A Hybrid-ARQ Protocol Using Noncoherent Orthogonal Modulation

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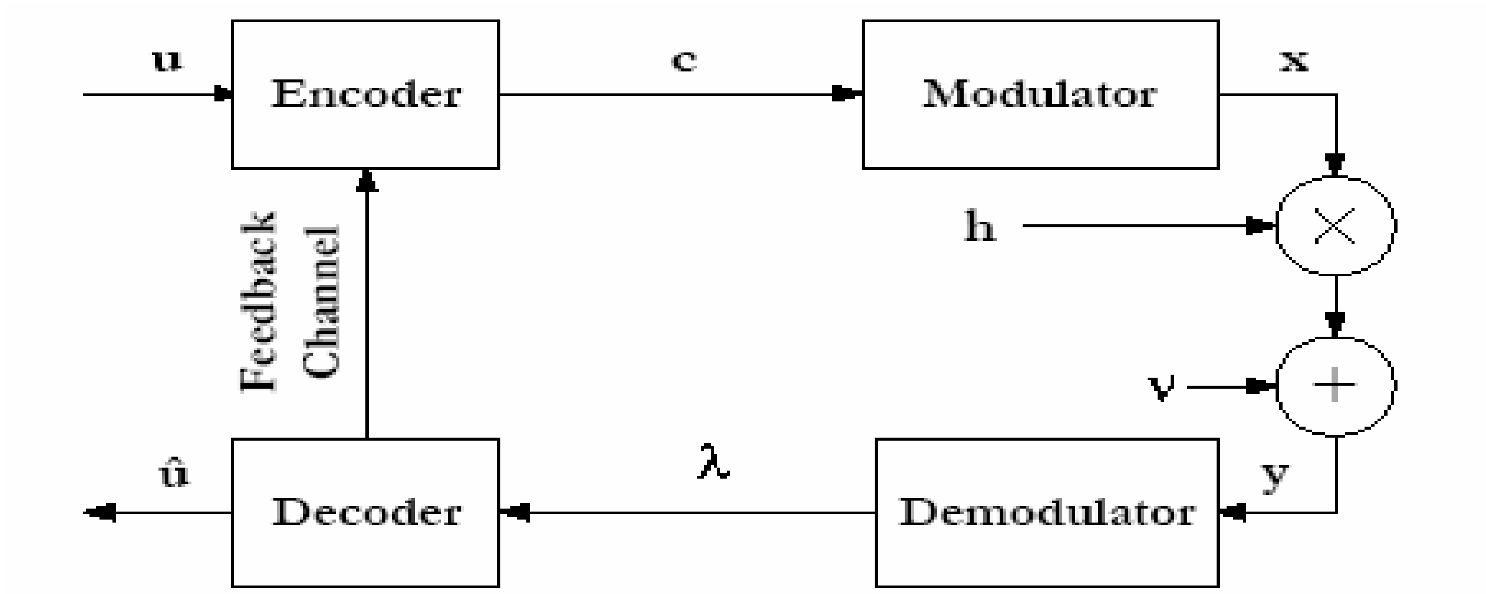
Overview

- **Noncoherent orthogonal modulation**
  - Energy detection based scheme
  - Pseudo-orthogonal modulation used in Bluetooth, Zigbee etc.
- **Hard Decision Decoding with Reed Solomon codes**
  - Channel Coding and modulation can be made independent
  - Appropriate bound is the Discrete Memoryless Capacity (DMC)
- **Hybrid-ARQ technique**
  - Combines FEC with ARQ
  - Codeword broken into B blocks; incremental redundancy and code-combining
- **Relaying**
  - Exploits spatial diversity from redundant nodes
  - Hybrid-ARQ generalized; destination can receive blocks from multiple nodes

