Noncoherent Digital Network Coding using M-ary CPFSK Modulation

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Outline

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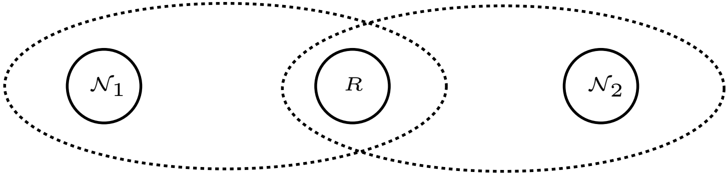
- *Network coding* is a high-throughput relaying technique which increases throughput over store-and-forward relaying.

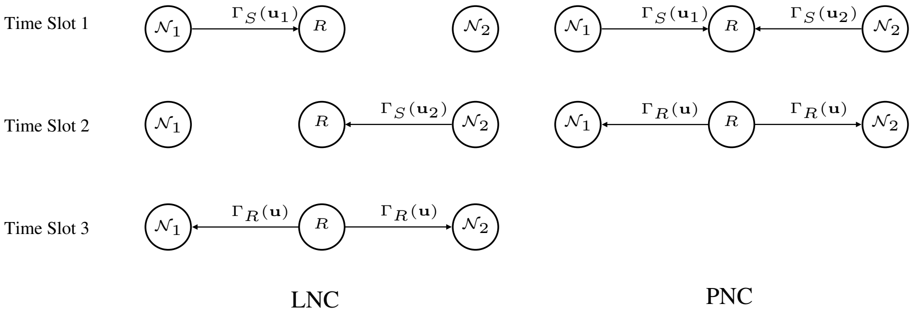
- Network coding may be implemented at the *link* or *physical* layer.
  - Using *link-layer* network coding (LNC), received symbols are combined after performing demodulation and detection.

  - Using *physical-layer network coding* (PNC) the network coding is performed on the received sum of electromagnetic signals.

  - *Digital* network coding (DNC) is an instance of PNC in which the relay performs network coding during demodulation and detection.



Two-way Relay Channel



- LNC requires three time slots for relaying.
- PNC only requires two.

- The primary contribution of this work is a soft-output M-ary CPFSK demodulator implementing DNC, and a throughput comparison against LNC. Previous work [1] considered binary CPFSK.
- CPFSK is an attractive modulation for applications in which coherent demodulation is not practical.
- Simulated error-rate performance is presented for modulation orders 2 and 4.
- Increasing the modulation order from 2 to 4 provides a higher data rate at the same spectral efficiency, with improved energy efficiency.

[1] M. C. Valenti, D. Torrieri, and T. Ferrett, "Noncoherent physical-layer network coding with FSK Modulation: Relay Receiver Design Issues," *IEEE Trans. Commun.*, Sept. 2011.

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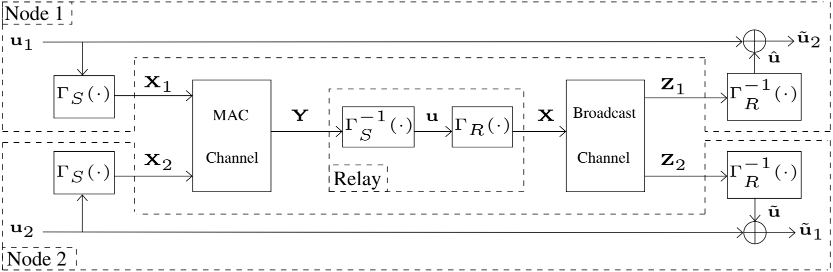


Discrete-time system model under DNC operation

- Considering the MAC phase,
  - A length-K information sequence is generated at each end node.
  - When no channel code is applied,
    - The information sequence is divided into K/μ sets of bits, mapped to M-ary CPFSK symbols, and transmitted to the relay, where $\mu = \log_2 M$.
  - When a channel code is applied,
    - Identical Turbo channel codes are applied to the information sequences at rate is r_S.
    - The codeword is divided into N_c/μ sets of bits, mapped to M-ary CPFSK symbols, and transmitted to the relay, where $\mu = \log_2 M$.
- Under LNC, the end nodes transmit to the relay in separate time slots, while under DNC, the end nodes transmit simultaneously.
- All channels are modeled as flat-fading channels with independent gains for every signaling interval.
- The broadcast phase contains conventional point-to-point links, and is not analyzed in this work.

