Constellation Shaping for Communication Channels with Quantized Outputs

Chandana Nannapaneni, Dr. Matthew C. Valenti and Xingyu Xiang

Lane Department of Computer Science and Electrical Engineering
West Virginia University

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Outline

1. Introduction
2. Constellation Shaping
3. Quantization
4. Discrete Memoryless Channel
5. Optimization Results
6. Implementation
7. Conclusion

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Transmitter Side Optimization

"Constellation Shaping"
-
Stephane Y. Le Goff
2007 IEEE , T. Wireless

Receiver Side Optimization

"Quantizer Optimization"
-
Jaspreet Singh
2008 ISIT

Our Contribution
-
Joint optimization
CISS 2011

Mutual Information(MI) and Channel Capacity

- MI between two random variables, X and Y is given by,

$$I(X;Y) = E\left[\log\left(\frac{p(Y|X)}{p(Y)}\right)\right]$$

- Channel Capacity is the highest rate at which information can be transmitted over the channel with low error probability. Given the channel and the receiver, capacity is defined as

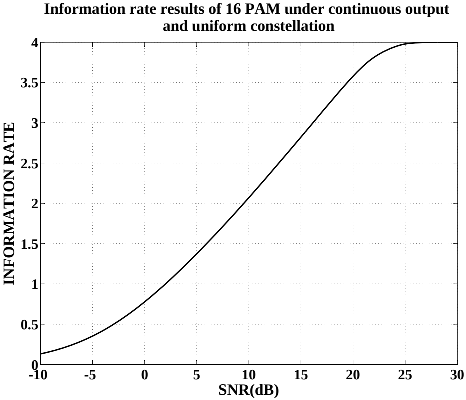
$$C = \max_{p(x)} I(X;Y)$$

The mutual information between output Y and input X is

$$I(X;Y) \;=\; \sum_{j=0}^{M-1} p(x_j) \int p(y|x_j) \log_2 \frac{p(y|x_j)}{p(y)} dy.$$

M - number of input symbols.

This can be solved using Gauss - Hermite Quadratures.



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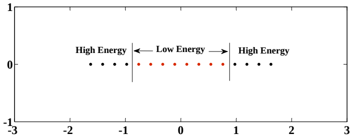
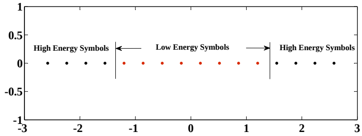
Constellation Shaping for PAM

- Our strategy is from S. LeGoff, IEEE T. Wireless, 2007.
- Transmit low-energy symbols more frequently than high-energy symbols.
- Shaping encoder helps in achieving the desired symbol distribution.
- For a fixed average energy, shaping spreads out the symbols with uniform spacing maintained.



(a) Probability of picking low-energy subconstellation = 0.5



(b) Probability of picking low-energy subconstellation = 0.9

$$(\ \mathcal{E}_s = \Sigma_{i=0}^{M-1} p(x_i)\mathcal{E}_i = 1)$$

Shaping Encoder

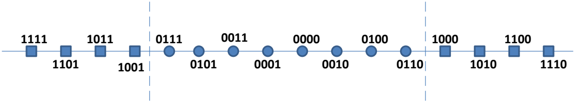
We design the shaping encoder to output more zeros than ones. One example is,

Table: (5,3) shaping code.

3 input bits	5 output bits
0 0 0	0 0 0 0 0
0 0 1	0 0 0 0 1
0 1 0	0 0 0 1 0
0 1 1	0 0 1 0 0
1 0 0	0 1 0 0 0
1 0 1	1 0 0 0 0
1 1 0	0 0 0 1 1
1 1 1	1 0 1 0 0

If p_0, p_1 represents the probability of the shaping encoder giving out a zero and one respectively, then from the table,
$p_0 = \frac{31}{40}$ and $p_1 = \frac{9}{40}$

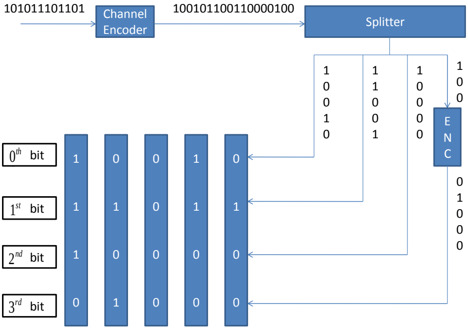
16-PAM Constellation and its symbol-labeling map