System Model



Noncoherent orthogonal modulation

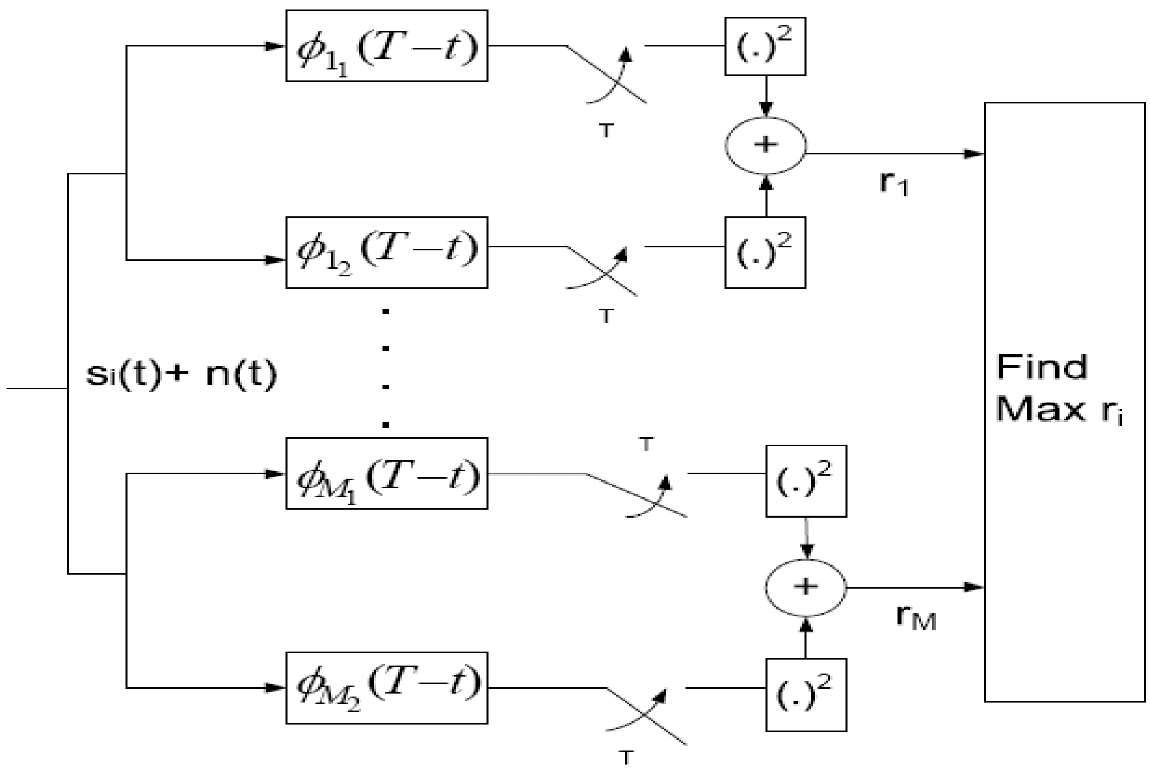
- **Phase information not required**

- **M-ary modulation**
  – M basis functions; each function orthogonal to others

- **Analysis in vector space**
  – Block of size of N symbols represented by M x N matrix
  – Symbol S_n represented by n^{th} column vector; all elements zero except row m (set to one) for m^{th} basis function
  – Constant fading coefficient **h** over a block; complex Gaussian noise added to each matrix element **y**[m,n]
  – **y**[m,n] = **h**.**x**[m,n] + **v**[m,n]

Demodulation for Noncoherent FSK

- $\phi_{i_1}(T-t)$ is match-filtered to $\cos(2\pi f_i t)$ (inphase), $\phi_{i_2}(T-t)$ corresponds to $\sin(2\pi f_i t)$ (quadrature) while f_i is the i^th^ frequency tone



Discrete Memoryless Channel (DMC) Capacity

- Mutual information

$$I(X,Y) = \sum_y \sum_x p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)} \qquad C = \max_{p(x)} I(X,Y)$$

- M basis functions have zero correlation. Apriori probability of any symbol is 1/M so an error implies possibility of *M-1* equi-probable symbols

