Constellation Labeling Maps for Low Error Floors

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Abstract

A constellation labeling map is the assignment of a bit pattern to each symbol in a signal-set constellation. In a system with bit-interleaved coded-modulation and iterative decoding and demodulation (BICM-ID), the error floor of the bit error rate is highly dependent on the labeling map. A simple class of labeling maps that significantly lower the error floors is presented. Examples show the applications of the proposed mapping to phase-shift keying (PSK), quadrature amplitude modulation (QAM), and continuous-phase frequency-shift keying (CPFSK). Simulation results indicate that the proposed labeling maps are comparable to or better than other labeling maps in providing a low error floor. A major advantage of the proposed labeling maps is that they are easily generated.

I. Introduction

Bit-interleaved coded-modulation (BICM) increases the time diversity of a communication system and, hence, improves its performance over a fading channel by using bit interleaving instead of symbol interleaving [1]. As a result, BICM has become a standard feature in cellular, satellite, and wireless network systems. In a system with BICM and iterative decoding and demodulation (BICM-ID), soft-decision information is exchanged between the demodulator and the decoder [2], [3], which itself may be internally iterative. The iterative decoding and demodulation minimizes any performance degradation experienced by BICM over the AWGN channel.

Plots of the bit error rate for BICM-ID systems generally exhibit a *waterfall region,* which is characterized by a rapid decrease in the bit error rate as the signal-to-noise ratio increases, and an *error-floor region,* in which the bit error rate decreases much more slowly. The choice of the labeling map has a major impact on both regions. In this paper, we present a simple method for constructing labeling maps that produce low error floors for an arbitrary constellation and error-control code. A low error floor may be important for radio-relay communications, space-ground communications, or when an automatic-repeat request is not feasible because of the variable delays.

Methods for generating good labeling maps for low error floors have been previously described [4], [5], [6]. These methods entail computer searches based on approximate upper bounds on the bit error rate. The new labeling maps presented in this paper are based on the Euclidean distances in the signal-set constellations and are much simpler to generate. Simulation results indicate that these labeling maps produce an error floor at least as low as those produced by other proposed labeling maps.

In addition to a suitable choice of the labeling map, there are several independent strategies for lowering the error floor. One can lower the error floor by strengthening or appropriately selecting the code. As examples, one can use a turbo code instead of a convolutional code or more powerful component codes within the turbo code. One can use a regular low-density parity-check (LDPC) code instead of a comparable irregular one to lower the error floor. Bit interleavers that ensure the unequal protection of bits can be designed to provide low error floors for both bit-interleaved turbo-coded systems [7] and LDPC-coded systems [8]. The new labeling maps can supplement any of the other methods of lowering the error floor and lower it further.

A Gray labeling map minimizes the number of bit errors that occur if an adjacent symbol of a received symbol is assigned the highest likelihood or largest metric by the decoder. Thus, a Gray labeling map will provide an early onset of the waterfall region, but produces a relatively high error floor primarily determined by the minimum Euclidean distance of the symbol set. In contrast, the new *TV labeling map* described in Section II lowers the error floor at the cost of an adversely shifted waterfall region.

II. Labeling Maps

The major components of a BICM-ID system are diagrammed in Fig. 1. In the transmitter, message bits are encoded, bit-interleaved, and then applied to the modulator. A *constellation labeling* or *labeling map* is the mapping of a bit pattern to each symbol or point in a signal-set constellation. Each set of $m = \log_2 q$ consecutive bits in the input $\mathbf{b} = \{b_0, ..., b_{m-1}\} \in [0,1]^m$ is mapped into a q-ary symbol $\mathbf{s} = \mu(\mathbf{b})$, where $\mu(\mathbf{b})$ is the labeling map, and the set of constellation symbols has cardinality q. In the receiver, the demodulator converts the received signal into a sequence of received symbols. A demapper within the demodulator processes each received symbol to produce a vector of bit metrics. This vector provides extrinsic information that is interleaved and passed to the decoder. The demapper and decoder exchange extrinsic information until bit decisions are made.

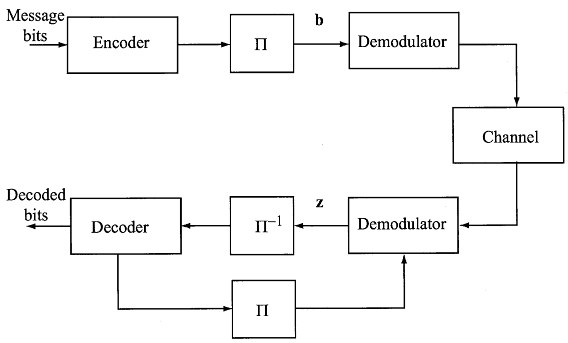

Fig 1. BICM-ID system. Π denotes an interleaver, and Π^{-1} denotes a deinterleaver.

