The Capacity of Noncoherent Continuous-Phase Frequency Shift Keying

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Outline

1. Motivation
2. Noncoherent CPFSK
3. Capacity Analysis under Bandwidth Constraint
4. Application
5. Conclusion

Bandwidth Efficiency

$$x_i(t) = \frac{1}{\sqrt{T_S}} e^{j\frac{\pi(2k-(M-1))t}{T_S}},\ \ k = 0, 1, \cdots, M-1$$

- Orthogonal FSK
  - Known to achieve Gaussian capacity when the number of tones M goes to infinity
  - Adjacent frequency tones are at least $1/T_S$ apart.
- Nonorthogonal FSK
  - Nonorthogonal FSK saves the bandwidth by using modulation index $h < 1$. Adjacent frequency tones are h/T_S apart.
  - Full response CPM with rectangular pulse shape, also called CPFSK
  - Partial response CPM has more compact bandwidth, but leads to complex signal processing

Bandwidth Efficiency

$$x_i(t) = \frac{1}{\sqrt{T_S}} e^{j\left(\frac{h\pi(2k-(M-1))t}{T_S}+\phi\right)}, \; k = 0, 1, \cdots, M-1$$

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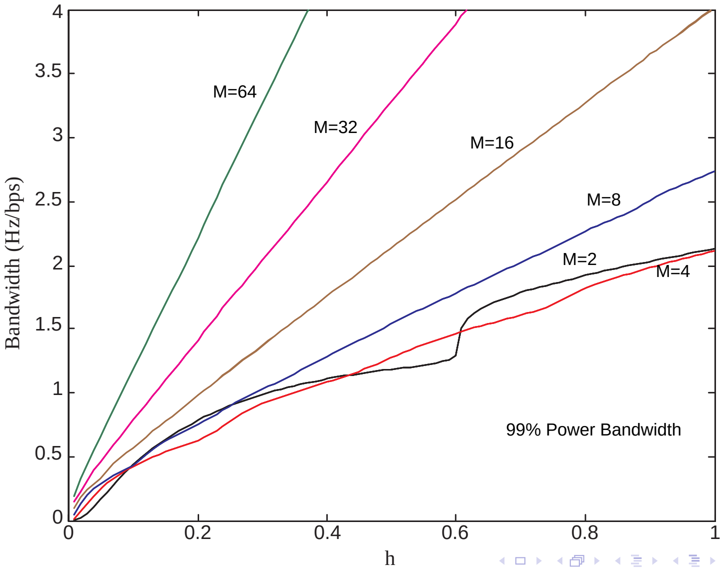
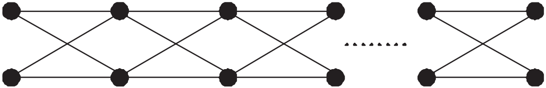
Bandwidth of CPFSK



CPFSK Detection

- Coherent Detection
  - Decoding through trellis
  - h needs to be rational
  - Phase synchronization



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  - Symbol by symbol noncoherent detection

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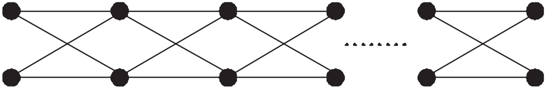

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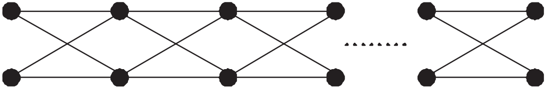

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Noncoherent CPFSK Discrete Time Model

$$\mathbf{y} = ae^{j\theta}\sqrt{\mathcal{E}_s}\mathbf{x} + \mathbf{n}$$

- $\mathbf{n}$ is colored noise, with $E(\mathbf{n}\mathbf{n}^H) = N_0\mathbf{K}$
- $\mathbf{x}$ is chosen from columns of $\mathbf{K} = [\mathbf{k}_0, \mathbf{k}_1, \cdots, \mathbf{k}_{M-1}]$
- $p(\mathbf{y}|\mathbf{x} = \mathbf{k}_\nu) \propto I_0\left(2\frac{a\sqrt{\mathcal{E}_s}}{N_0}|y_\nu|\right)$

A Binary Example: Minimum Shift Keying (MSK)

