The Impact of an Antenna Array in a Relay Network

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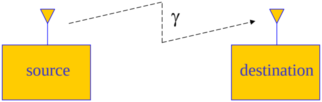
Outline

Information Outages in Direct SISO Links



- The unconstrained capacity of a link with SNR γ is

$$C = \log_2(1+\gamma).$$

- Information outage
  - Since γ is random, so is C.
  - If $C < R$, then an information outage occurs.
- In quasi-static Rayleigh fading,
  - γ is exponential with $E[\gamma] = \Gamma$.
  - The average information outage probability is

$$\begin{aligned} P_0 &= P[\log_2(1+\gamma) < R] \\ &= 1 - \exp\left\{\frac{1-2^R}{\Gamma}\right\}. \end{aligned}$$

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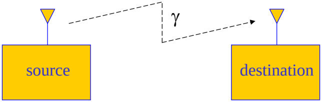

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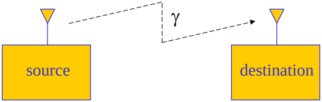

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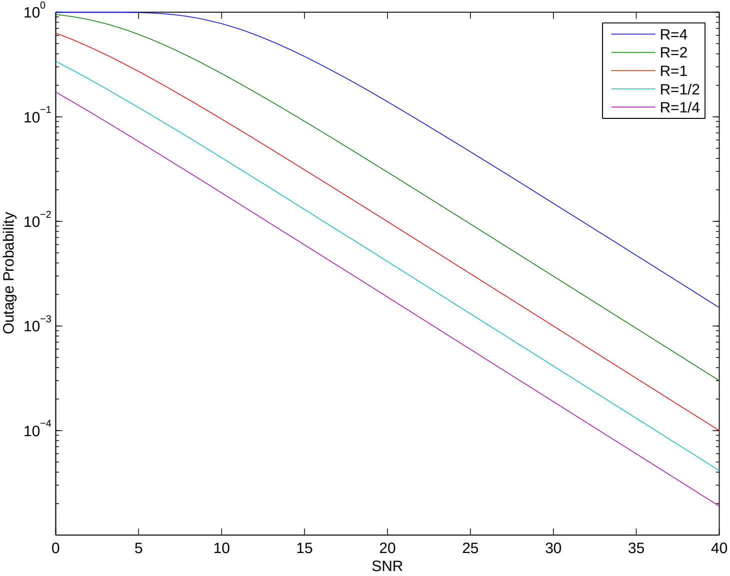
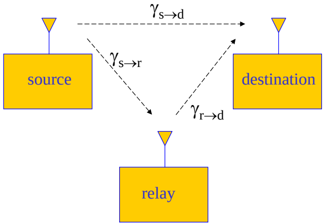
Outage Probability in Rayleigh Fading

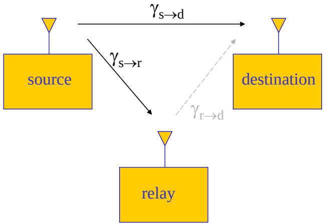


A Network with One Relay



- A relay can be used to increase diversity.
  - Also called *cooperative diversity*.
- The radios may each have one or more antennas.
  - This paper derives closed-form expressions for the case that there is an array at one of the radios.
- First, let's review performance with single-antenna radios.

The First Orthogonal Slot



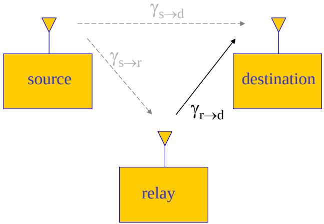
- Time divided into two equal-length orthogonal slots.
- In first slot, source broadcasts to both relay and destination.
  - Message can be decoded by destination if

$$C_{s\to d} \quad > \quad 2R$$

  - Message can be decoded at relay if

$$C_{s\to r} \quad > \quad 2R$$

The Second Orthogonal Slot



- If relay could decode, then it retransmits in the second slot.
  - Decode-and-forward with repetition coding.
- Can be decoded if

$$C_{s+r\to d} \quad > \quad 2R$$

where

$$C_{s+r\to d} \quad = \quad \log_2(1 + \underbrace{\gamma_{s\to d} + \gamma_{r\to d}}_{\text{Diversity combining}}).$$

