Noncoherent Digital Network Coding using M-ary CPFSK Modulation

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System Model

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System Model

Digital Network Coding Relay Receiver
  Matched Filter Output Distributions
    Coherent Reception
    Noncoherent Reception with CSI
    Noncoherent Reception without CSI
  DNC Soft-Demapper
  Network Coding Module

Simulation Study
  Error-rate performance without an error-correcting code
  Error-rate performance with outer Turbo code
  Throughput comparison - DNC and LNC

Conclusion
Network coding is a high-throughput relaying technique which increases throughput over store-and-forward relaying.

Network coding may be implemented at the link or physical layer.

- Using link-layer network coding (LNC), received symbols are combined after performing demodulation and detection.

- Using physical-layer network coding (PNC) the network coding is performed on the received sum of electromagnetic signals.

- Digital network coding (DNC) is an instance of PNC in which the relay performs network coding during demodulation and detection.

![Two-way Relay Channel Diagram]
LNC requires three time slots for relaying.

PNC only requires two.
The primary contribution of this work is a soft-output $M$-ary CPFSK demodulator implementing DNC, and a throughput comparison against LNC. Previous work $^1$ considered binary CPFSK.

CPFSK is an attractive modulation for applications in which coherent demodulation is not practical.

Simulated error-rate performance is presented for modulation orders 2 and 4.

Increasing the modulation order from 2 to 4 provides a higher data rate at the same spectral efficiency, with improved energy efficiency.

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Discrete-time system model under DNC operation
Considering the MAC phase,

- A length-$K$ information sequence is generated at each end node.
- When no channel code is applied,
  - The information sequence is divided into $K/\mu$ sets of bits, mapped to $M$-ary CPFSK symbols, and transmitted to the relay, where $\mu = \log_2 M$.
- When a channel code is applied,
  - Identical Turbo channel codes are applied to the information sequences at rate is $r_S$.
  - The codeword is divided into $N_c/\mu$ sets of bits, mapped to $M$-ary CPFSK symbols, and transmitted to the relay, where $\mu = \log_2 M$.

- Under LNC, the end nodes transmit to the relay in separate time slots, while under DNC, the end nodes transmit simultaneously.

- All channels are modeled as flat-fading channels with independent gains for every signaling interval.

- The broadcast phase contains conventional point-to-point links, and is not analyzed in this work.
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Consider a single pair of symbols transmitted by the end nodes, \( q_1 \) by \( \mathcal{N}_1 \) and \( q_2 \) by \( \mathcal{N}_2 \), where \( q_1, q_2 \in \{0, ..., M - 1\} \).

The vector model of the received signal at the relay is

\[
y = h_1 x_1 + h_2 x_2 + n
\]

where \( h_1 = \alpha_1 e^{j\phi_1} \) and \( h_2 = \alpha_2 e^{j\phi_2} \) are complex-valued channel gains, \( x_1 \) and \( x_2 \) are the vector representations of \( q_1 \) and \( q_2 \), and \( n \) is circularly-symmetric complex Gaussian noise.

We desire the expressions:

\[
\Lambda(b_k) = \log \left[ \frac{P(b_k = 1|y)}{P(b_k = 0|y)} \right], \quad k \in \{0, ..., \mu - 1\}
\]

where \( \Lambda(b_k) \) is the log-likelihood ratio of the network coded bit \( b_k = b_{k,1} \oplus b_{k,2} \), and \( b_{k,1} \) and \( b_{k,2} \), are the \( k \)-th bit of each symbol.
Computation of the log-likelihood ratio of the network coded bit at the relay is broken into three sub-computations,

- Probability of the received signal conditioned on the symbols transmitted by the end nodes and channel information.

- Probability of the received signal conditioned on the pair of bits mapped to the {$k^{th}$} position of the received symbols.

- Log-likelihood ratios of the network-coded bits.

\[
\Lambda(b_k) = p(y|b_{k,1}, b_{k,2}) \prod P[b_{k,1}] P[b_{k,2}]
\]
The pdf of the received signal at the relay under coherent reception is

\[ p(y|m_{i,j}) = \left( \frac{1}{\pi N_0} \right)^M \exp \left\{ -\frac{1}{N_0} \|y - m_{i,j}\|^2 \right\} \]

where the means are defined as

\[ m_{i,j} = h_1 x_1 + h_2 x_2 \quad i, j \in \{0, ..., M - 1\} \]

and the subscripts \( i, j \) denote the transmission of symbol \( q_1 = i \) by \( \mathcal{N}_1 \) and \( q_2 = j \) by \( \mathcal{N}_2 \).
When the phases of the fading coefficients are unknown at the relay (partial CSI), the conditional pdf of the received signal becomes

\[ p(y|\mu_{i,j}) = \int_0^{2\pi} \int_0^{2\pi} p(\phi_i, \phi_j)p(y|m_{i,j})d\phi_id\phi_j \]

Where \( \mu_{i,j} = |m_{i,j}| \), and the phases are uniformly distributed.