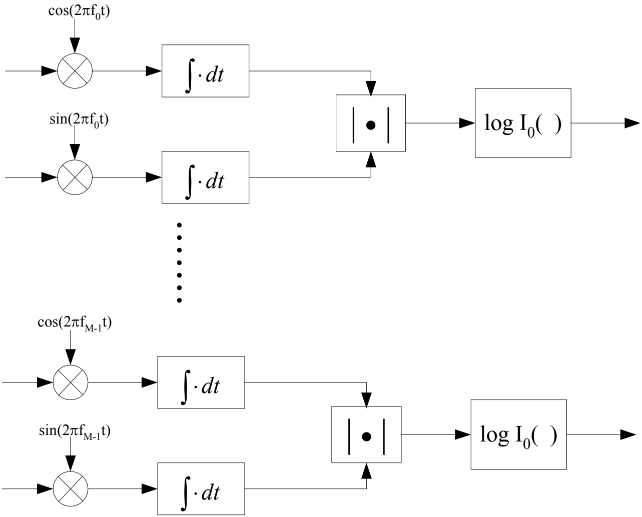
- MSK: $M = 2, h = 1/2$
- The normalized correlation matrix for $\mathbf{n}$ is

$$\mathbf{K} = \begin{bmatrix} 1 & -0.6366j \\ 0.6366j & 1 \end{bmatrix}$$

- The modulator selects the columns of $\mathbf{K}$, depending on the M-ary input d.

$d = 0$	$d = 1$
$\mathbf{x} = \begin{bmatrix} 1 \\ -0.6366j \end{bmatrix}$	$\mathbf{x} = \begin{bmatrix} 0.6366j \\ 1 \end{bmatrix}$

Noncoherent CPFSK Detector Diagram



Capacity Calculation

- Instantaneous capacity

$$i(\mathbf{x};\mathbf{y}) \quad = \quad \log M - \log \frac{\sum_{\mathbf{x}' \in \mathcal{S}} p(\mathbf{y}|\mathbf{x}')}{p(\mathbf{y}|\mathbf{x})}.$$

- Capacity

$$I(\mathbf{x};\mathbf{y}) \quad = \quad \log M - E_{\mathbf{x}} \left[\log \frac{\sum_{\mathbf{x}' \in \mathcal{S}} p(\mathbf{y}|\mathbf{x}')}{p(\mathbf{y}|\mathbf{x})} \right].$$

- Monte Carlo simulation

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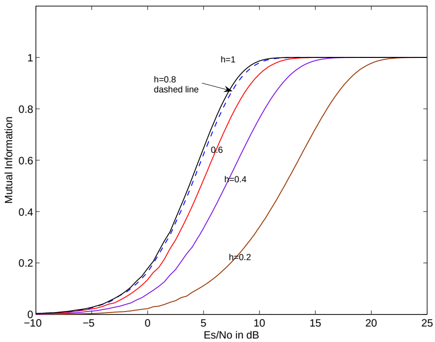
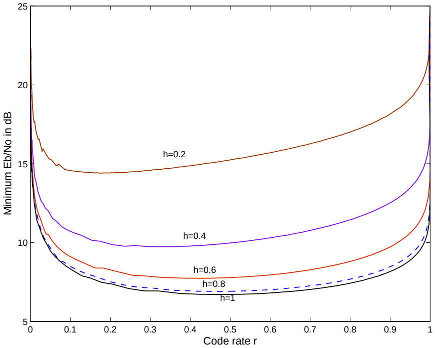
Binary Noncoherent CPFSK Capacities in AWGN