Information Outage Probability

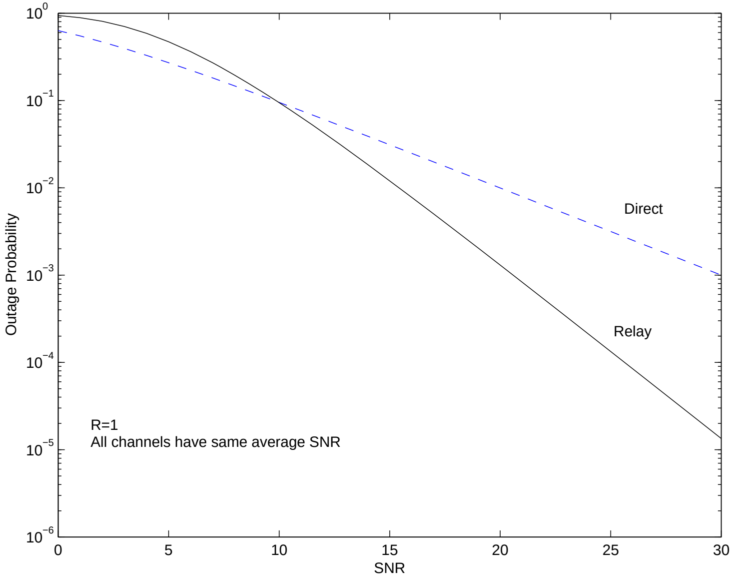
- The outage event is

$$\left[\underbrace{\{C_{s\to r} < 2R\}}_{\text{Relay out.}} \cap \underbrace{\{C_{s\to d} < 2R\}}_{\text{S-D out.}}\right] \cup \left[\underbrace{\{C_{s\to r} > 2R\}}_{\text{Relay decodes}} \cap \underbrace{\{C_{s+r\to d} < 2R\}}_{\text{Destination out.}}\right]$$

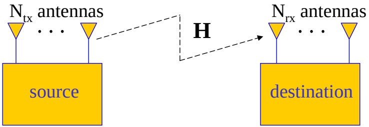
- The average information outage probability is:

$$\begin{aligned} P_0 &= P[C_{s\to r} < 2R]P[C_{s\to d} < 2R] \\ &\quad + P[C_{s\to r} > 2R]P[C_{s+r\to d} < 2R] \end{aligned}$$

Outage Probability of the SISO-Relay Channel



Capacity of a Point-to-Point MIMO Link



- MIMO channel model

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

where $\mathbf{H}$ is a N_{tx} by N_{rx} matrix.

- Capacity

$$C = \log\det\left[\mathbf{I}_{N_{rx}} + \frac{\Gamma}{N_{tx}}\mathbf{H}\mathbf{H}^{\dagger}\right]$$

Outage Probability of SIMO and MISO Links

- The CDF of a normalized χ^2 random variable with $2L$ degrees of freedom is

$$F_L(x) = 1 - e^{-x} \sum_{k=0}^{L-1} \frac{x^k}{k!}.$$

- If the channel is SIMO $(N_{tx} = 1)$ or MISO $(N_{rx} = 1)$ then

$$P_0 = F_L\left(N_{tx} z\right)$$

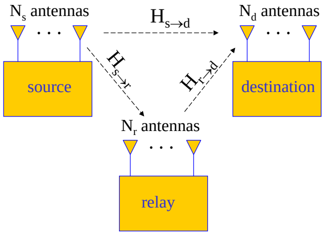
where

$$z = \frac{2^R - 1}{\Gamma}$$

and

$$L = \max\left\{N_{tx}, N_{rx}\right\}$$

Relay Channel with MIMO Links



- Overall outage probability can still be found using

$$P_0 = P[C_{s\to r} < 2R]P[C_{s\to d} < 2R] + P[C_{s\to r} > 2R]P[C_{s+r\to d} < 2R]$$

- Now, must use the appropriate MIMO capacities for each of the links.
- Because of the orthogonal time slots, redefine

$$z = \frac{2^{2R}-1}{\Gamma}$$

Array at Destination

- $s \to r$ is 1×1 channel.

$$P[C_{s\to r} < 2R] \;=\; F_1(z)$$

- $s \to d$ is $1 \times N_d$ channel.

