Iterative Multisymbol Noncoherent Reception of Coded CPFSK

Matthew C. Valenti[1], Shi Cheng[2], and Don Torrieri[3]

[1]West Virginia University

[2]Arraycomm LLC

[2]US Army Research Lab

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Outline

CPFSK: Definition

- *Continuous-phase frequency-shift keying* (CPFSK) is *continuous-phase modulation* (CPM) with a full-response rectangular frequency pulse (LREC-1).
- The transmitted signal for $iT_S \leq t \leq (i+1)T_S$ is:

$$\sqrt{2P}\cos\left[2\pi\left(f_c + \frac{q_i h}{T_s}\right)(t - iT_s) + \phi_i\right]$$

  where:
  - f_c is the carrier frequency.
  - T_s is the symbol duration.
  - h is the modulation index.
  - $q_i \in \{0, ..., M-1\}$ is the information symbol at time i.
  - ϕ_i assures a continuous phase transition from symbol to symbol and is accumulated as:

$$\phi_{i+1} = 2\pi q_i h + \phi_i.$$

    with initial condition $\phi_0 = 0$.
  - $P = \mathcal{E}_s / T_s$ is the power.

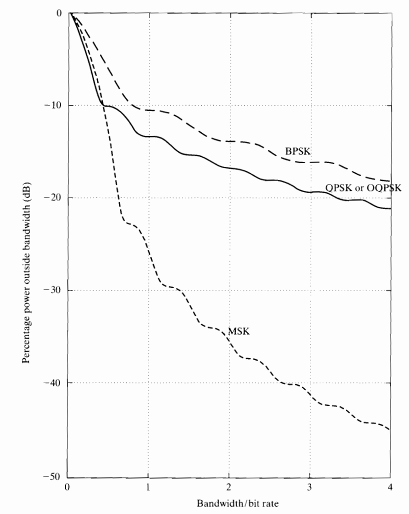
CPFSK: Benefits

The benefits of CPFSK are:

- Constant amplitude (unity PAPR) for efficient amplification.
- Low spectral sidelobes for reduced ACI.
- Suitable for noncoherent reception.

Source of figure:

R.E. Ziemer and R.L. Peterson, *Introduction to Digital Communication*, second edition, Prentice Hall, 2001.



Discrete-time Model

- The received signal is downconverted and passed through a bank of M pairs of matched filters.
- The MF outputs are placed into a vector

$$\mathbf{y} \quad = \quad a e^{j(\phi+\theta)} \sqrt{\mathcal{E}_s} \mathbf{x} + \mathbf{n}$$

where

- The subscript i has been dropped for clarity.
- a is the fading amplitude.
- θ is the phase shift due to fading and oscillator offsets.
- $\mathbf{x}$ is the q^{th} column of the correlation matrix $\mathbf{K}$ with entries:

$$K_{\ell,m} \quad = \quad \frac{\sin(\pi(m-\ell)h)}{\pi(m-\ell)h} e^{j\pi(m-\ell)h}$$

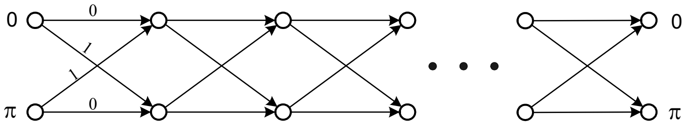
- $\mathbf{n}$ is colored noise, with $E(\mathbf{n}\mathbf{n}^H) = N_0 \mathbf{K}$.

Coherent Detection

- The decoding metric for each postulated symbol $q = \{0, ..., M-1\}$ is

$$p(\mathbf{y}|q, a\sqrt{\mathcal{E}_s}, \psi) \quad \propto \quad \exp\left(2\frac{a\sqrt{\mathcal{E}_s}}{N_0}\mathrm{Re}\left\{e^{-j\psi}y_q\right\}\right)$$

  where $\psi = \theta + \phi$.
- Trellis-based demodulation:
  - If $h = P/Q$, then ML demodulation may be performed on a trellis with Q states.
  - The states represent the integer multiples of $2\pi/Q$.
  - For MSK ($M = 2$ and $h = 1/2$), the trellis is:



Noncoherent Detection

- Assume that ψ is uniform, and marginalize over it.
- The decoding metric for each postulated symbol $q = \{0, ..., M-1\}$ is

$$p(\mathbf{y}|q, a\sqrt{\mathcal{E}_s}) \quad \propto \quad I_0\left(2\frac{a\sqrt{\mathcal{E}_s}}{N_0}|y_q|\right)$$

  where $I_0(\cdot)$ is the 0^{th} order Bessel function of the first kind.
- Note that this metric still requires *channel state information* (CSI) in the form of an accurate estimate of $a\sqrt{\mathcal{E}_s}/N_0$.