- Symbol error probability

$$P_s = \sum_{k=1}^{M-1} \binom{M-1}{k} \log_2 \frac{(-1)^{k+1}}{k+1} \exp\left[-\frac{k}{k+1}\frac{Es}{No}\right]$$

DMC Capacity

- Using P_s the $M \times M$ symbol transition matrix T_s is generated

$$T_s = \begin{bmatrix} 1-P_s & \frac{P_s}{M-1} & . & \frac{P_s}{M-1} \\ \frac{P_s}{M-1} & 1-P_s & . & . \\ . & . & . & . \\ \frac{P_s}{M-1} & . & . & 1-P_s \end{bmatrix}$$

- $T_s(i,j)$ = **p(j|i)** conditional probability that ith symbol sent and jth received
- Using T_s Mutual information (i.e. DMC capacity) is computed as

$$C = \log_2 M + P_s \log_2 \frac{P_s}{M-1} + (1-P_s)\log_2(1-P_s)$$

Block Fading

- **Slow fading environment (Erdogic capacity does not exist)**
  - Encoded codeword broken into B blocks each with rate R_B
  - SNR exponentially distributed due to Rayleigh fading
  - Code-combined capacity is given by:

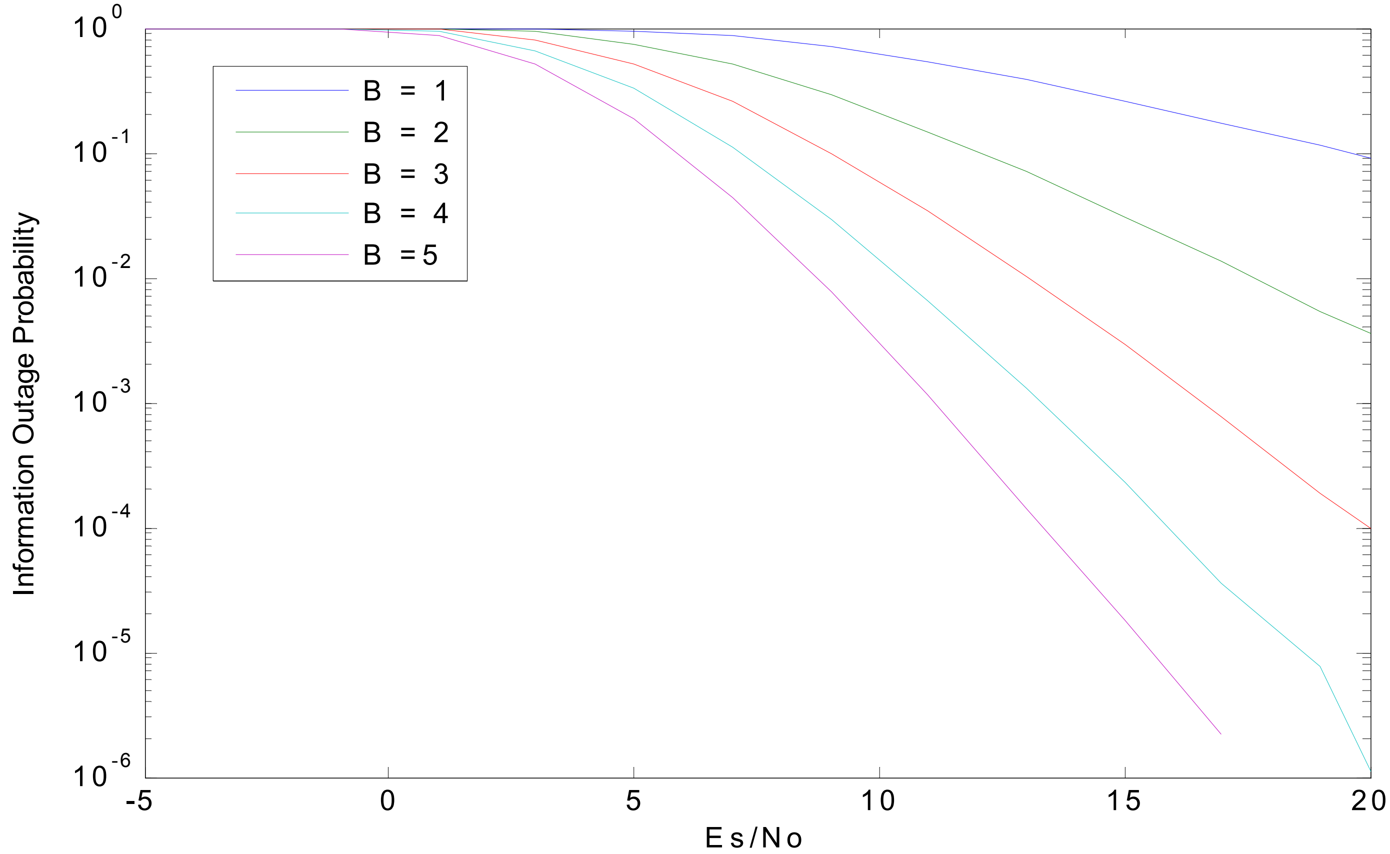
$$C_B = \log_2 M + \overline{P_s} \log_2 \frac{\overline{P_s}}{M-1} + (1-\overline{P_s}) \log_2 (1-\overline{P_s})$$