Outline

- Consider a single pair of symbols transmitted by the end nodes, q_1 by $\mathcal{N}_1$ and q_2 by $\mathcal{N}_2$, where $q_1, q_2 \in \{0, ..., M-1\}$.

- The vector model of the received signal at the relay is

$$\mathbf{y} = h_1\mathbf{x}_1 + h_2\mathbf{x}_2 + \mathbf{n}$$

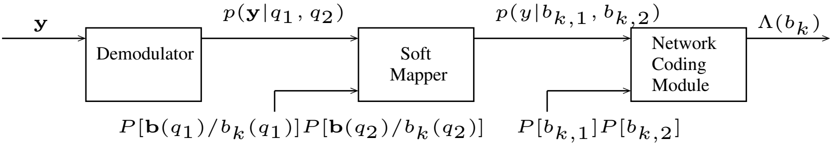
- where $h_1 = \alpha_1 e^{j\phi_1}$ and $h_2 = \alpha_2 e^{j\phi_2}$ are complex-valued channel gains, $\mathbf{x}_1$ and $\mathbf{x}_2$ are the vector representations of q_1 and q_2, and $\mathbf{n}$ is circularly-symmetric complex Gaussian noise.

- We desire the expressions:

$$\Lambda(b_k) = \log\left[\frac{P(b_k = 1|\mathbf{y})}{P(b_k = 0|\mathbf{y})}\right], \; k \in \{0, ..., \mu - 1\}$$

- where $\Lambda(b_k)$ is the log-likelihood ratio of the network coded bit $b_k = b_{k,1} \oplus b_{k,2}$, and $b_{k,1}$ and $b_{k,2}$, are the k-th bit of each symbol.

- Computation of the log-likelihood ratio of the network coded bit at the relay is broken into three sub-computations,
  - Probability of the received signal conditioned on the symbols transmitted by the end nodes and channel information.
  - Probability of the received signal conditioned on the pair of bits mapped to the k^{th} position of the received symbols.
  - Log-likelihood ratios of the network-coded bits.



Relay Receiver Block Diagram

- The pdf of the received signal at the relay under coherent reception is

$$p(\mathbf{y}|\mathbf{m}_{i,j}) = \left(\frac{1}{\pi N_0}\right)^M \exp\left\{-\frac{1}{N_0}||\mathbf{y}-\mathbf{m}_{i,j}||^2\right\}$$

- where the means are defined as

$$\mathbf{m}_{i,j} = h_1\mathbf{x}_1 + h_2\mathbf{x}_2 \quad i,j \in \{0,...,M-1\}$$

- and the subscripts i,j denote the transmission of symbol $q_1 = i$ by $\mathcal{N}_1$ and $q_2 = j$ by $\mathcal{N}_2$.

- When the phases of the fading coefficients are unknown at the relay (partial CSI), the conditional pdf of the received signal becomes

$$p(\mathbf{y}|\mu_{i,j}) = \int_0^{2\pi}\int_0^{2\pi} p(\phi_i,\phi_j)p(\mathbf{y}|\mathbf{m}_{i,j})d\phi_i d\phi_j$$

- Where $\mu_{i,j} = |\mathbf{m}_{i,j}|$, and the phases are uniformly distributed.

- When the end nodes transmit different tones,

$$p(\mathbf{y}|\mu_{i,j}) = \exp\left\{-\frac{\alpha_1^2+\alpha_2^2}{N_0}\right\} I_0\left(\frac{2|y_i|\alpha_1}{N_0}\right) I_0\left(\frac{2|y_j|\alpha_2}{N_0}\right)$$

- When the end nodes transmit the same tone,

$$p(\mathbf{y}|\mu_{i,j}) = \exp\left\{-\frac{\alpha^2}{N_0}\right\} I_0\left(\frac{2|y_i|\alpha}{N_0}\right)$$

- When the phases and fading amplitudes are not known at the relay (no CSI), and the sources transmit different tones, the conditional pdf of the received signal becomes

$$p(\mathbf{y}|\mathcal{E}_1,\ \mathcal{E}_2) = \int_0^{2\pi}\int_0^{2\pi} p(\alpha_1,\alpha_2)p(\mathbf{y}|\mu_{i,j})d\alpha_1 d\alpha_2$$

  - where $\mathcal{E}_i$ is the symbol energy utilized at end node $\mathcal{N}_i$.