The Euclidean distance is a measure of the separation between two constellation points. An *adjacent constellation symbol* of symbol i is one at the minimum Euclidean distance $d_{e1}(i)$ from symbol i. A *second closest constellation symbol* of symbol i is one at the second shortest Euclidean distance $d_{e2}(i)$ from symbol i. A symbol is said to be *selected* if the bits of its label are the decoder output bits. A set of bits is said to be *essentially known* if the decoder has assigned them very high likelihood ratios that are fed back to the demodulator.

Due to its larger Euclidean distance, a second closest symbol has a lower probability of incorrect selection by the decoder than an adjacent symbol when all the bits are unknown. When the SNR is high enough and there are enough decoder iterations, some bits are essentially known by the decoder. The symbols that include the essentially known bits constitute a subset A of the constellation. Consider a demodulator iteration after receiving extrinsic information produced by the decoder. Let $\mathbf{y} = \alpha\mathbf{s} + \mathbf{n}$ denote an arbitrary received symbol, where $\mathbf{s}$ is the transmitted symbol, α is the fading amplitude, and $\mathbf{n}$ is complex Gaussian noise with variance $N_0/2$ per component. Let $\mathbf{s}_i$, $1 \leq i \leq q$, denote symbol i of the constellation, and $b_k(\mathbf{s}_i)$ denote bit k of $\mathbf{s}_i$. If a bit is essentially known, the bit metric produced by the demodulator for that bit no longer significantly affects the computation of the decoder metric. However, if a bit is unknown, then the demodulator bit metric has a significant effect on the decoder bit metric produced by the next decoder iteration. The demodulator metric for bit k is [9], [10]

$$z_k = \log \frac{\sum_{\mathcal{D}_k^{(1)}} p(\mathbf{y}|\mathbf{s}_i) \prod_{\substack{j=0 \\ j\neq k}}^{m-1} \exp[b_j(\mathbf{s}_i)v_j]}{\sum_{\mathcal{D}_k^{(0)}} p(\mathbf{y}|\mathbf{s}_i) \prod_{\substack{j=0 \\ j\neq k}}^{m-1} \exp[b_j(\mathbf{s}_i)v_j]} \tag{1}$$

where $\mathcal{D}_k^{(b)} = \{\mathbf{s}_i : b_k(\mathbf{s}_i) = b\}$ contains all symbols labeled with $b_k = b$, and v_j is the extrinsic log-likelihood ratio for bit j that is produced by the decoder and fed back to the demodulator. When bit k is unknown, then in both summations in (1), any term for which $\mathbf{s}_i \notin A$ is negligible compared with terms for which $\mathbf{s}_i \in A$. Therefore, for an unknown bit k,

$$z_k = \log \frac{\sum_{\mathcal{D}_k^{(1)} \cap A} p(\mathbf{y}|\mathbf{s}_i) \prod_{\substack{j=0 \\ j\neq k}}^{\mu-1} \exp[b_j(\mathbf{s}_i)v_j]}{\sum_{\mathcal{D}_k^{(0)} \cap A} p(\mathbf{y}|\mathbf{s}_i) \prod_{\substack{j=0 \\ j\neq k}}^{\mu-1} \exp[b_j(\mathbf{s}_i)v_j]}. \tag{2}$$

If a labeling map can ensure that the set A does not include symbols that are adjacent in the constellation, then (2) indicates that z_k for unknown bit k is independent of adjacent symbols at the minimum distance $d_{e1}(i)$ and, hence, the effective minimum distance for the demodulator iteration is at least $d_{e2}(i)$. Thus, subsequent decoder iterations are less likely to result in a symbol or bit error, and the error floor is lower for this labeling map than for the Gray labeling map. A *TV labeling map* is defined as a labeling map for symbol sets with $m \geq 3$ such that adjacent symbols are absent from any set of symbols with κ known bits if $2 \leq \kappa \leq m-1$.

Proposition 1: A labeling map for a constellation with $m \geq 3$ is a TV labeling map if and only if the minimum Hamming distance $d_a(i)$ from a constellation symbol i to its adjacent ones is at least $m-1$.