where $$\overline{P}_s = \frac{P_{s_1} + P_{s_2} + ..P_{s_B}}{B}$$

- **Information Outage Probability $P_0(B)$**
  - $P_0(B) = P[C_B<R_B]$
  - For example, for uncoded 16-ary NFSK, $R_B = \log_2 16 = 4$

Code-combining in Block Fading

- Monte-Carlo simulations used. In a (51, 45) block code, rate $R_B = 4 \times (45/51)$



Hybrid-ARQ

- Combines **Forward Error Correction** (FEC) with **Automatic Repeat Request** (ARQ).
- A message of length *k* is encoded to a *n* length codeword. Codeword is broken into B_{max} blocks.
- Each of the blocks *incrementally* sent over the channel until the destination able to decode the message or all have been sent.
- Types of hybrid-ARQ

  Type I: Individual blocks decoded but not combined

  Type II: All accumulated blocks used for decoding but cannot be decoded on their own.

  Type III: Individual blocks also decode-able

Reed Solomon as Channel Code

- **RS (n,k)**
  - RS codes are nonbinary.
  - can correct $t = (n-k)/2$ symbol errors
  - n is number of encoded symbols
  - K is number of message symbols
  - For 256-ary RS symbols each symbol corresponds to one byte of data.
  - k bytes are input to the encoder, where $k \leq 51$.
  - Output of encoder is a 255 byte long mother code word
  - Our hybrid-ARQ protocol breaks it into 5 blocks, each of length 51 bytes.
  - In 16-ary modulation each RS symbol matched to two modulation symbols