- And the joint pdf of the fading amplitudes α_1, α_2 is

$$p(\alpha_1,\alpha_2) = \left(\frac{2\alpha_1}{\mathcal{E}_1}\exp\left\{-\frac{\alpha_1^2}{\mathcal{E}_1}\right\}\right)\left(\frac{2\alpha_2}{\mathcal{E}_2}\exp\left\{-\frac{\alpha_2^2}{\mathcal{E}_2}\right\}\right)$$

- When the phases and fading amplitudes are not known at the relay, and the sources transmit the same tones, the conditional pdf of the received signal becomes

$$p(\mathbf{y}|\mathcal{E}_1,\ \mathcal{E}_2) = \int_0^{2\pi} p(\alpha)p(\mathbf{y}|\mu_{i,j})d\alpha$$

- And the joint pdf of the fading amplitude α is

$$p(\alpha) = \frac{2\alpha}{\mathcal{E}_1 + \mathcal{E}_2} \exp\left\{-\frac{\alpha^2}{\mathcal{E}_1 + \mathcal{E}_2}\right\}$$

- When the sources transmit the same tone,

$$p(\mathbf{y}|\mathcal{E}_1, \mathcal{E}_2) = \left(\frac{1}{\mathcal{E}_1 + \mathcal{E}_2}\right)\left(\frac{1}{\mathcal{E}_1 + \mathcal{E}_2} + \frac{1}{N_0}\right)^{-1} \exp\left\{\frac{|y_i|^2(\mathcal{E}_1 + \mathcal{E}_2)}{N_0^2 + N_0(\mathcal{E}_1 + \mathcal{E}_2)}\right\}$$

- When the sources transmit different tones,

$$p(\mathbf{y}|\mathcal{E}_1, \mathcal{E}_2) = \left[\left(\frac{1}{\mathcal{E}_1\mathcal{E}_2}\right)\left(\frac{1}{\mathcal{E}_1} + \frac{1}{N_o}\right)\left(\frac{1}{\mathcal{E}_2} + \frac{1}{N_0}\right)\right]^{-1} \exp\left\{\frac{|y_i|^2\mathcal{E}_1}{N_o(N_0 + \mathcal{E}_1)} + \frac{|y_j|^2\mathcal{E}_2}{N_0(N_0 + \mathcal{E}_2)}\right\}$$

- The soft demapper stage computes the probabilities of the received signal conditioned on the k^{th} bit of the received symbols.

- The soft mapper takes two inputs,
  1. The set of received signal probabilities conditioned on all possible combinations of received symbols,

$$\{p(\mathbf{y}|q_1, q_2) : (q_1,\ q_2) \in \mathcal{D} \times \mathcal{D}\}$$

    - where $\mathcal{D}$ is the set of all possible CPFSK symbols.
  2. The set of *a-priori* probabilities of the code bits transmitted by the sources, excluding the k^{th} bit

$$P[\mathbf{b}(q_1)\backslash b_k(q_1)]P[\mathbf{b}(q_2)\backslash b_k(q_2)]$$

- where the function $\mathbf{b}(q_i)$ selects all code bits associated with symbol q_i, and $b_k(q_i)$ selects the k^{th} bit associated symbol q_i.

- The output of the soft demapper is the set of received signal probabilities conditioned on the bits transmitted by the sources

$$\{p(\mathbf{y}|b_{k,1}, b_{k,2}) : (b_{k,1}, b_{k,2}) \in \mathcal{B} \times \mathcal{B}\}$$

  - where $\mathcal{B}$ the set of bits $\{0, 1\}$.

- The pdf of the received signal conditioned on the k-th bit of the received symbols is

$$p(\mathbf{y}|b_{k,1} = m, b_{k,2} = n) = \sum_{\substack{q_1 : b_k(q_1) = m \\ q_2 : b_k(q_2) = n}} p(\mathbf{y}|q_1, q_2) P[\mathbf{b}_1(q_1) \backslash b_k(q_1)] P[\mathbf{b}_2(q_2) \backslash b_k(q_2)]$$

- Applying Bayes' rule to the output probabilities of the soft demapper,

$$P(b_{k,1}, b_{k,2}|\mathbf{y}) = \frac{p(\mathbf{y}|b_{k,1}, b_{k,2})P(b_{k,1})P(b_{k,2})}{p(\mathbf{y})}$$