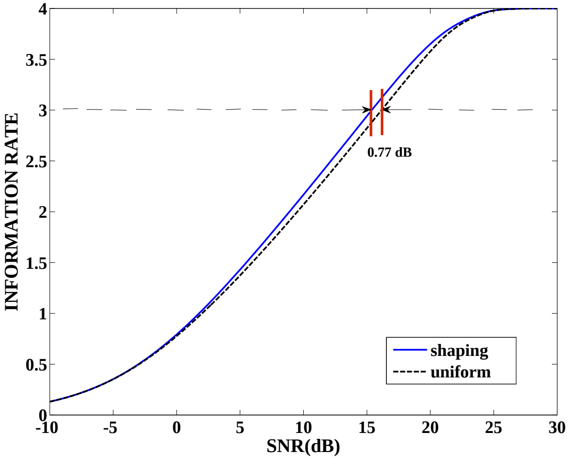
Each symbol is selected with probability $\frac{p_0}{8}$

Each symbol is selected with probability $\frac{1-p_0}{8}$

Shaping Operation



16-PAM results with continuous output optimized over p_0



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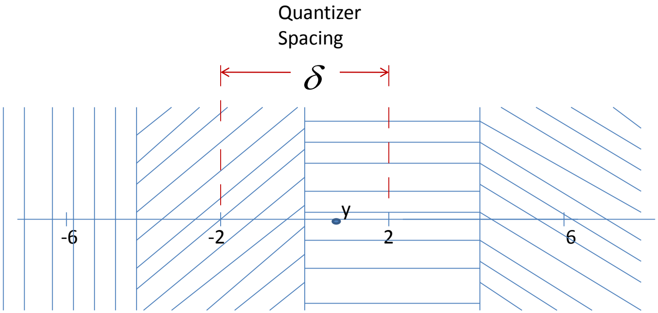
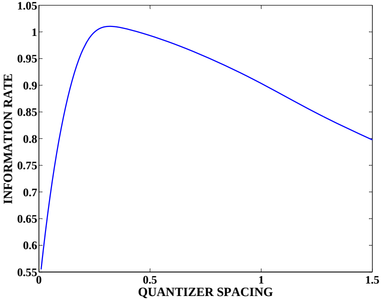
Quantization Basics

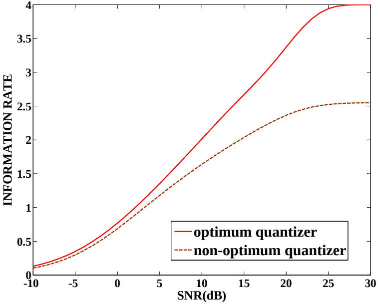
- The output of most communications channels must be *quantized* prior to processing.
- Quantizer
  - approximates its input to one of the predefined levels.
  - results in loss of precision.
- The idea of improving information rate by optimizing the quantizer is from the ISIT 2008 paper by Jaspreet Singh.



Importance of Quantizer Spacing



(c) Information Variation with quantizer spacing at SNR = 10dB, uniformly-distributed inputs and 16 quantization levels



(d) Variation of information rate with SNR under quantizer spacing = 0.1, 16 quantization levels and uniformly-distributed inputs

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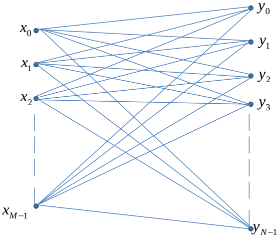
Discrete Memoryless Channel

- AWGN channel with discrete inputs and outputs can be modelled by a DMC.
- Channel described by transition or crossover probabilities.



$$p(y_i|x_j) = \int_{b_i}^{b_{i+1}} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(y-x_j)^2}{2\sigma^2}\right) dy$$

where b_i, b_{i+1} are the boundaries of the quantization region associated with level y_i.