Proof: **Necessity**. Assume that a labeling map is such that some symbol i is at Hamming distance $d_a(i) \leq m-2$ from one or more adjacent ones. Therefore, at least two bits are common to the symbol i and one or more of its adjacent symbols. If the common bits are known, symbol i and one or more of its adjacent symbols are members of the set of symbols with the *two* known bits. Thus, the labeling map cannot be a TV labeling map. **Sufficiency**. Assume that a labeling map has $d_a(i) \geq m-1$ for any symbol i. Let $d_2(i, A)$ denote the Hamming distance between symbol i and another member of a set A of symbols with κ known bits. If $2 \leq \kappa \leq m-1$, then $d_2(i, A) \leq m - \kappa < m-1 \leq d_a(i)$ for any symbol i. Since $d_2(i, A) < d_a(i)$, set A cannot include both symbol i and its adjacent symbols. ■

The proposition implies that *a TV labeling map can be constructed by assigning bit patterns to symbols such that the Hamming distance to adjacent symbols always is at least* $m-1$. In the following labeling algorithm, the symbols are labeled sequentially. After each symbol labeling, the unused bit patterns are called the *remaining labels*.

Labeling Algorithm: Two tables are associated with each symbol i. The *adjacent-symbol table* $S(i)$ is a list of symbols that are adjacent to symbol i. The *adjacent-label table* $L(i)$ is a list of labels that could be used by adjacent symbols. This table is initially empty. After symbol i is labeled, $L(i)$ is a list of remaining labels that are at Hamming distance $m-1$ or m from the label of symbol i. As successive symbols are labeled, $L(i)$ is shortened. One symbol is selected to be the first labeled symbol and is labeled arbitrarily.

Using the adjacent-symbol tables, arbitrarily select one of the unlabeled symbols that have the largest number of labeled adjacent symbols. This selected symbol is labeled with one of the remaining labels that is common to all the adjacent-label tables of its labeled adjacent symbols. If there is more than one such label, one of them is chosen arbitrarily. The process terminates when every symbol has been labeled. If at any step, no further symbol labeling is possible, then the algorithm returns to the last arbitrary choice, erases all labels subsequent to this choice, makes a different choice, and then continues with the sequential labeling. □

A TV labeling map exists only if every constellation symbol has $m+1$ or fewer adjacent symbols. Many TV labeling maps exist for most practical constellations. If an m-digit binary number is modulo-2 added to all the bit labels of one TV labeling map, then another TV labeling map is produced. Furthermore, rotated and reflected versions of a TV labeling map can be constructed.

Example 1: For multiple phase-shift keying (MPSK), let $\mathbf{s}_i = \exp(j2\pi i/q)$, $i = 0, 1, \ldots, q-1$, denote the complex value of constellation symbol i, where $j = \sqrt{-1}$ and $q = 2^m$. For the small alphabet with $m = 3$, some previously described labeling maps [3] , [6] belong to the class of TV labeling maps. For $m = 3$, a TV labeling map is the following.

symbol	bit label	symbol	bit label
0	010	4	110
1	001	5	101
2	100	6	000
3	011	7	111

The minimum Hamming distance between symbols 0 and 7 and all other adjacent symbols is equal to or greater than 2. Suppose that the first two bits of the map are essentially known to be 10. Then the decoder will use the demodulator metrics for symbol 2 and symbol 5, which are not adjacent symbols, to make a decision on the third bit. For $m = 4$, a TV labeling map is the following.

symbol	bit label	symbol	bit label
0	0 0 0 0	8	1 0 1 0
1	1 1 1 1	9	0 1 0 1
2	0 0 0 1	10	1 0 1 1
3	1 1 1 0	11	0 1 0 0
4	1 0 0 1	12	0 0 1 1
5	0 1 1 0	13	1 1 0 0
6	1 0 0 0	14	0 0 1 0
7	0 1 1 1	15	1 1 0 1

□

Example 2: In Fig. 2 for quadrature amplitude modulation (QAM), the bit labels of a TV labeling map for the 16-QAM constellation are expressed in a decimal format. The complex-valued symbols have 2, 3, or 4 adjacent symbols. The application of the labeling algorithm is illustrated by its first few steps. The constellation symbols are denoted by 0, 1, ..., 15 from left to right and from top to bottom starting in the upper left corner of the figure. Symbol 0 is labeled 0000. Table $L(0)$ lists the labels 1111, 1110, 1101, 1011, and 0111. Since both $S(1)$ and $S(4)$ list symbol 0, symbol 1 is arbitrarily selected to be the next labeled symbol, and 1101 (13) is arbitrarily chosen from $L(0)$ as its label. This label is deleted from $L(0)$. Symbol 4 is arbitrarily selected to be the next labeled symbol, and 0111 (7) is arbitrarily chosen from $L(0)$ as its label. Symbol 5 is the only unlabeled symbol with 2 labeled adjacent symbols, so it is the next labeled symbol. Since 1010 is the only label common to both $L(1)$ and $L(4)$, symbol 5 is labeled 1010 (10). The labeling algorithm is continued until we obtain the TV labeling map of Fig. 2.