When the end nodes transmit different tones,

\[ p(y|\mu_{i,j}) = \exp \left\{ -\frac{\alpha_1^2 + \alpha_2^2}{N_0} \right\} I_0 \left( \frac{2|y_i|\alpha_1}{N_0} \right) I_0 \left( \frac{2|y_j|\alpha_2}{N_0} \right) \]

When the end nodes transmit the same tone,

\[ p(y|\mu_{i,j}) = \exp \left\{ -\frac{\alpha^2}{N_0} \right\} I_0 \left( \frac{2|y_i|\alpha}{N_0} \right) \]

\[ -\frac{\alpha^2}{N_0} \]
When the phases and fading amplitudes are not known at the relay (no CSI), and the sources transmit different tones, the conditional pdf of the received signal becomes

\[
p(y|E_1, E_2) = \int_0^{2\pi} \int_0^{2\pi} p(\alpha_1, \alpha_2) p(y|\mu_{i,j}) d\alpha_1 d\alpha_2
\]

where \(E_i\) is the symbol energy utilized at end node \(N_i\).

And the joint pdf of the fading amplitudes \(\alpha_1, \alpha_2\) is

\[
p(\alpha_1, \alpha_2) = \left( \frac{2\alpha_1}{E_1} \exp \left\{ -\frac{\alpha_1^2}{E_1} \right\} \right) \left( \frac{2\alpha_2}{E_2} \exp \left\{ -\frac{\alpha_2^2}{E_2} \right\} \right)
\]
When the phases and fading amplitudes are not known at the relay, and the sources transmit the same tones, the conditional pdf of the received signal becomes

\[ p(y|\mathcal{E}_1, \mathcal{E}_2) = \int_0^{2\pi} p(\alpha)p(y|\mu_{i,j})d\alpha \]

And the joint pdf of the fading amplitude \( \alpha \) is

\[ p(\alpha) = \frac{2\alpha}{\mathcal{E}_1 + \mathcal{E}_2} \exp\left\{ -\frac{\alpha^2}{\mathcal{E}_1 + \mathcal{E}_2} \right\} \]
When the sources transmit the same tone,

\[ p(y|\mathcal{E}_1, \mathcal{E}_2) = \left( \frac{1}{\mathcal{E}_1 + \mathcal{E}_2} \right) \left( \frac{1}{\mathcal{E}_1 + \mathcal{E}_2} + \frac{1}{N_0} \right)^{-1} \exp \left\{ \frac{|y_i|^2(\mathcal{E}_1 + \mathcal{E}_2)}{N_0^2 + N_0(\mathcal{E}_1 + \mathcal{E}_2)} \right\} \]

When the sources transmit different tones,

\[ p(y|\mathcal{E}_1, \mathcal{E}_2) = \left[ \left( \frac{1}{\mathcal{E}_1 \mathcal{E}_2} \right) \left( \frac{1}{\mathcal{E}_1} + \frac{1}{N_0} \right) \left( \frac{1}{\mathcal{E}_2} + \frac{1}{N_0} \right) \right]^{-1} \exp \left\{ \frac{|y_i|^2 \mathcal{E}_1}{N_0(N_0 + \mathcal{E}_1)} + \frac{|y_j|^2 \mathcal{E}_2}{N_0(N_0 + \mathcal{E}_2)} \right\} \]
The soft demapper stage computes the probabilities of the received signal conditioned on the $k^{th}$ bit of the received symbols.

The soft mapper takes two inputs,

1. The set of received signal probabilities conditioned on all possible combinations of received symbols,
\[
\{p(y|q_1, q_2) : (q_1, q_2) \in D \times D\}
\]

where $D$ is the set of all possible CPFSK symbols.