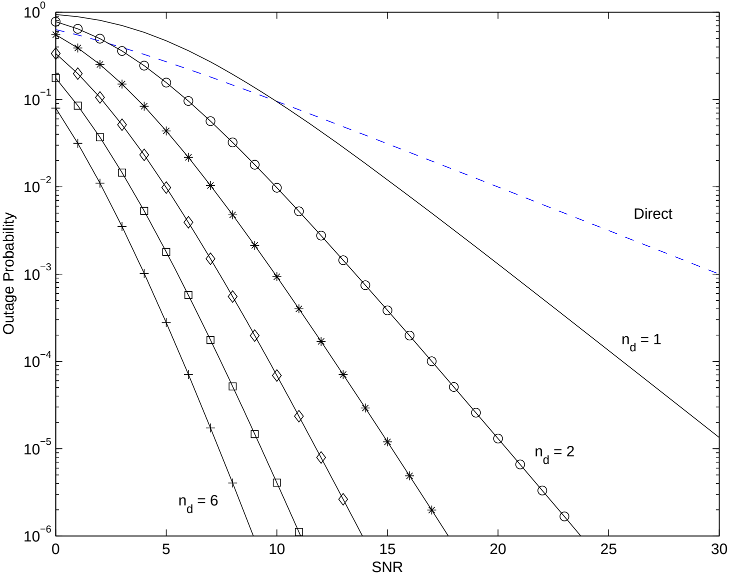
$$P[C_{s\to d} < 2R] \;=\; F_{N_d}(z)$$

- $s + r \to d$ is two parallel $1 \times N_d$ channels.

$$P[C_{s+r\to d} < 2R] \;=\; F_{2N_d}(z)$$

- The above assumes $\Gamma_{s\to d} = \Gamma_{r\to d}$.

Outage Probability with Array at Destination



Array at Relay

- $s \to r$ is $1 \times N_r$ channel.

$$P[C_{s\to r} < 2R] = F_{N_r}(z)$$

- $s \to d$ is 1×1 channel.

$$P[C_{s\to d} < 2R] = F_1(z)$$

- $s + r \to d$ is a 1×1 channel in parallel with a $N_r \times 1$ channel.

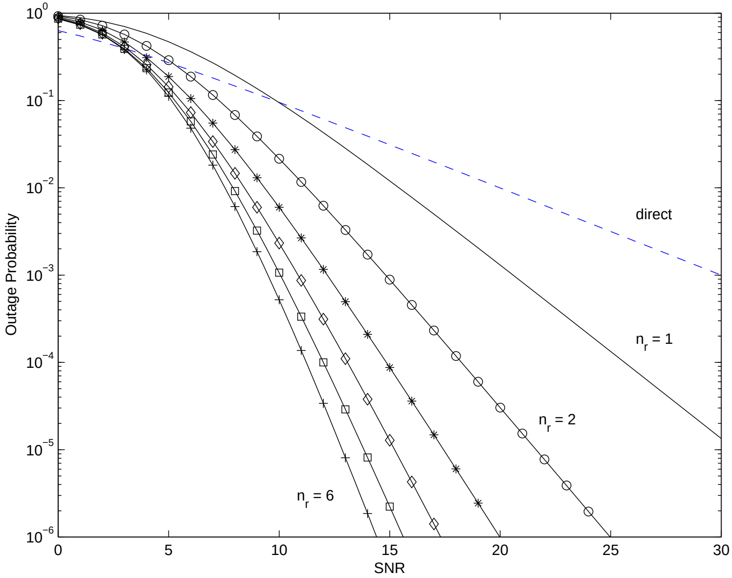
$$P[C_{s+r\to d} < 2R] = \frac{L^L}{(L-1)!}\int_0^z x^{L-1}e^{-Lx}[1 - e^{x-z}]dx,$$

where $L = N_r$.

- The above integral can be solved in closed form using

$$\int_0^z x^n e^{ax} dx = \frac{(-1)^{n+1}n!}{a^{n+1}} + e^{az}\sum_{k=0}^{n}\frac{(-1)^k n! z^{n-k}}{(n-k)!a^{k+1}}$$

Outage Probability with Array at Relay



Array at Source

- $s \to r$ is $N_s \times 1$ channel.

$$P[C_{s\to r} < 2R] = F_{N_s}(N_s z)$$

- $s \to d$ is $N_s \times 1$ channel.