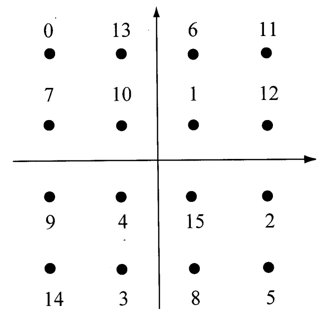

Fig. 2. Labeling map for 16-QAM constellation.

The "D5 mapping" of [11], which is proposed as an approximation of an anti-Gray map, belongs to the class of TV labeling maps for 64-QAM. In Fig. 3, the bit labels of a TV labeling map for the 64-QAM constellation are expressed in a decimal format. □

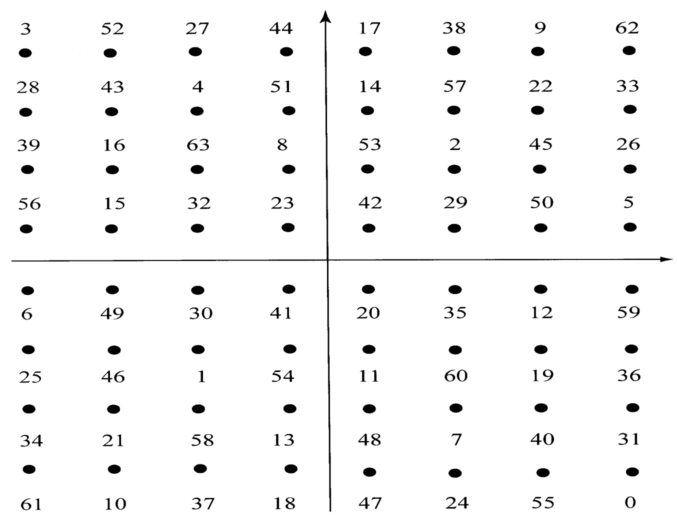

Fig. 3. Labeling map for 64-QAM constellation.

Example 3: Consider a noncoherent q-ary continuous-phase frequency-shift keying (CPFSK) system with modulation index $h > 0$ that does not exploit the memory due to the continuous phase of the signal. Then the signal-set constellation consists of q symbols representing the signals $s_l(t) = e^{j2\pi lht/T}/\sqrt{T}$, $0 \le t \le T$, $l = 1, 2, \ldots, q$, where T is the signal duration. The square of the Euclidean distance between symbols i and j is

$$\begin{aligned} d^2(|i-j|) &= \int_0^T |s_i(t) - s_j(t)|^2 \, dt \\ &= 2\mathcal{E}_s \left[1 - \frac{\sin[(i-j)h\pi]}{(i-j)h\pi} \right] \end{aligned} \tag{3}$$

where $\mathcal{E}_s$ is the signal energy, and the Euclidean distance is a function of $|i-j|$ because $\sin x/x$ is an even function of x.

If $0 < h \leq 1$, an adjacent symbol corresponds to the signal closest in frequency to the signal associated with a specified symbol, which is true because the minimum Euclidean distance of the constellation is $d(1) < d(k)$ for any integer $k \geq 2$. The latter follows from the fact that $\sin h\pi/h\pi > \sin kh\pi/kh\pi$, $0 < h \leq 1$, for $k \geq 2$, which may be proved by showing that $k \sin h\pi - \sin kh\pi > 0$, $0 < h \leq 1$, for $k \geq 2$. The proof is by mathematical induction. Assume that $0 < h \leq 1$. Since $\sin 2h\pi = 2 \sin h\pi \cos h\pi$, the inequality is true for $k = 2$. Assume that $k \sin h\pi - \sin kh\pi > 0$ is true for some $k \geq 2$. Using this inequality, we obtain $(k+1)\sin h\pi - \sin(k+1)h\pi = (k+1)\sin h\pi - \sin kh\pi \cos h\pi - \sin h\pi \cos kh\pi > \sin h\pi(k+1-k\cos h\pi - \cos kh\pi) > 0$, which completes the proof. Since $d(1)$ is the minimum Euclidean distance of the constellation, each constellation symbol has either one or two adjacent symbols.

