The Capacity of Noncoherent Continuous-Phase Frequency Shift Keying

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Outline

1 Motivation
2 Noncoherent CPFSK
3 Capacity Analysis under Bandwidth Constraint
4 Application
5 Conclusion
Motivation

Bandwidth Efficiency

\[ x_i(t) = \frac{1}{\sqrt{T_S}} e^{j \frac{\pi (2k - (M-1)) t}{T_S}}, \quad k = 0, 1, \ldots, M - 1 \]

- **Orthogonal FSK**
  - Known to achieve Gaussian capacity when the number of tones \( M \) goes to infinity
  - Adjacent frequency tones are at least \( 1/T_S \) apart.

- **Nonorthogonal FSK**
  - Nonorthogonal FSK saves the bandwidth by using modulation index \( h < 1 \). Adjacent frequency tones are \( h/T_S \) apart.
  - Full response CPM with rectangular pulse shape, also called CPFSK
  - Partial response CPM has more compact bandwidth, but leads to complex signal processing
Motivation

Bandwidth Efficiency

\[ x_i(t) = \frac{1}{\sqrt{T_S}} e^{j\left(\frac{h\pi(2k-(M-1))t}{T_S} + \phi\right)}, \quad k = 0, 1, \ldots, M - 1 \]

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The Capacity of Noncoherent CPFSK

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Bandwidth of CPFSK

The Capacity of Noncoherent CPFSK

Motivation

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**CPFSK Detection**

- **Coherent Detection**
  - Decoding through trellis
  - $h$ needs to be rational
  - Phase synchronization

- **Noncoherent Detection**
  - Symbol by symbol noncoherent detection
CPFSK Detection

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Noncoherent CPFSK Discrete Time Model

\[ y = ae^{j\theta} \sqrt{E_s} x + n \]

- \( n \) is colored noise, with \( E(nn^H) = N_0 K \)
- \( x \) is chosen from columns of \( K = [k_0, k_1, \cdots, k_{M-1}] \)
- \( p(y|x = k_\nu) \propto I_0 \left( 2 \frac{a \sqrt{E_s}}{N_0} |y_\nu| \right) \)
A Binary Example: Minimum Shift Keying (MSK)

- MSK: $M = 2, h = 1/2$
- The normalized correlation matrix for $n$ is
  \[
  K = \begin{bmatrix}
  1 & -0.6366j \\
  0.6366j & 1
  \end{bmatrix}
  \]
- The modulator selects the columns of $K$, depending on the M-ary input $d$.

<table>
<thead>
<tr>
<th></th>
<th>$d = 0$</th>
<th>$d = 1$</th>
</tr>
</thead>
</table>
| $x$   | $\begin{bmatrix}
  1 \\
  -0.6366j
  \end{bmatrix}$ | $\begin{bmatrix}
  0.6366j \\
  1
  \end{bmatrix}$ |
Noncoherent CPFSK Detector Diagram

\[
\int \cos(2\pi f_0 t) \cdot dt \\
\int \sin(2\pi f_0 t) \cdot dt \\
\int \cos(2\pi f_{M-1} t) \cdot dt \\
\int \sin(2\pi f_{M-1} t) \cdot dt \\
\int \log I_0( ) \\
\int \cos(2\pi f_{M-1} t) \cdot dt \\
\int \sin(2\pi f_{M-1} t) \cdot dt \\
\int \log I_0( )
\]
Capacity Calculation

- **Instantaneous capacity**

\[
i(x; y) = \log M - \log \frac{\sum_{x' \in S} p(y|x')}{p(y|x)}.
\]

- **Capacity**

\[
l(x; y) = \log M - E_x \left[ \log \frac{\sum_{x' \in S} p(y|x')}{p(y|x)} \right].
\]

- **Monte Carlo simulation**
Capacity Calculation

- **Instantaneous capacity**
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- **Monte Carlo simulation**
Binary Noncoherent CPFSK Capacities in AWGN

(a) channel capacity versus $\mathcal{E}_S/N_0$

(b) minimum $\mathcal{E}_b/N_0$ versus coding rate
Binary Noncoherent CPFSK Capacities in AWGN under Bandwidth Constraint

η (bits/s/Hz) is the Bandwidth Efficiency

η = 0
η = 1/3
η = 1/2
η = 1
η (bits/s/Hz) is the Bandwidth Efficiency
Noncoherent CPFSK Capacities in AWGN Channel

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The Capacity of Noncoherent CPFSK Capacities

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Noncoherent CPFSK Capacities in AWGN Channel

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Noncoherent CPFSK Capacities in Rayleigh Channel

M = 2

Minimum Eb/No in dB

η = 1/2 (solid lines)
η = 0 (dashed lines)
Noncoherent CPFSK Capacities in Rayleigh Channel

\[ \text{Minimum } \frac{E_b}{N_0} \text{ in dB} \]

- M=64
- M=32
- M=16
- M=8
- M=4
- M=2

\[ \eta \text{ in bps/Hz} \]

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FH Network Scenario

- Multiple access interference
- Tradeoff on the bandwidth of each sub-channel and the possibility of interference
- Equal received average SNR, Rayleigh fading, Interference nodes subject to log normal shadowing $\sigma = 8 dB$
FH Network Scenario

- Total bandwidth $W = \frac{2000}{T_u}$

<table>
<thead>
<tr>
<th>$h$</th>
<th>$M$</th>
<th>Coding rate</th>
<th>Number of channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2048/6144</td>
<td>315</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>2048/6144</td>
<td>244</td>
</tr>
<tr>
<td>0.46</td>
<td>4</td>
<td>2048/3456</td>
<td>1000</td>
</tr>
<tr>
<td>0.32</td>
<td>8</td>
<td>2048/3840</td>
<td>1000</td>
</tr>
</tbody>
</table>

- All users use the same modulation and coding strategy

- Channel estimator using EM algorithm, the same as the one of orthogonal FSK
Noncoherent CPFSK FH Network against MAI

SNR collected at BER = $10^{-4}$

UMTS turbo code

32 hops
This paper outlines a methodology for finding the coded modulation (CM) capacity of CPFSK with noncoherent detection. For a specific number of tones $M$ and spectral efficiency $\eta$, the minimum required $E_b/N_0$ can be optimized over the modulation index $h$ and coding rate $r$. For a fixed spectral efficiency, it is better to use a higher order $M$ and smaller $h$. However, when the spectral efficiency is sufficiently high, the benefit is small by using $M > 8$. 
Thank you