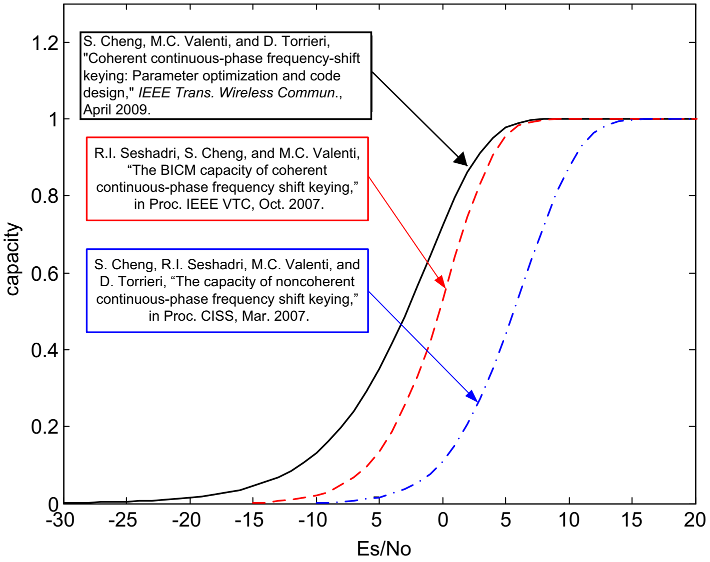
Symmetric Information Rate

- The (average) *mutual information* between the length N channel input $\mathbf{x}_1^N$ and output $\mathbf{y}_1^N$ is:

$$I\left(\mathbf{x}_1^N;\mathbf{y}_1^N\right) = E\left[\log\frac{p\left(\mathbf{x}_1^N,\mathbf{y}_1^N\right)}{p\left(\mathbf{x}_1^N\right)p\left(\mathbf{y}_1^N\right)}\right] \quad (1)$$

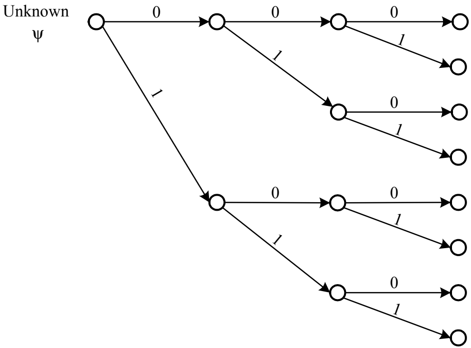
- The *capacity* of the channel is (1) maximized over the input distribution $p\left(\mathbf{x}_1^N\right)$.
- The *symmetric information rate* is (1) under a uniform input distribution.
  - Also called the *i.u.d. capacity* and denoted C.
  - May be estimated through Monte Carlo simulation.
  - Is a bound on performance when a capacity-approaching code is used.

Symmetric Information Rate: MSK



Multi-symbol Noncoherent Detector

- Assume that channel constant for N symbols, and only *initial* phase is unknown.
- Demodulation can be performed over the block of N symbols (Simon and Divsalar 1993).
- The demodulator operates over a tree structure:



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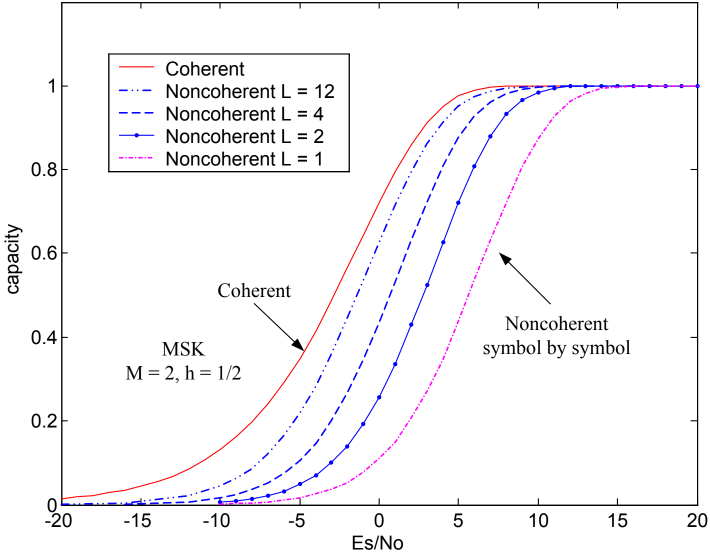
$$p(\mathbf{y}_1^N|\mathbf{q}, a\sqrt{\mathcal{E}_s}) \propto I_0\left(2\frac{a\sqrt{\mathcal{E}_s}}{N_0}|\mu(\mathbf{q})|\right)$$

where

$$\mu(\mathbf{q}) = \sum_{i=1}^{N} y_{q_i} \exp\left\{-2h\pi \sum_{k=1}^{i} q_k\right\}.$$