The TV labeling map for $m = 3$ in example 1 is applicable to noncoherent CPFSK with $q = 8$. Symbol i has adjacent symbols $i-1$ and $i+1$, $1 \leq i \leq 6$, symbol 0 has adjacent symbol 1, and symbol 7 has adjacent symbol 6. Symbols 0 and 7 are not adjacent. Since symbol 6 is the only symbol adjacent to symbol 7, the labeling map indicates that $d_a(7) = 3$. $\square$

If all the bits in a symbol are known except one, then the dominant influence on the bit error probability is the minimum Euclidean distance between the two constellation symbols that differ in that one bit [1], [2]. Thus, the *asymptotic error floor* of the bit error probability is determined by the minimum Euclidean distance D_e between constellation symbols that differ in a single bit of their labels. The asymptotic error floor with its extremely low bit error probability is unlikely to be reached by most practical communication systems. The error floor that is reached when at least two decoded bits are essentially known is a more realistic goal. The latter error floor is minimized by the TV labeling map.

Consider a TV labeling map and constellation symbols s_1 and s_2 that differ in a single bit and that do not differ in $m-1$ *identical* bits. Let β denote the number of branches in a path from s_1 to s_2 that passes through successive adjacent symbols. For example, starting from the symbol labeled 0 in Fig. 2, the symbol labeled 1, which differs in a single bit of its label, can be reached by passing through the 3 successive adjacent symbols labeled 13, 6, and 1.

Proposition 2: If m is an even number and the TV labeling map has adjacent symbols that differ in exactly $m-1$ bits, then $\beta \geq m-1$. Furthermore, a square QAM constellation has

$$D_e \geq d_{e1} \left(\frac{m^2}{2} - m + 1 \right)^{1/2} \tag{4}$$

where d_{e1} is the Euclidean distance between adjacent symbols.

Proof: Each of the identical bits must change an even number of times as the path between symbols s_1 and s_2 that differ in a single bit is traversed. If β is an odd number, each of the $m-1$ identical bits must be unchanged in traversing at least one branch of the path. Since adjacent symbols separated by a branch have at most one bit that is unchanged, $\beta \geq m-1$. Suppose that m is an even number and the TV labeling map has adjacent symbols that differ in exactly $m-1$ bits. The total number of bit changes in a path from s_1 to s_2 must be an odd number and also must equal $(m-1)\beta$. Therefore, β must be an odd number, and hence $\beta \geq m-1$. For a square QAM constellation with m even, the minimum Euclidean distance for a path with $\beta = m-1$ occurs when there are $m/2$ branches in one direction and $m/2 - 1$ branches in the orthogonal direction. Since $\beta \geq m-1$, the Pythagorean theorem implies (4). ■

A TV labeling map may have relatively low values of β and D_e. However, if m is an even number, the *constrained labeling algorithm* prevents the occurrence of this undesirable feature by constraining each adjacent-channel table $L(i)$ to have labels only at Hamming distance $m-1$ from the symbol i.

Example 4: Consider the 16-QAM constellation. For the TV labeling map in Fig. 2, $D_e = \sqrt{5}d_{e1}$, which is the lower bound of (4). The SGHB map proposed by Schreckenbach, Gortz, Hagenauer, and Bauch [4] and the HR map proposed by Huang and Ritcey [5] have $D_e = 2d_{e1}$, and the Gray map has $D_e = d_{e1}$. Therefore, the TV labeling map provides a lower asymptotic error floor for 16-QAM than these other maps. $\square$

0	239	24	199	120	151	40	223	48	143	96	191	80	167	72	247
251	20	227	60	131	108	211	36	203	116	155	68	171	92	179	12
132	107	156	67	252	19	172	91	180	11	228	59	212	35	204	115
122	149	98	189	2	237	82	165	74	245	26	197	42	221	50	141
129	110	153	70	249	22	169	94	177	14	225	62	209	38	201	118
254	17	230	57	134	105	214	33	206	113	158	65	174	89	182	9
5	234	29	194	125	146	45	218	53	138	101	186	85	162	77	242
248	23	224	63	128	111	208	39	200	119	152	71	168	95	176	15
3	236	27	196	123	148	43	220	51	140	99	188	83	164	75	244
253	18	229	58	133	106	213	34	205	114	157	66	173	90	181	10
6	233	30	193	126	145	46	217	54	137	102	185	86	161	78	241
121	150	97	190	1	238	81	166	73	246	25	198	41	222	49	142
135	104	159	64	255	16	175	88	183	8	231	56	215	32	207	112
124	147	100	187	4	235	84	163	76	243	28	195	44	219	52	139
130	109	154	69	250	21	170	93	178	13	226	61	210	37	202	117
127	144	103	184	7	232	87	160	79	240	31	192	47	216	55	136

Fig. 4. Labeling for 256-QAM constellation.