2. The set of $a$-priori probabilities of the code bits transmitted by the sources, excluding the $k^{th}$ bit
\[
P[b(q_1) \setminus b_k(q_1)]P[b(q_2) \setminus b_k(q_2)]
\]

where the function $b(q_i)$ selects all code bits associated with symbol $q_i$, and $b_k(q_i)$ selects the $k^{th}$ bit associated symbol $q_i$. 
The output of the soft demapper is the set of received signal probabilities conditioned on the bits transmitted by the sources

\[ \{ p(y|b_{k,1}, b_{k,2}) : (b_{k,1}, b_{k,2}) \in \mathcal{B} \times \mathcal{B} \} \]

where \( \mathcal{B} \) the set of bits \( \{0, 1\} \).

The pdf of the received signal conditioned on the \( k \)-th bit of the received symbols is

\[
p(y|b_{k,1} = m, b_{k,2} = n) = \sum_{q_1 : b_k(q_1) = m, q_2 : b_k(q_2) = n} p(y|q_1, q_2) P[b_1(q_1) \setminus b_k(q_1)] P[b_2(q_2) \setminus b_k(q_2)]
\]
Applying Bayes’ rule to the output probabilities of the soft demapper,

\[
P(b_{k,1}, b_{k,2}|y) = \frac{p(y|b_{k,1}, b_{k,2})P(b_{k,1})P(b_{k,2})}{p(y)}
\]

\[
(b_{k,1}, b_{k,2}) \in \mathcal{B} \times \mathcal{B}
\]

Denote all possible combinations of bits transmitted by the end nodes as

- \( E_1 = \{b_{k,1} = 0, b_{k,2} = 0\} \)
- \( E_2 = \{b_{k,1} = 1, b_{k,2} = 1\} \)
- \( E_3 = \{b_{k,1} = 0, b_{k,2} = 1\} \)
- \( E_4 = \{b_{k,1} = 1, b_{k,2} = 0\} \).

The log-likelihood ratio of the network coded bit is then expressed as

\[
\Lambda(b_k) = \log \left[ \frac{P(y|E_3)P(E_3) + P(y|E_4)P(E_4)}{P(y|E_1)P(E_1) + P(y|E_2)P(E_2)} \right]
\]
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Conclusion
This section contains simulated error-rate performance at the relay, and end-to-end throughput performance at the end nodes.

Error-rate performance is shown for detection of the network-coded bit at the relay

1. For DNC and LNC.
2. With and without Turbo channel coding.
3. For varying levels of channel state information at the relay.

In all simulation cases, the end nodes generate frames containing $K = 4500$ information bits.

The throughput of digital and link-layer network coding is compared.
Uncoded error-rate performance at the relay.
Coded error-rate performance at the relay using Turbo code rate $r_S = 4500/5000$. 
The throughput of DNC and LNC is compared by selecting channel code rates which equalize error performance for both systems.

The LNC system requires 2 time slots during the MAC phase to transmit $2K$ information bits to the relay, using length $N_L = 5000$ code bits at each end node.

The DNC system requires a single time slot during the MAC phase to transfer $2K$ information bits, using length $N_D$ code bits at each end node.

Both systems use $N_B = 5000$ channel code bits in the broadcast phase.

The proportional throughput increase $T_I$ of DNC over LNC is thus

$$T_I = \frac{2K/(N_D + N_B)}{2K/(2N_L + N_B)} = \frac{15000}{N_D + 5000}$$  

(1)
Coded error-rate performance used to compare DNC and LNC throughput, assuming no channel state information is available.
Coded error-rate performance used to compare DNC and LNC throughput, assuming partial channel state information is available.
The following table summarizes the throughput improvement of DNC over LNC.

<table>
<thead>
<tr>
<th>CSI</th>
<th>$M=2$</th>
<th>$M=4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>30.4%</td>
<td>32.7%</td>
</tr>
<tr>
<td>Partial</td>
<td>37.1%</td>
<td>41.0%</td>
</tr>
</tbody>
</table>

**Table:** Throughput Improvement - DNC over LNC
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This work presents a soft-output detector which implements DNC in the two-way relay channel.

Simulated error-rate and throughput performance for a system which utilizes DNC and LNC, 2 and 4-ary CPFSK modulation, Turbo channel coding, and a fully-interleaved Rayleigh fading channel model.

Increasing CPFSK modulation order from 2 to 4 improves DNC energy efficiency by $1 - 2$ dB, and decreases the energy efficiency gap between DNC and LNC by 1 dB.

DNC increases throughput over LNC by at least 30%, using 2-ary modulation and no channel state information, and by 41%, using 4-ary modulation and partial channel state information.

Potential avenues for future work include design of techniques to synchronize the frames transmitted by the end nodes, and implementation in a software radio platform.
Thank You!