]_{r,r}}\right). \tag{5}$$

$$\Pr[r \in \mathcal{D}(k)] = \Pr[I_{k,r} > R_{\text{CDMA}}] \tag{6}$$

- The outage probability of the channel between user k and base station

$$\Pr[I < R] = \sum_{\mathcal{D}(k)} \Pr[\mathcal{D}(k)]\Pr[I < R|\mathcal{D}(k)]. \tag{7}$$

Performance Under Code Combining

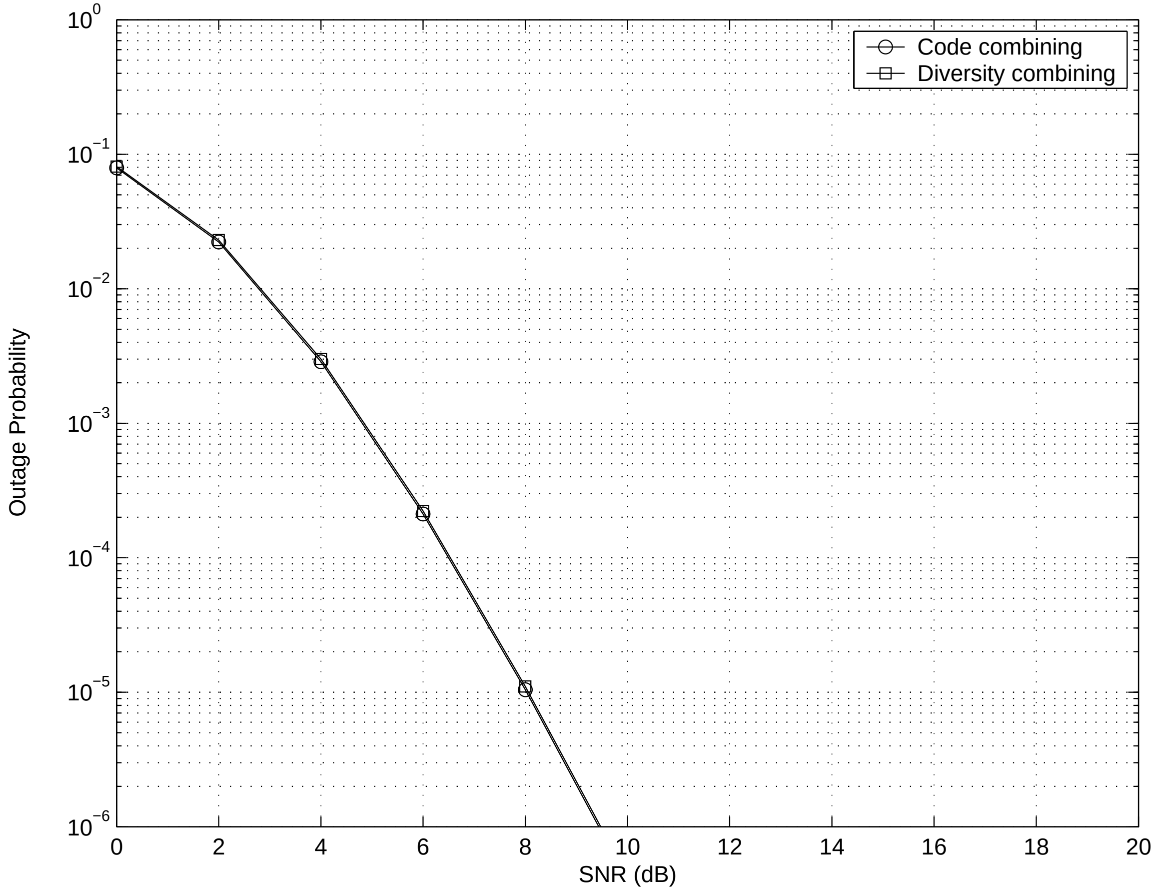
- The existing cooperative diversity schemes employing code combining require the existence of orthogonal (parallel) channels
- Decorrelating MUD in the proposed scheme forms interference free parallel channels with increased background noise
- Relays employ different Gaussian codebooks and transmit relayed information toward destination. This results in a code combining of the relayed transmissions at the base station.
- The mutual information under code combining and fully loaded CDMA system configuration, conditioned on a decoding set can be given by