Example 5: The TV labeling map of Fig. 3 for the 64-QAM constellation has $D_e = \sqrt{13}d_{e1}$. One of the TV labeling maps for the 256-QAM constellation with $D_e = 5d_{e1}$ is displayed in Fig. 4. The bit labels are expressed in a decimal format and the constellation points are not shown for simplicity.

III. Simulation Examples

The TV labeling maps are designed to increase the effective Euclidean distance once at least two bits are essentially known. Therefore, they are expected to be increasingly advantageous as the number of bits per symbol increases. The subsequent simulation results confirm this characteristic. The TV labeling map maximizes the number of bits that differ between adjacent symbols. Thus, the error floor will be low, but the onset of the waterfall region will be adversely shifted relative to other labeling maps.

To illustrate the effects of the TV labeling maps, simulations were conducted to generate plots of the bit error rate (BER) of several systems as a function of $\mathcal{E}_b/N_0$, where $\mathcal{E}_b$ is the bit energy. In Figures 5, 6, and 7, coherent QAM demodulation and a rate-1/2 convolutional code with constraint length $K = 3$ and octal generators (5,7) are used. Frames comprise 200 consecutive code bits, and the transmitted QAM symbols experience independent and identically distributed Rayleigh fading.

Figure 5 shows the results for 16-QAM and both BICM and BICM-ID systems. The BICM-ID plots are for 10 iterations of the demodulator and decoder. The four labeling maps in the figures are the TV map of Fig. 2, the SGHB map, the HR map, and the Gray map. All but the Gray labeling map are designed to produce a low error floor in a BICM-ID system. Although the Gray map exhibits an advantage for BICM, it is distinctly inferior to the other labeling maps for BICM-ID and bit error rates below 10^{-3}. The HR and SGHB maps have almost the same bit error rates. The bit error rates of the TV map initially drop more slowly as $\mathcal{E}_b/N_0$ increases but then become almost the same near the error-floor region as the bit error rates of the HR and SGHB maps. Since it is not feasible to simulate bit error rates below 10^{-8}, it is not possible to distinguish the error floors produced by the TV, HR, and SGHB labeling maps.

Figure 6 shows the results for 64-QAM, BICM, and BICM-ID with 12 iterations of the demodulator and decoder. The labeling maps are the TV map of Fig. 3, the HR map, and the Gray map. Again the Gray map exhibits an advantage for BICM, but is inferior to the TV and HR maps for BICM-ID and low bit error rates. The TV labeling map provides a lower error floor and a better performance than the HR labeling map when the bit error rate is below 10^{-7}. The cost is a significant degradation in the waterfall region relative to the performance of the HR map.

The heuristic searches required by the HR and SGHB maps are very complex and inefficient to implement for 128-QAM and larger constellations, whereas the TV map is easily generated for large constellations and ensures a low error floor.

Figure 7 shows the results for 256-QAM, BICM, and BICM-ID with 14 iterations of the demodulator and decoder. The labeling maps are the TV map of Fig. 4 and the Gray map. The onset of the waterfall region for the TV map is greatly delayed relative to the Gray map, but then the plot for the TV map shows a steep descent. For BICM-ID, the plot for the TV map crosses the plot for the Gray map at a bit error rate of 0.6×10^{-6} and is still falling rapidly to a very low error floor.

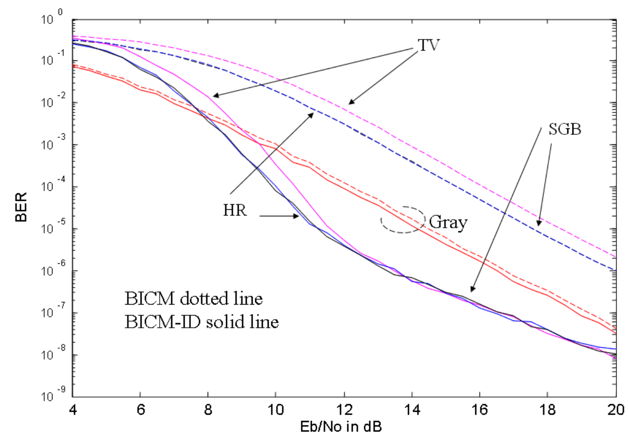

Fig. 5. Bit error rates for various 16-QAM systems.