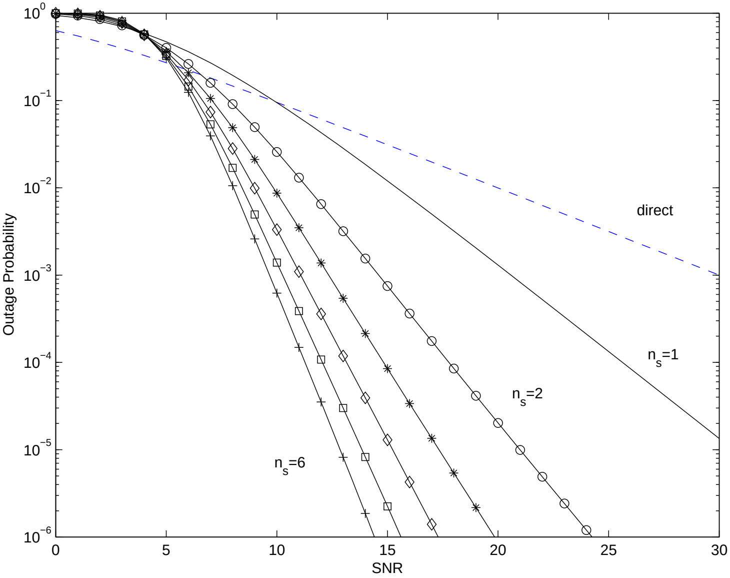
$$P[C_{s\to d} < 2R] = F_{N_s}(N_s z)$$

- $s + r \to d$ is a $N_s \times 1$ channel in parallel with a 1×1 channel.

$$P[C_{s+r\to d} < 2R] = \frac{L^L}{(L-1)!} \int_0^z x^{L-1} e^{-Lx} [1 - e^{x-z}] dx,$$

where $L = N_s$.

Outage Probability with Array at Source



Summary of Outage Probability

- At destination

$$P_0 = F_1(z)F_L(z) + (1 - F_1(z))\, F_{2L}(z)$$

- At relay

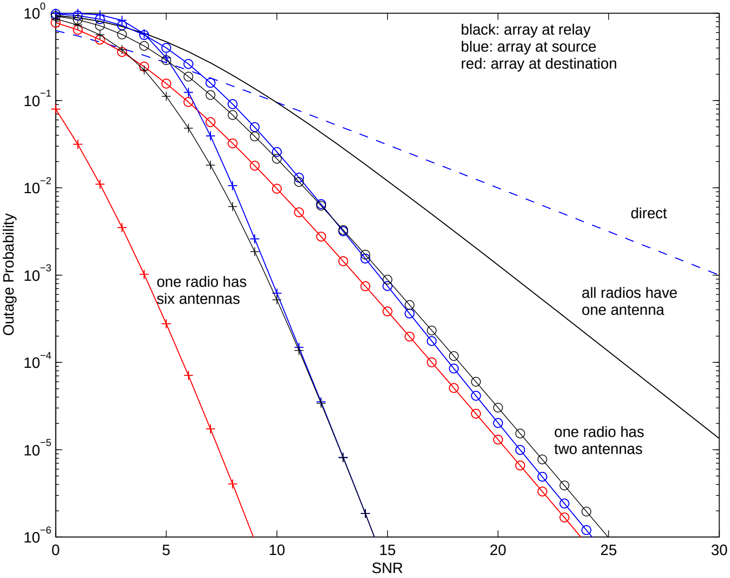
$$P_0 = F_L(z)F_1(z) + (1 - F_L(z))\, \frac{L^L}{(L-1)!} \int_0^z x^{L-1} e^{-Lx} [1 - e^{x-z}] dx$$

- At source

$$\begin{aligned} P_0 &= F_L(Lz)F_L(Lz) \\ &\quad + (1 - F_L(Lz))\, \frac{L^L}{(L-1)!} \int_0^z x^{L-1} e^{-Lx} [1 - e^{x-z}] dx \end{aligned}$$

- $L = \max\{N_s, N_r, N_d\}$

Outage Probability with Array at Different Locations



Conclusion

- Summary
  - Diversity order increased by using Tx or Rx antenna array.
  - Single-antenna relay increases diversity by one.
  - Antenna array can be placed at relay instead of source handset.
  - Closed form expressions can be found when just one antenna array.
- Future work
  - Array at 2 or 3 terminals.
  - Multiple relays.
  - Code combining.
  - Space-time modulation constraints.

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Questions?