The Impact of an Antenna Array in a Relay Network

Ramachandran Rajagopalan, Daryl Reynolds, Matthew C. Valenti, and Brian Woerner

Lane Department of Computer Science and Electrical Engineering
West Virginia University
Morgantown, WV
U.S.A.

mvalenti@wvu.edu

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Outline

1. Information Outage Probability
2. The Relay Channel
3. The MIMO Channel
4. The MIMO-Relay Channel
5. Conclusion
Information Outages in Direct SISO Links

- The unconstrained capacity of a link with SNR $\gamma$ is
  \[ C = \log_2(1 + \gamma). \]

- Information outage
  - Since $\gamma$ is random, so is $C$.
  - If $C < R$, then an information outage occurs.

- In quasi-static Rayleigh fading,
  - $\gamma$ is exponential with $E[\gamma] = \Gamma$.
  - The average information outage probability is
    \[ P_0 = P[\log_2(1 + \gamma) < R] = 1 - \exp\left\{ \frac{1 - 2^R}{\Gamma} \right\}. \]
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Outage Probability in Rayleigh Fading

SNR

Outage Probability

R=4
R=2
R=1
R=1/2
R=1/4

Outage Probability

SNR
A Network with One Relay

- A relay can be used to increase diversity.
  - Also called cooperative diversity.
- The radios may each have one or more antennas.
  - This paper derives closed-form expressions for the case that there is an array at one of the radios.
- First, let’s review performance with single-antenna radios.
The First Orthogonal Slot

- Time divided into two equal-length orthogonal slots.
- In first slot, source broadcasts to both relay and destination.
  - Message can be decoded by destination if
    \[ C_{s \rightarrow d} > 2R \]
  - Message can be decoded at relay if
    \[ C_{s \rightarrow r} > 2R \]
If relay could decode, then it retransmits in the second slot.
- **Decode-and-forward** with **repetition coding**.

Can be decoded if

$$C_{s+r \rightarrow d} > 2R$$

where

$$C_{s+r \rightarrow d} = \log_2(1 + \gamma_{s \rightarrow d} + \gamma_{r \rightarrow d})$$

Diversity combining
The outage event is:

\[
\{ C_{s\rightarrow r} < 2R \} \cap \{ C_{s\rightarrow d} < 2R \} \cup \{ C_{s\rightarrow r} > 2R \} \cap \{ C_{s+r\rightarrow d} < 2R \}
\]

- Relay out.
- S-D out.
- Relay decodes
- Destination out.

The average information outage probability is:

\[
P_0 = P[C_{s\rightarrow r} < 2R]P[C_{s\rightarrow d} < 2R] + P[C_{s\rightarrow r} > 2R]P[C_{s+r\rightarrow d} < 2R]
\]
Outage Probability of the SISO-Relay Channel

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Capacity of a Point-to-Point MIMO Link

- **MIMO channel model**

  \[ \mathbf{y} = \mathbf{Hs} + \mathbf{n} \]

  where \( \mathbf{H} \) is a \( N_{tx} \) by \( N_{rx} \) matrix.

- **Capacity**

  \[ C = \log \det \left( \mathbf{I}_{N_{rx}} + \frac{\Gamma}{N_{tx}} \mathbf{HH}^\dagger \right) \]
The CDF of a normalized $\chi^2$ random variable with $2L$ degrees of freedom is

$$F_L(x) = 1 - e^{-x} \sum_{k=0}^{L-1} \frac{x^k}{k!}.$$  

If the channel is SIMO ($N_{tx} = 1$) or MISO ($N_{rx} = 1$) then

$$P_0 = F_L (N_{tx} z)$$

where

$$z = \frac{2^R - 1}{\Gamma}$$

and

$$L = \max \{ N_{tx}, N_{rx} \}$$
Overall outage probability can still be found using

\[
P_0 = P[C_{s\rightarrow r} < 2R]P[C_{s\rightarrow d} < 2R] + P[C_{s\rightarrow r} > 2R]P[C_{s+r\rightarrow d} < 2R]
\]

Now, must use the appropriate MIMO capacities for each of the links.