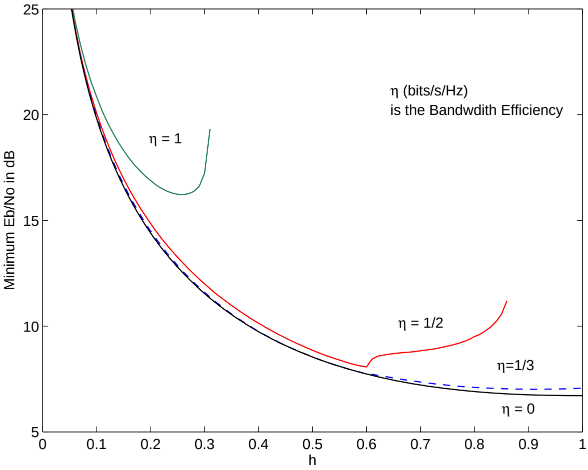


(a) channel capacity versus $\mathcal{E}_S/N_0$

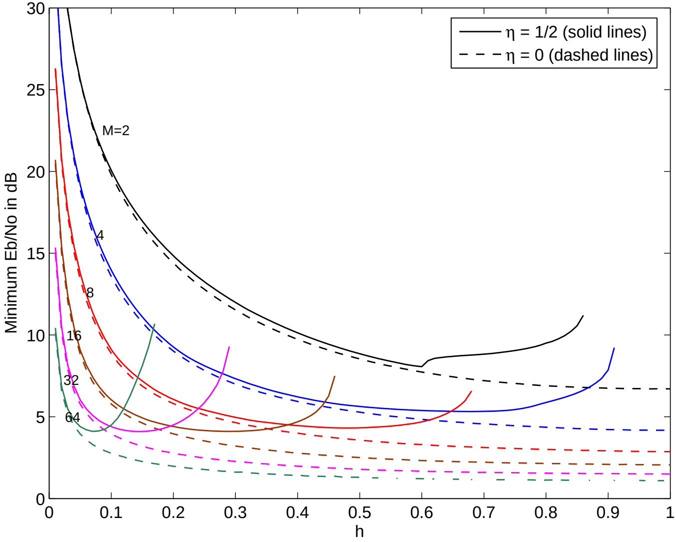


(b) minimum $\mathcal{E}_b/N_0$ versus coding rate

Binary Noncoherent CPFSK Capacities in AWGN under Bandwidth Constraint



Noncoherent CPFSK Capacities in AWGN Channel



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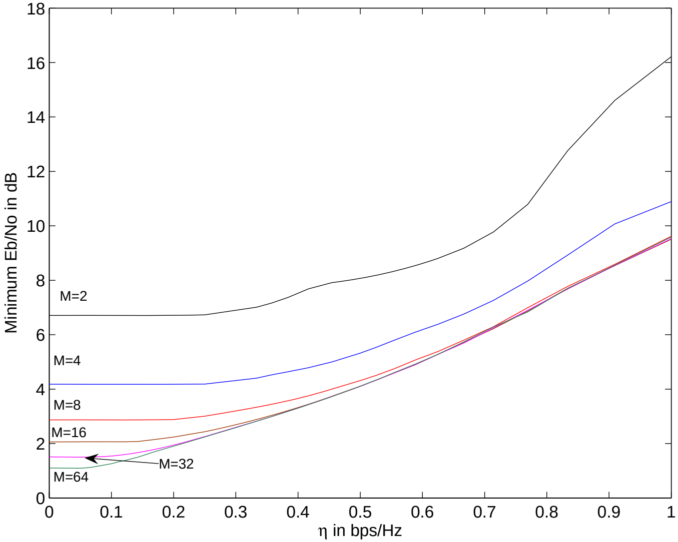
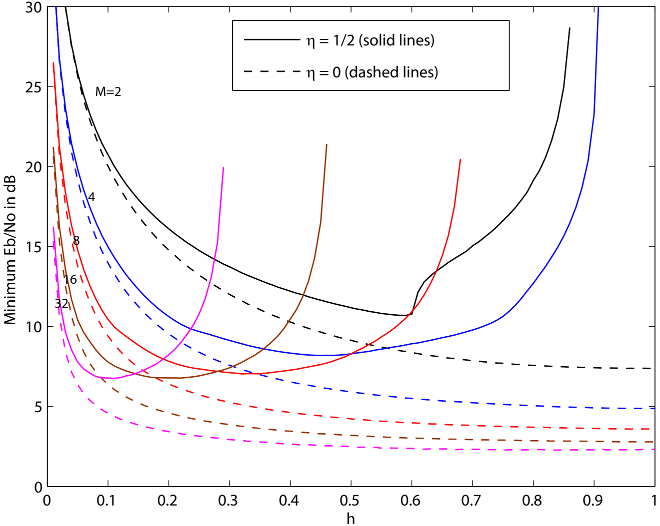