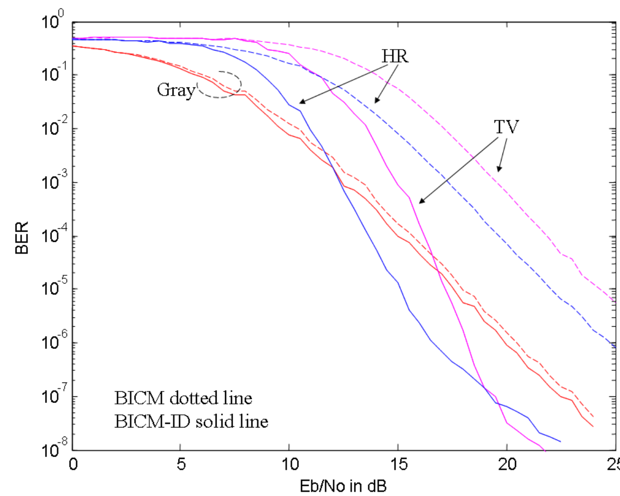

Fig. 6. Bit error rates for various 64-QAM systems.

Figures 8 and 9 show the bit error rates for noncoherent CPFSK with $q = 8$, $h = 0.32$, 2048 message bits, and code symbols that experience independent and identically distributed Rayleigh fading. In Fig. 8, a rate-1/2 convolutional code with constraint length $K = 7$ and octal generators (133,171) is used with BICM and BICM-ID, and the Gray map, the natural map, and the TV labeling map of example 1. For BICM-ID, there are 8 demodulator and decoder iterations. The figure illustrates the dramatic improvement in the waterfall region and the lowering of the error floor when the TV labeling map is used with BICM-ID instead of BICM. For bit error rates below 10^{-3}, the TV labeling map has an expanding advantage that is nearly 2 dB relative to the natural labeling map when the bit error rate is 10^{-5}.

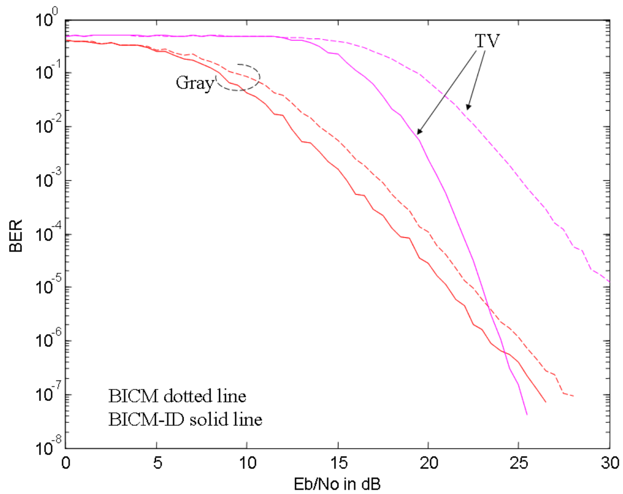

Fig. 7. Bit error rates for various 256-QAM systems.

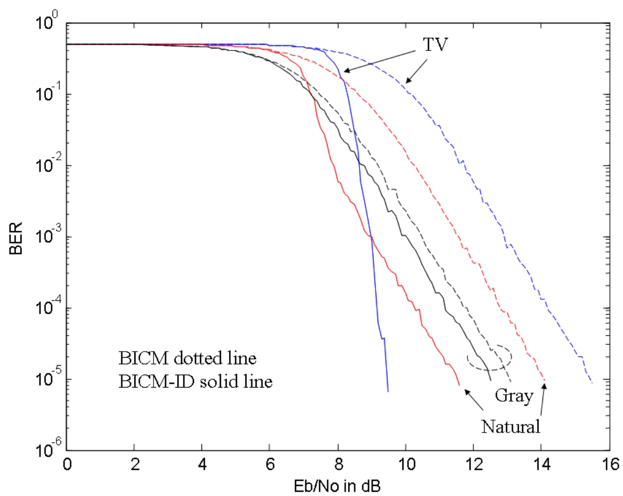

Fig. 8. Bit error rates for convolutionally coded CPFSK with q=8 and constraint length K=7.