Symbol Errors Decoding

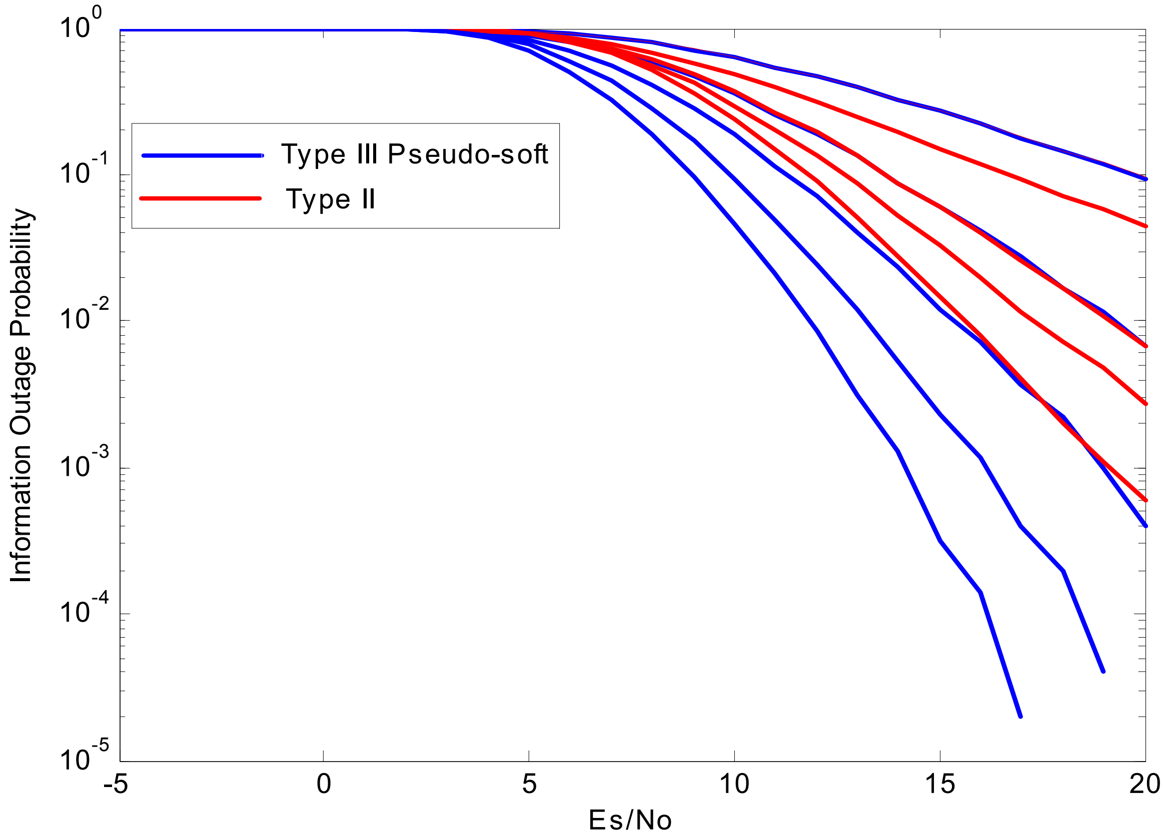
- Consider k=45
  - The number of correctable errors after each block transmission is:

block	n	t
1	51	3
2	102	28
3	153	54
4	204	79
5	255	105

  - Performance can be poor if one block has many errors
  - Iteratively decode error blocks by trying different combinations; we can call this selective-combining technique ***Pseudo-soft*** decoding
- For example by B = 4, block errors are 14, 51, 21, 17. Since total Errors are 104 the codeword is uncorrectable
- Pseudo-soft approach decodes 53 errors for blocks 1, 3 and 4

Gains from Selective Code-Combining

- Results for a (255, 45) RS mother code using noncoherent orthogonal modulation (M=16)



Throughput Derivation

- Pmf of B, the number of hybrid-ARQ transmissions until successful decoding is denoted by

$$p_B[b] = \begin{cases} \xi(1-p_o(b))\prod_{i=1}^{b-1} p_o(i) & \text{for } 1 \le b \le \mathrm{B}_{\max} \end{cases}$$