Noncoherent CPFSK Capacities in Rayleigh Channel



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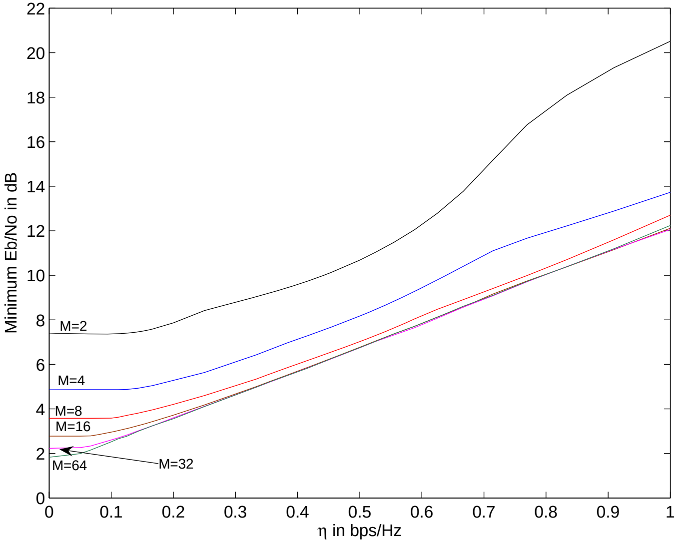
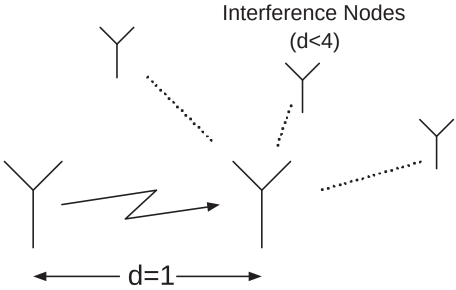


FH Network Scenario



- Multiple access interference
- Tradeoff on the bandwidth of each sub-channel and the possibility of interference
- Equal received average SNR, Rayleigh fading, Interference nodes subject to log normal shadowing $\sigma = 8dB$

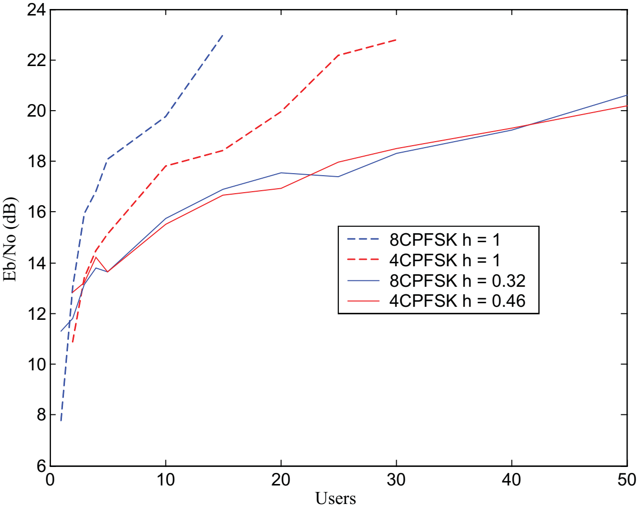
FH Network Scenario

- Total bandwidth $W = 2000/T_u$

h	M	Coding rate	Number of channels
1	4	2048/6144	315
1	8	2048/6144	244
0.46	4	2048/3456	1000
0.32	8	2048/3840	1000

- All users use the same modulation and coding strategy
- Channel estimator using EM algorithm, the same as the one of orthogonal FSK

Noncoherent CPFSK FH Network against MAI



- SNR collected at BER $= 10^{-4}$
- UMTS turbo code
- 32 hops

Conclusion

- This paper outlines a methodology for finding the coded modulation (CM) capacity of CPFSK with noncoherent detection.
- For a specific number of tones M and spectral efficiency η, the minimum required $\mathcal{E}_b/N_0$ can be optimized over the modulation index h and coding rate r.
- For a fixed spectral efficiency, it is better to use a higher order M and smaller h. However, when the spectral efficiency is sufficiently high, the benefit is small by using $M > 8$.

Thank you