$$I_{\text{f-CDMA}} = \frac{1}{2}\log\left(1 + \frac{2\mathsf{SNR}}{K}\frac{|\alpha_{k,d}|^2}{[\boldsymbol{R}^{-1}]_{1,1}}\right) + \sum_{r\in\mathcal{D}(k)}\frac{1}{2}\log\left(1 + \frac{2\mathsf{SNR}}{K}\frac{|\alpha_{r,d}|^2}{[\boldsymbol{R}^{-1}]_{r,r}}\right). \quad (8)$$

System Parameters

- Use of random spreading codes
- Delays are uniformly distributed within one symbol interval
- Processing gain N =8
- K = total number of users =8
- Rayleigh fading channel

Conditional outage probability performance comparison of diversity combining and code combining schemes



The outage probability is conditioned on $\boldsymbol{R}$. The threshold spectral efficiency is $R = 1$ bit/sec/Hz. Code combining is 0.01 dB better than the diversity combining and so the plots are almost indistinguishable.

Channel Capacity Under Modulation Constraints : Point-to-Point Case

- Suppose $\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{n}$ where $\boldsymbol{n} \sim \mathcal{N}_c(0, \sigma^2 \boldsymbol{I})$, $\boldsymbol{x}$ being a modulated symbol alphabet
- The modulation-constrained capacity is [Caire *et al.* IT 1998]

$$C = \left(m - \mathsf{E}_{x,y} \left[\log \frac{\sum_{\boldsymbol{z} \in \chi} p(\boldsymbol{y}|\boldsymbol{z})}{p(\boldsymbol{y}|\boldsymbol{x})} \right] \right) \tag{9}$$

where $m = \log_2 M$, M being the signal constellation size, χ denotes the signal set, and $p(\boldsymbol{y}|\boldsymbol{x})$ is the transition probability density function between input $\boldsymbol{x}$ and the output $\boldsymbol{y}$

Performance Under Modulation Constraints

- We model the received signal at the destination during two time-phases as follows.
- In the first phase, user k transmits toward the base station. The received signal at the base station during first phase after decorrelating multiuser detection can be written as

$$y_1 = \frac{\alpha_{k,d}}{\sqrt{[\boldsymbol{R}]^{-1}_{1,1}}} x + n \tag{10}$$

where $n \sim \mathcal{N}_c(0, N_0)$, x is a modulated symbol drawn from the uniform probability distribution with $\mathsf{E}\{x\}^2 = 2P/K$.

- The mutual information under modulation constraints between k-th user and the base station during phase I conditioned on a decoding set $\mathcal{D}(k)$ is

$$I_1 = \frac{1}{2}\left(m - \mathsf{E}_{x,y_1}\left[\log \frac{\sum_{z \in \chi} p(y_1|z)}{p(y_1|x)}\right]\right). \tag{11}$$

Performance Under Modulation Constraints Cont'd

- The received signal at the base station during second phase under modulation constraints due to retransmissions from K' relays can be modeled as