Figure 9 shows the bit error rates when rate-1/2 convolutional codes with constraint lengths $K = 7$ and $K = 4$ and a rate-8/15 UMTS turbo code with $K = 4$ are used with BICM-ID. For the convolutional codes, there are 8 demodulator and decoder iterations. For the turbo code, there are 16 global demodulator and decoder iterations with one decoder iteration per global iteration. The TV labeling map enables the $K = 4$ convolutional code, which has octal generators (13, 15), to track the performance of the turbo code for bit error rates above 10^{-3} despite the fact that the convolutional code is much simpler and less expensive to implement.

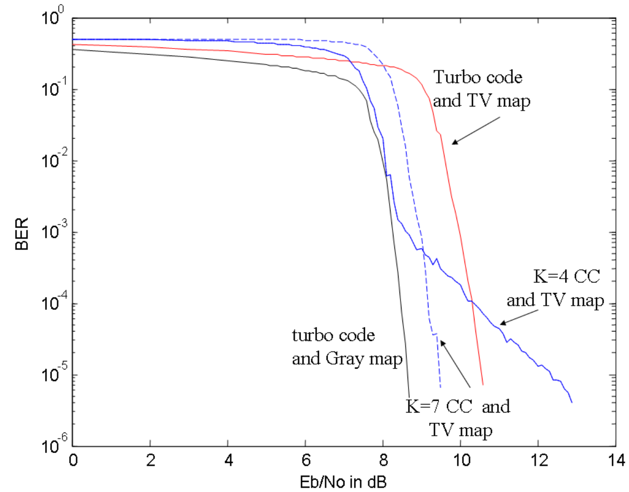

Fig. 9. Bit error rates for CPFSK with q=8, BICM-ID, and turbo and convolutional codes.

IV. Conclusions

The class of TV labeling maps for signal-set constellations and BICM-ID has been derived and applied to a variety of specific communication systems that operate in the presence of ergodic Rayleigh fading. Simulation results indicate that the TV labeling maps are at least as good as other proposed labeling maps in providing a low error floor. The cost is an adverse shift in the onset of the waterfall region of the bit error rate. A major advantage of the TV labeling maps is that they are easily generated.

References

[1] G. Caire, G. Taricco, and E. Biglieri, “Bit-interleaved coded modulation,” *IEEE Trans. Inform. Theory*, vol. 44, pp. 927–946, May 1998.

[2] A. Chindapol and J. A. Ritcey, “Design, analysis and performance evaluation for BICM-ID with square QAM constellations in Rayleigh fading channels," *IEEE J. Select. Areas Commun.*, vol. 19, pp. 944-957, May 2001.

[3] X. Li, A. Chindapol, and J. A. Ritcey, “Bit-interleaved coded modulation with iterative decoding and 8PSK modulation,” *IEEE Trans. Commun.*, vol. 50, pp. 1250-1257, Aug. 2002.

[4] F. Schreckenbach, N. Gortz, J. Hagenauer, and G. Bauch,"Optimization of symbol mappings for bit-interleaved coded modulation with iterative decoding," *IEEE Commun. Lett.,* vol. 7, pp.593-595, Dec. 2003.

[5] Y. Huang and J. A. Ritcey, "Optimal constellation labeling for iteratively decoded bit-interleaved space-time coded modulation," *IEEE Trans Inf. Theory,* vol. 51, pp. 1865-1871, May 2005.

[6] J. Tan and G. L. Stuber, "Analysis and Design of Symbol Mappers for Iteratively Decoded BICM," *IEEE Trans. Wireless Commun.,* vol. 4, pp. 662-672, March 2005.

[7] E. Rosnes and O. Ytrehus, "On the design of bit-interleaved turbo-coded modulation with low error floors," *IEEE Trans. Commun.,* vol. 54, pp. 1563-1573, Sept. 2006.

[8] R. D. Maddock and A. H. Banihashemi, "Reliability-based coded modulation with low-density parity-check codes," *IEEE Trans. Commun.,* vol. 54, pp. 403-406, March 2006.

[9] M. C. Valenti and S. Cheng, “Iterative demodulation and decoding of turbo coded M-ary noncoherent orthogonal modulation,” *IEEE J. Select. Areas Commun.*, vol. 23, pp. 1738–1747, Sept. 2005.

[10] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, “A soft-input soft-output APP module for iterative decoding of concatenated codes,” *IEEE Commun. Lett.*, vol. 1, pp. 22–24, Jan. 1997.

[11] S. Pfletschinger and F. Sanzi, "Error floor removal for bit-interleaved coded modulation with iterative detection," *IEEE Trans. Wireless Commun.,* vol. 5, pp. 3174-3181, Sept. 2006.