where

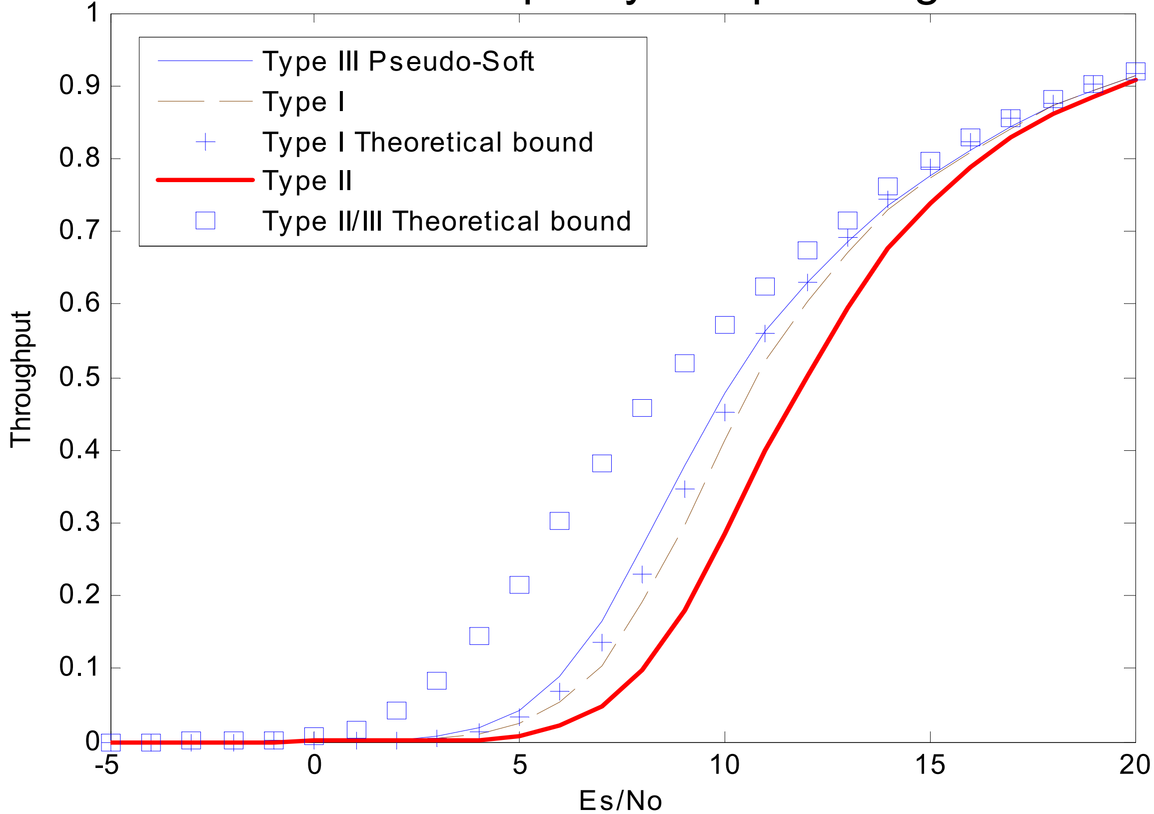
$$\xi = \left[\sum_{i=1}^{B_{\max}} (1 - p_b(b)) \prod_{i=1}^{b-1} p_b(b) \right]$$

- Throughput efficiency is the ratio of correct bits to transmitted bits

$$\eta_{eff} = \frac{1 - p_0(B_{\max})}{E[B]}$$

Throughput Performance Comparison

- Noncoherent 16-ary orthogonal modulation based hybrid-ARQ with a (255, 45) Reed Solomon mother code assumed. The *Theoretical bound* curves represent the results for DMC capacity compared against rate R_B= (45/51)x4