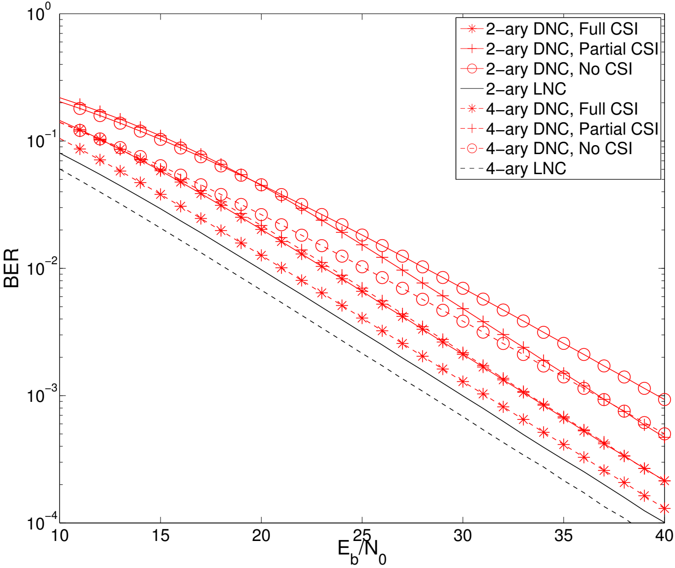
$$(b_{k,1}, b_{k,2}) \in \mathcal{B} \times \mathcal{B}$$

- Denote all possible combinations of bits transmitted by the end nodes as
  - $\mathbb{E}_1 = \{b_{k,1} = 0, b_{k,2} = 0\}$
  - $\mathbb{E}_2 = \{b_{k,1} = 1, b_{k,2} = 1\}$
  - $\mathbb{E}_3 = \{b_{k,1} = 0, b_{k,2} = 1\}$
  - $\mathbb{E}_4 = \{b_{k,1} = 1, b_{k,2} = 0\}$.
- The log-likelihood ratio of the network coded bit is then expressed as

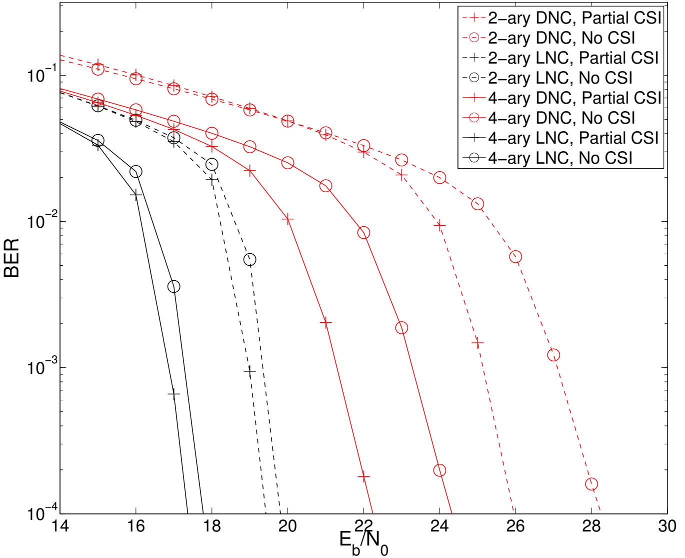
$$\Lambda(b_k) = \log\left[\frac{P(\mathbf{y}|\mathbb{E}_3)P(\mathbb{E}_3) + P(\mathbf{y}|\mathbb{E}_4)P(\mathbb{E}_4)}{P(\mathbf{y}|\mathbb{E}_1)P(\mathbb{E}_1) + P(\mathbf{y}|\mathbb{E}_2)P(\mathbb{E}_2)}\right]$$

Outline

- This section contains simulated error-rate performance at the relay, and end-to-end throughput performance at the end nodes.
- Error-rate performance is shown for detection of the network-coded bit at the relay
  1. For DNC and LNC.
  2. With and without Turbo channel coding.
  3. For varying levels of channel state information at the relay.
- In all simulation cases, the end nodes generate frames containing $K = 4500$ information bits.
- The throughput of digital and link-layer network coding is compared.



Uncoded error-rate performance at the relay.



Coded error-rate performance at the relay using Turbo code rate $r_S = 4500/5000$.