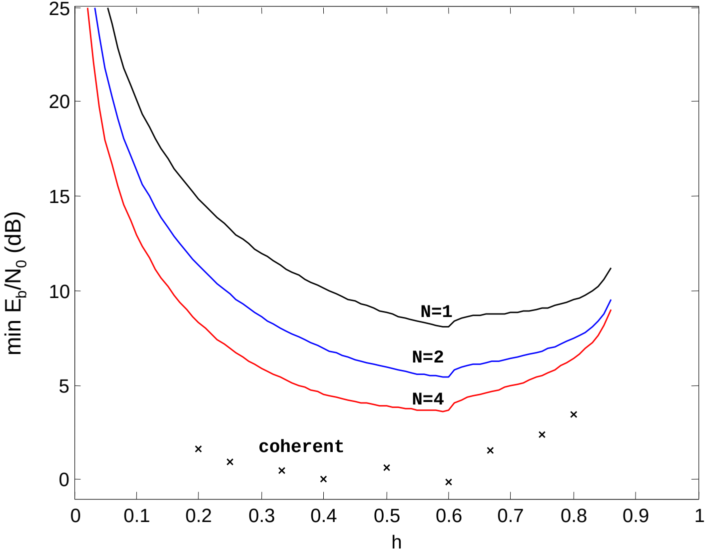
MSK: Multi-symbol Noncoherent vs. Coherent Detection



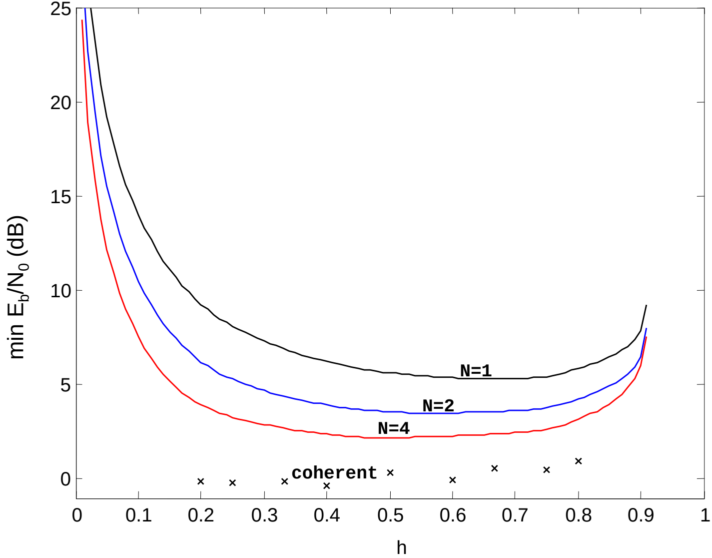
Optimization of R and h

- There is a tradeoff between h and the code rate R.
  - Lower R to increase coding gain.
  - Increase h (up to unity) to decrease inter-tone interference.
  - Increase R or decrease h to improve spectral efficiency.
- If (99% power) bandwidth is constrained, then there will be an *optimal* combination of R and h.
- Optimization procedure:
  1. Pick value of h.
  2. Determine corresponding R such that the BW constraint is satisfied.
  3. Generate a curve showing C as a function of $\mathcal{E}_s/N_0$.
  4. From the curve, determine value of $\mathcal{E}_s/N_0$ such that the $C = R$.
  5. Determine corresponding value of $\mathcal{E}_b/N_0 = (\mathcal{E}_s/N_0)/R$.
  6. Repeat for all feasible h.