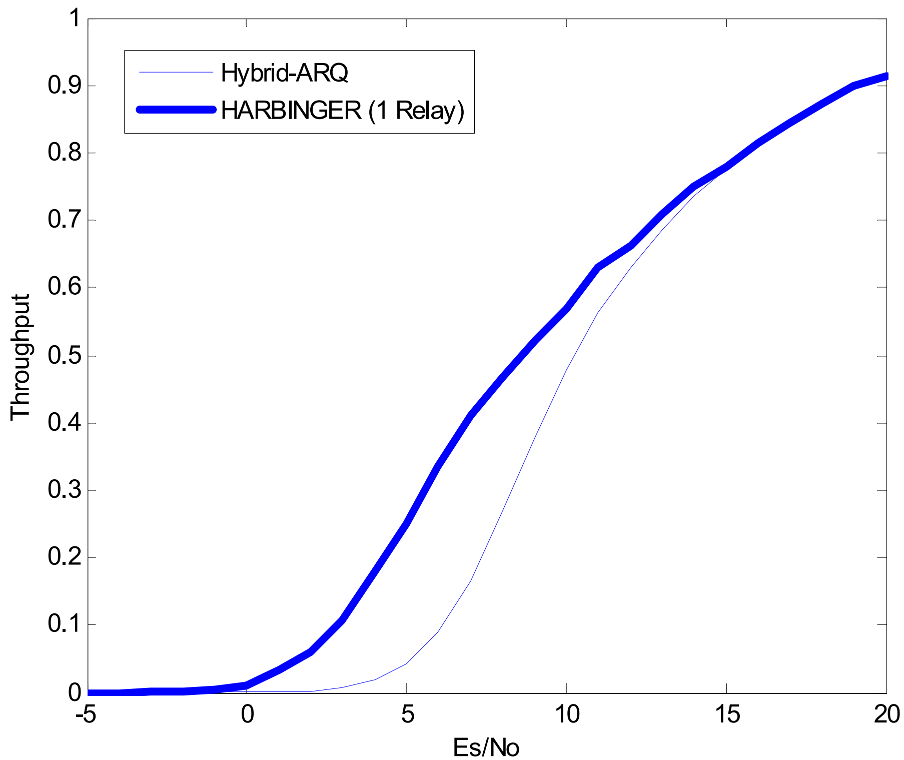
Network Relaying

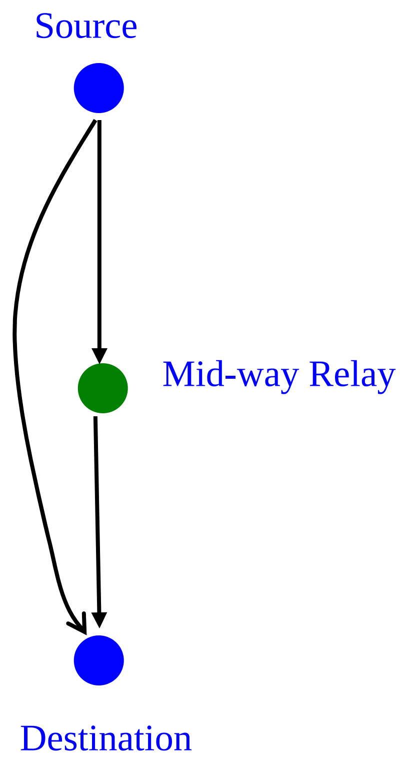
- **Decode and Forward relaying**
  - After each ARQ transmission, some of the intermediate nodes could "overhear" the transmissions.
  - Overhearing nodes that correctly decode could serve as ***relays***. The ARQ retransmission could come from a relay instead of the source.

- **HARBINGER** (Hybrid-ARQ based intra-cluster geographic relaying)
  - can Source broadcasts first packet
  - Relays that can decode are added to the ***decoding set*** *D.* The source is already in *D*
  - Next packet sent by a node in *D* geographically closest to destination
  - Process continues until the destination can decode or all blocks sent.

Relaying Gains

- Performance of HARBINGER with a path loss coefficient of 2. Results with modulation index M=16, RS codeword (255, 45) and Pseudo-soft decoding shown below.





Applications to Zigbee

- **IEEE 802.15.4/ Zigbee standard**
  - Pseudo-orthogonal modulation
  - Offers peer-to-peer topology
  - CSMA-CA mechanism for medium access control
  - Assigns ID to cluster nodes

- **Programming the protocol in Applications layer**
  - Addresses of source, destination and the message encoded in a codeword. Its first block is transported as payload of **data frame** to lower layers
  - Subsequent blocks sent in newer **data frames** in case of decoding failures

Conclusions

- A practical hybrid-ARQ protocol presented:
  - Noncoherent orthogonal modulation
  - Reed Solomon coding
  - Pseudo-soft decoding
- Information theoretic comparisons with DMC capacity more appropriate since off-the-shelf transceivers do not use matched filter outputs.
- Protocol used for relay networks
- Relevance to Zigbee-based networks shown

Thank you

Questions?