Information Rate Evaluation

For the DMC and one-dimensional modulation,

$$I(X;Y) = \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} p(x_j) p(y_i|x_j) \log_2 \left(\frac{p(y_i|x_j)}{p(y_i)} \right)$$

where,

- $p(y_i)$ is the probability of observing output y_i. For finding $p(y_i)$, we use,

$$p(y_i) = \sum_{j=0}^{M-1} p(y_i|x_j) p(x_j)$$

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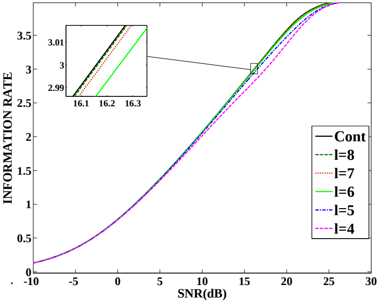
Joint Optimization(Our Contribution)

The variation of information rate with quantizer spacing follows a pattern. We have two values to optimize over, δ and p_0, given the SNR and number of quantization bits($\ell = \log_2(N)$).

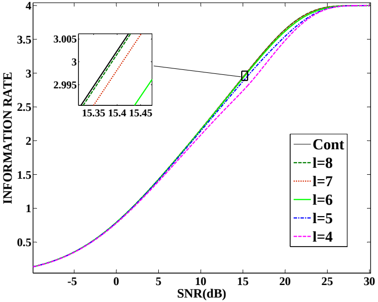
The algorithm used is,

1. Fix the SNR and number of quantization bits(ℓ).
2. Vary p_0 from 0.5 to 0.99 in increments of 0.005.
3. For each value of p_0, find the optimum quantizer spacing and compute the corresponding information rate.
4. By the end of step 3, we have an array of information rate values. We then go over the array and find the highest information rate that can be achieved, and the combination of p_0 and δ that will produce it.