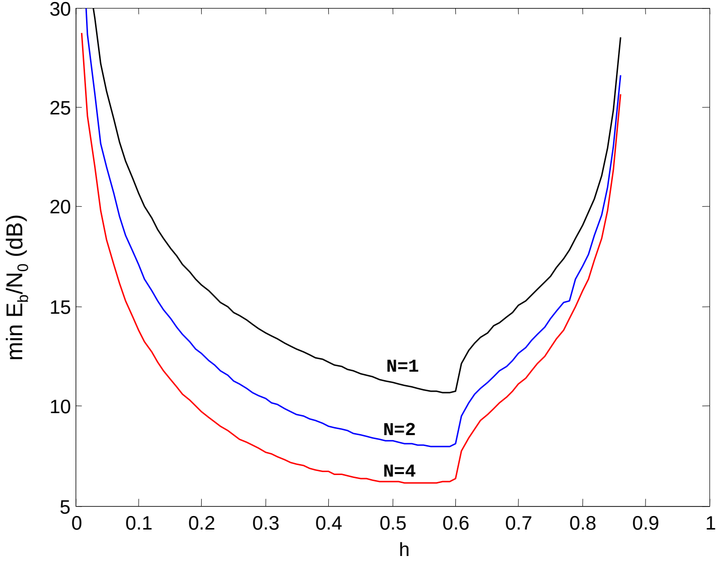
$M = 2$, 2 bps/Hz, AWGN



$M = 4$, 2 bps/Hz, AWGN



$M = 2$, 2 bps/Hz, Rayleigh Fading



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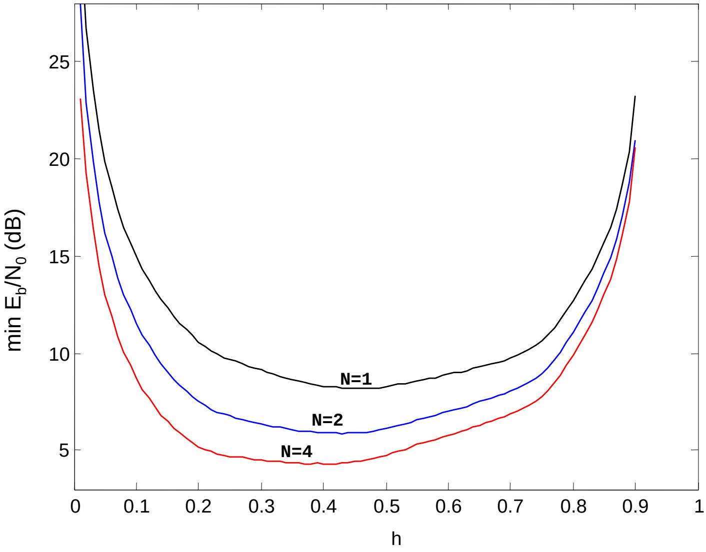
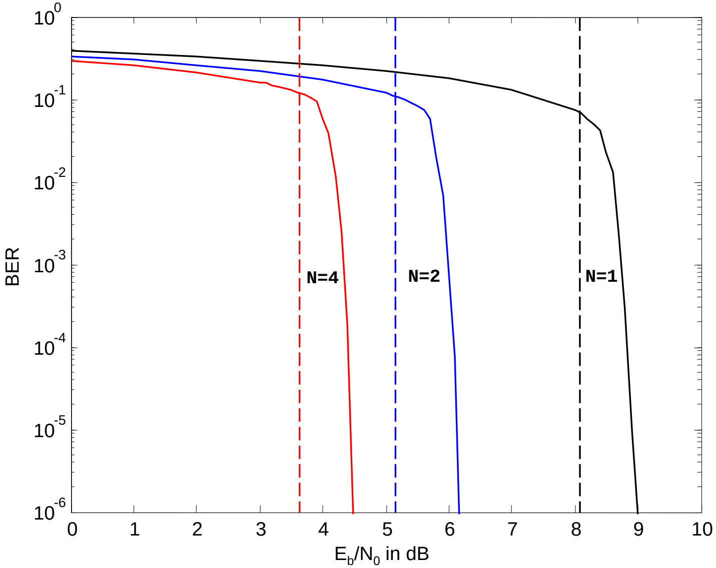