Because of the orthogonal time slots, redefine

\[
z = \frac{2^{2R} - 1}{\Gamma}
\]
Array at Destination

- $s \rightarrow r$ is $1 \times 1$ channel.

\[
P[C_{s\rightarrow r} < 2R] = F_1(z)
\]

- $s \rightarrow d$ is $1 \times N_d$ channel.

\[
P[C_{s\rightarrow d} < 2R] = F_{N_d}(z)
\]

- $s + r \rightarrow d$ is two parallel $1 \times N_d$ channels.

\[
P[C_{s+r\rightarrow d} < 2R] = F_{2N_d}(z)
\]

- The above assumes $\Gamma_{s\rightarrow d} = \Gamma_{r\rightarrow d}$. 
Outage Probability with Array at Destination

- Direct
- $n_d = 1$
- $n_d = 2$
- $n_d = 6$

SNR

Outage Probability

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Array at Relay

- $s \rightarrow r$ is a $1 \times N_r$ channel.
  \[ P[C_{s\rightarrow r} < 2R] = F_{N_r}(z) \]

- $s \rightarrow d$ is a $1 \times 1$ channel.
  \[ P[C_{s\rightarrow d} < 2R] = F_1(z) \]

- $s + r \rightarrow d$ is a $1 \times 1$ channel in parallel with a $N_r \times 1$ channel.
  \[ P[C_{s+r\rightarrow d} < 2R] = \frac{L^L}{(L-1)!} \int_0^z x^{L-1} e^{-Lx}[1 - e^{x-z}]dx, \]
  where $L = N_r$.

- The above integral can be solved in closed form using
  \[ \int_0^z x^n e^{ax} dx = \frac{(-1)^{n+1}n!}{a^{n+1}} + e^{az} \sum_{k=0}^n \frac{(-1)^k n! z^{n-k}}{(n-k)! a^{k+1}} \]
Outage Probability with Array at Relay

![Graph showing outage probability with varying SNR and different numbers of receive antennas (nr = 1, nr = 2, nr = 6).]
Array at Source

- $s \rightarrow r$ is $N_s \times 1$ channel.
  \[ P[C_{s \rightarrow r} < 2R] = F_{N_s}(N_sz) \]

- $s \rightarrow d$ is $N_s \times 1$ channel.
  \[ P[C_{s \rightarrow d} < 2R] = F_{N_s}(N_sz) \]

- $s + r \rightarrow d$ is a $N_s \times 1$ channel in parallel with a $1 \times 1$ channel.
  \[ P[C_{s+r \rightarrow d} < 2R] = \frac{L^L}{(L - 1)!} \int_0^z x^{L-1}e^{-Lx}[1 - e^{x-z}]dx, \]

  where $L = N_s$. 
Summary of Outage Probability

- At destination

\[ P_0 = F_1(z)F_L(z) + (1 - F_1(z))F_{2L}(z) \]

- At relay

\[ P_0 = F_L(z)F_1(z) + (1 - F_L(z)) \frac{L^L}{(L - 1)!} \int_0^z x^{L-1}e^{-Lx}[1 - e^{x-z}]dx \]

- At source

\[ P_0 = F_L(Lz)F_L(Lz) + (1 - F_L(Lz)) \frac{L^L}{(L - 1)!} \int_0^z x^{L-1}e^{-Lx}[1 - e^{x-z}]dx \]

- \( L = \max\{N_s, N_r, N_d\} \)
Outage Probability with Array at Different Locations

- **SNR**
- **Outage Probability**

- **black:** array at relay
- **blue:** array at source
- **red:** array at destination

- **one radio has six antennas**
- **one radio has two antennas**
- **all radios have one antenna**

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Summary

- Diversity order increased by using Tx or Rx antenna array.
- Single-antenna relay increases diversity by one.
- Antenna array can be placed at relay instead of source handset.
- Closed form expressions can be found when just one antenna array.

Future work

- Array at 2 or 3 terminals.
- Multiple relays.
- Code combining.
- Space-time modulation constraints.
Conclusion

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Questions?