- The throughput of DNC and LNC is compared by selecting channel code rates which equalize error performance for both systems.

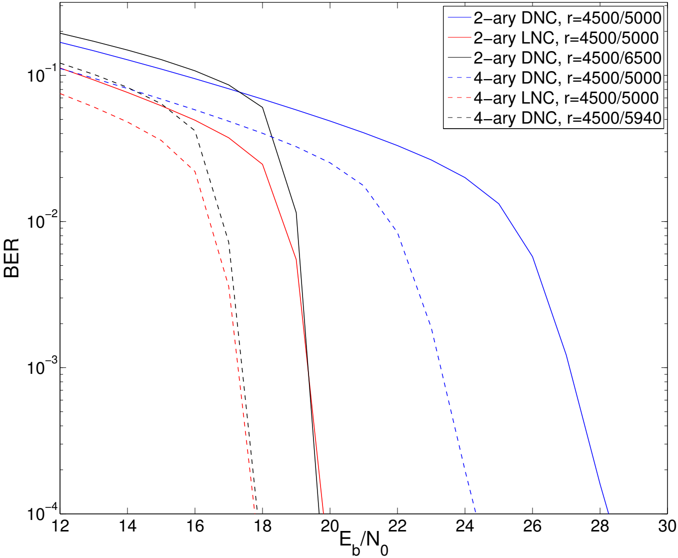
- The LNC system requires 2 time slots during the MAC phase to transmit $2K$ information bits to the relay, using length $N_L = 5000$ code bits at each end node.

- The DNC system requires a single time slot during the MAC phase to transfer $2K$ information bits, using length N_D code bits at each end node.

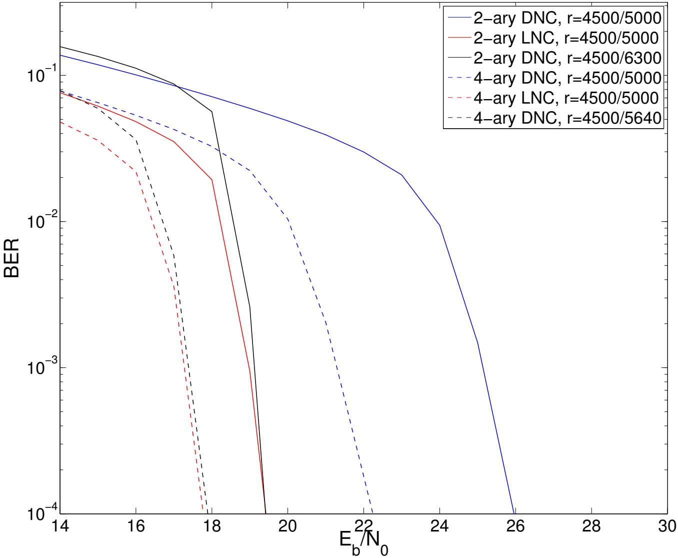
- Both systems use $N_B = 5000$ channel code bits in the broadcast phase.

- The propotional throughput increase T_I of DNC over LNC is thus

$$T_I = \frac{2K/(N_D + N_B)}{2K/(2N_L + N_B)} = \frac{15000}{N_D + 5000} \tag{1}$$



Coded error-rate performance used to compare DNC and LNC throughput, assuming no channel state information is available.



Coded error-rate performance used to compare DNC and LNC throughput, assuming partial channel state information is available.

- The following table summarizes the throughput improvement of DNC over LNC.

Throughput Improvement - T_P		
CSI	**M=2**	**M=4**
None	30.4%	32.7%
Partial	37.1%	41.0%

Table: Throughput Improvement - DNC over LNC

Outline

- This work presents a soft-output detector which implements DNC in the two-way relay channel.
- Simulated error-rate and throughput performance for a system which utilizes DNC and LNC, 2 and 4-ary CPFSK modulation, Turbo channel coding, and a fully-interleaved Rayleigh fading channel model.
- Increasing CPFSK modulation order from 2 to 4 improves DNC energy efficiency by $1-2$ dB, and decreases the energy efficiency gap between DNC and LNC by 1 dB.
- DNC increases throughput over LNC by at least 30%, using 2-ary modulation and no channel state information. and by 41%, using 4-ary modulation and partial channel state information.
- Potential avenues for future work include design of techniques to synchronize the frames transmitted by the end nodes, and implementation in a software radio platform.

Thank You!