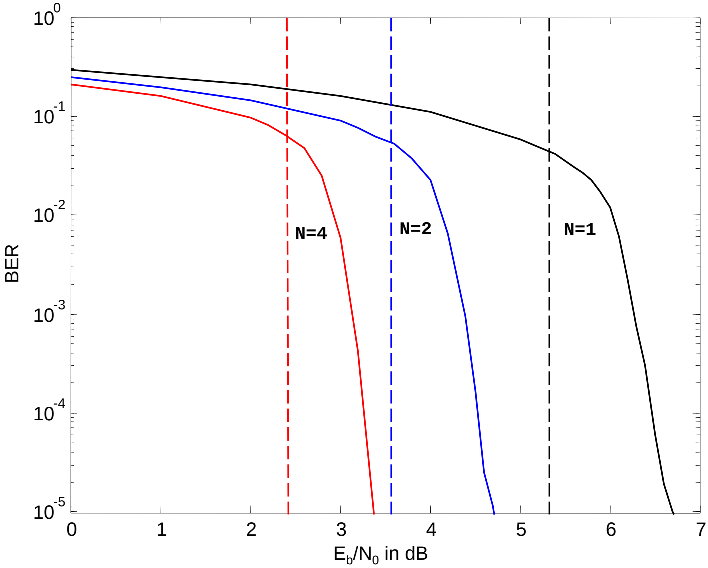
Turbo Coded Performance

- To confirm analytical results, simulated an actual system using the UMTS turbo code.
- Considered $M = \{2, 4\}$ and $N = \{1, 2, 4\}$
- Used parameters optimized for 2 bps/Hz.
- When $M^N > 2$, used iterative demodulation and decoding.

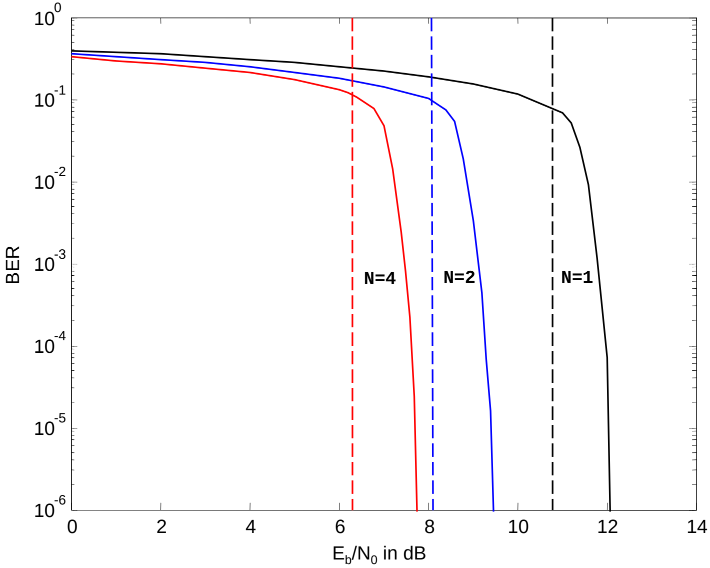
$M = 2$, AWGN


BER
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
0 1 2 3 4 5 6 7 8 9 10
Eb/N0 in dB
N=4
N=2
N=1

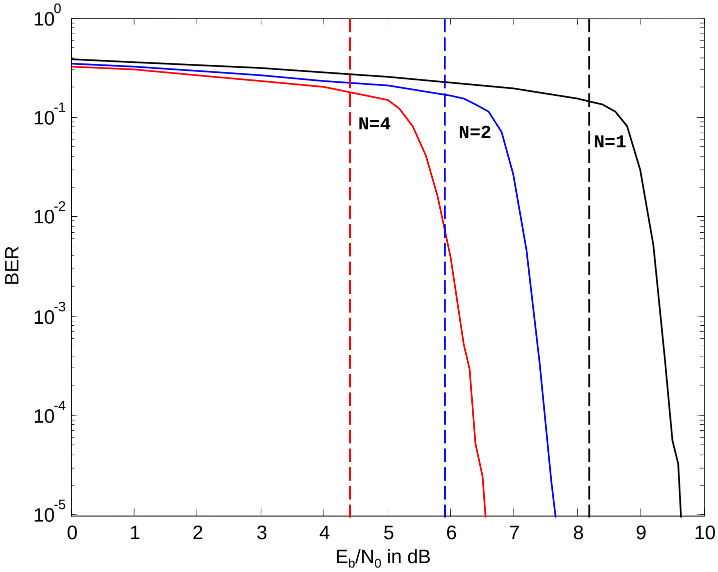
$M = 4$, AWGN



$M = 2$, Rayleigh Fading



$M = 4$, Rayleigh Fading



Conclusion

- Multisymbol noncoherent demodulation is an attractive compromise between coherent demodulation and single-symbol noncoherent demodulation.
- Symmetric-information rate may be used to jointly optimize the code rate R and modulation index h under a bandwidth constraint.
- Performance with an off-the-shelf turbo code is within within 1 or 2 dB of the limit predicted by information-theory.

Thank you