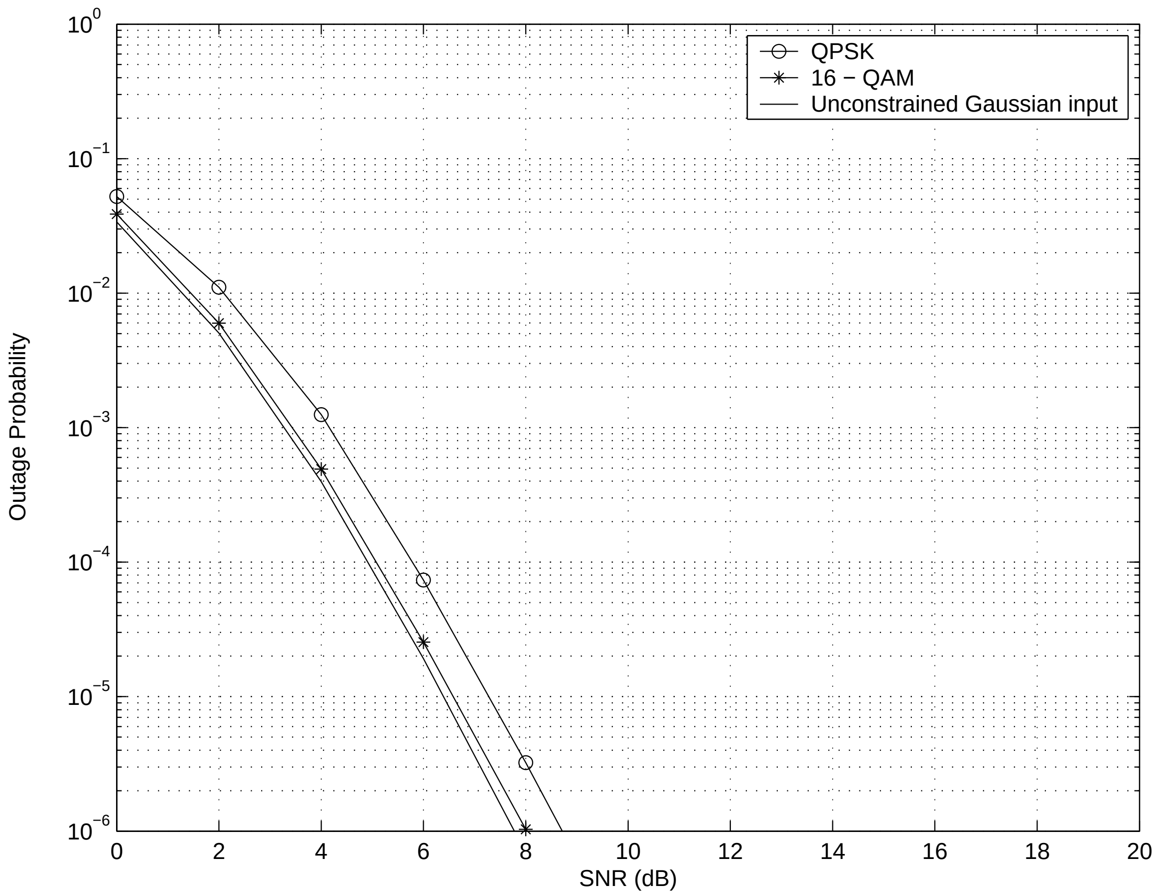
$$\boldsymbol{y} = \begin{bmatrix} \alpha_{2,d}/\sqrt{[\boldsymbol{R}]^{-1}_{2,2}} \\ \vdots \\ \alpha_{K',d}/\sqrt{[\boldsymbol{R}]^{-1}_{K',K'}} \end{bmatrix} x + \boldsymbol{n}. \tag{12}$$

- The mutual information under uniform input probability conditioned on a decoding set $\mathcal{D}(k)$ is

$$I_2 = \frac{1}{2}\left(m - \mathsf{E}_{x,\boldsymbol{y}}\left[\log \frac{\sum_{z\in\chi} p(\boldsymbol{y}|z)}{p(\boldsymbol{y}|x)}\right]\right). \tag{13}$$

- The overall mutual information conditioned on a decoding set between k-th user and the base station is $I_m = I_1 + I_2$.

Conditional outage probability performance comparison under the constraint of uniform input probability and unconstrained Gaussian input.



The outage probability is conditioned on $\boldsymbol{R}$. The threshold spectral efficiency is $R = 0.8$ bits/sec/Hz.

Conclusions

- In multi-user cooperation and under the system parameters considered in this work, diversity combining yields almost the same outage probability performance as code combining
  - Because not all users in the system act as relays all the time.
  - Hence the probabilities of the decoding sets turn out to be a prominent factor in deciding which combining scheme to use at the base station.
- It is observed that increasing the signal constellation size while keeping the target rate constant, we can approach the outage probability performance of a cooperation scheme that uses Gaussian inputs.

The performance loss incurred (with respect to their counterparts) by making the system design much simpler and more practical, for e.g., using diversity combining (instead of code combining), and a 16-symbol alphabet, is relatively small.

Thank you!