Capacity Results

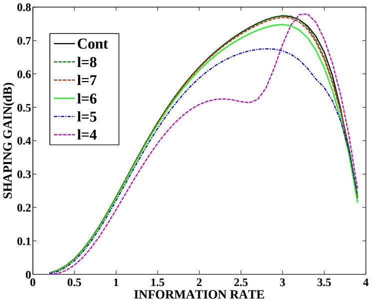


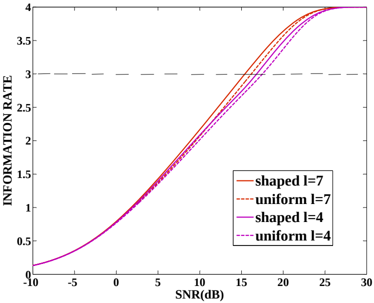
(e) Uniform Distribution



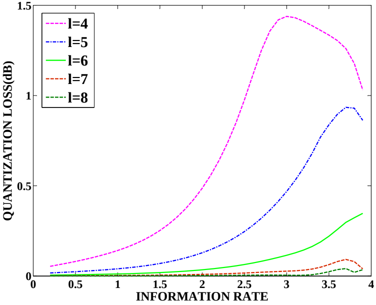
(f) Shaped Distribution

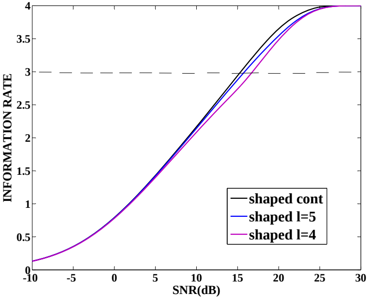
Shaping Gain and its evaluation





Quantization Loss and its evaluation





(i) Shaped Distribution

Quantization Loss

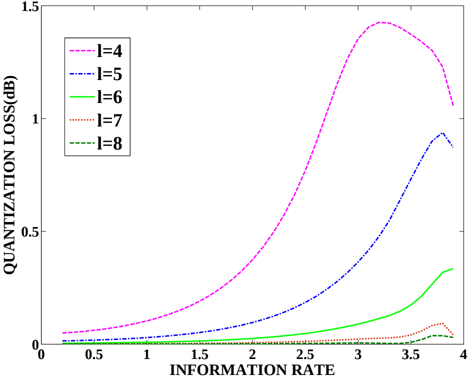
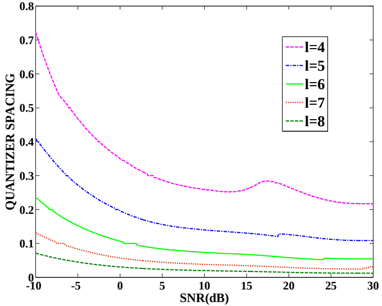
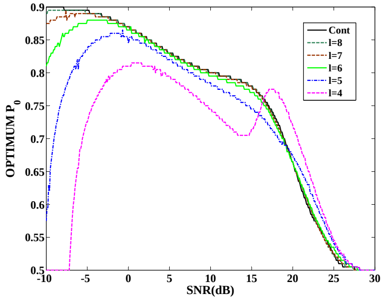


Figure: Uniform Distribution

Optimization Results



(a) Optimal Quantizer Spacing



(b) Optimal p_0 (probability of selecting lower-energy subconstellation)

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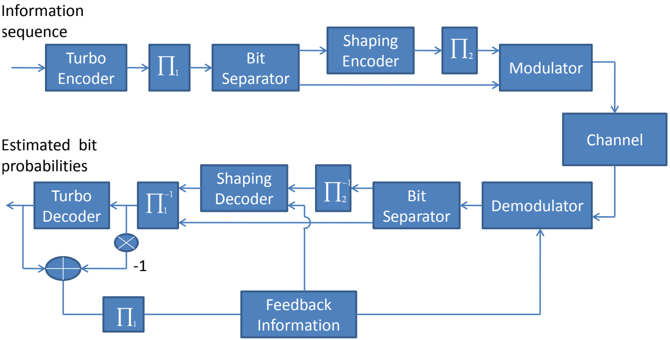
Implementation

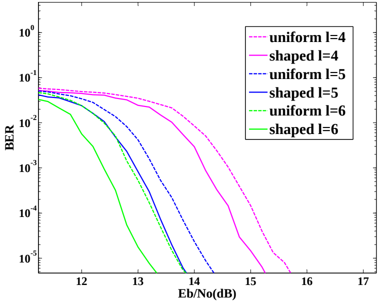
BICM - ID System



BER Curves

Table: Parameters used for simulation of a 16PAM system.

	R	r_c	r_s	Quantizer Spacing				
				$\ell = 8$	$\ell = 7$	$\ell = 6$	$\ell = 5$	$\ell = 4$
Uniform	2.9940	2000/2672	1	0.0139	0.0272	0.0534	0.1058	0.2165
Shaping	2.9836	2000/2479	7/10	0.0172	0.0339	0.0670	0.1270	0.2781



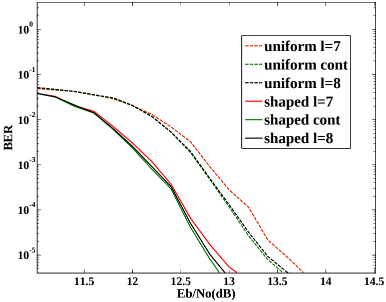


Table: SNR required by 16PAM at rate $R = 3$.

		$\mathcal{E}_b/N_0$ in dB					
		Cont	$\ell = 8$	$\ell = 7$	$\ell = 6$	$\ell = 5$	$\ell = 4$
Theoretical	Uniform	11.387	11.393	11.410	11.476	11.751	12.74
	Shaping	10.613	10.618	10.640	10.728	11.082	12.051
Actual	Uniform	13.378	13.394	13.585	13.703	14.184	15.524
	Shaping	12.789	12.811	12.922	13.154	13.742	15.112

The $\mathcal{E}_b/N_0$ values in actual case are taken at BER $= 10^{-5}$

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Conclusion

- The simple constellation-shaping strategy considered in this paper can achieve shaping gains of over 0.7 dB.
- When a finite-resolution quantizer is used, there will necessarily be a quantization loss. The loss can be minimized by using an optimal quantizer spacing.
- When properly optimized, a resolution of 8 bits is sufficient to provide performance that is very close to that of an unquantized system.
- This work can be further extended to a more-complex two-dimensional modulations like 16-APSK and to higher-dimensional modulations.
- Using a non-uniform vector quantizer can also be investigated for such systems.