Distributed Parameter Estimation in Wireless Sensor Networks Using Fused Local Observations

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Wireless Sensing, Localization, and Processing VII
SPIE Defense, Security, and Sensing, Baltimore, MD

April 25, 2012

Outline

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What Is Distributed Estimation in WSNs?

- Distributed sensors observe their surrounding environment.
- Process their local observations.
- Send their processed data to a fusion center (FC).
- FC performs the ultimate global estimation.

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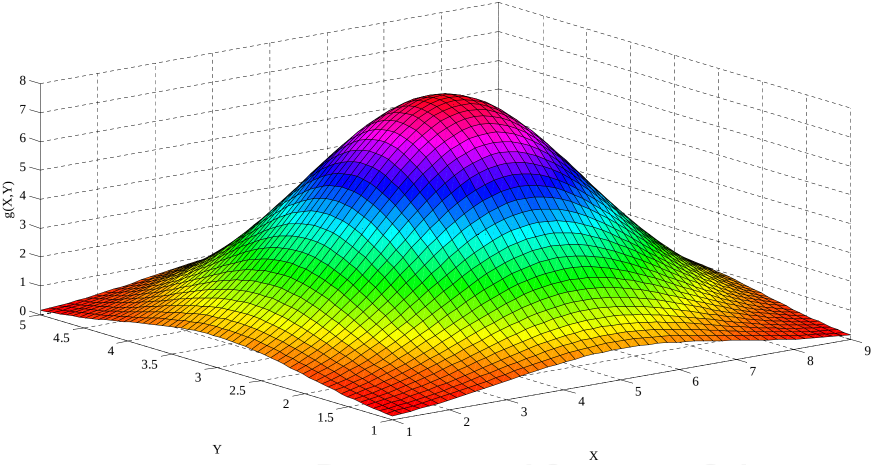
Goal of Our Paper

Reliably estimating a *vector* of unknown parameters of a *deterministic* function at the fusion center of a WSN from its distributed noisy samples observed by local sensors and communicated through parallel block-fading channels.

Simple Conceptual Example

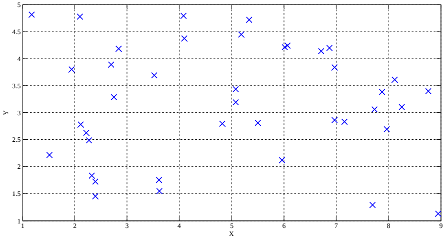
Two-dimensional Gaussian-shaped function completely known except for a set of unknown deterministic parameters:

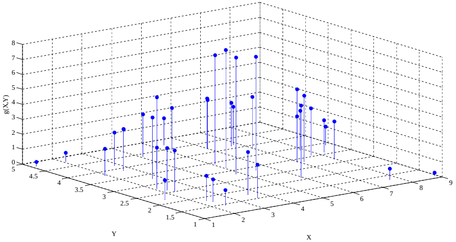
hight, center, variance in different directions.



Simple Conceptual Example

$K = 40$ sensors are randomly placed in the domain of the function to accumulate its noisy samples at their locations.

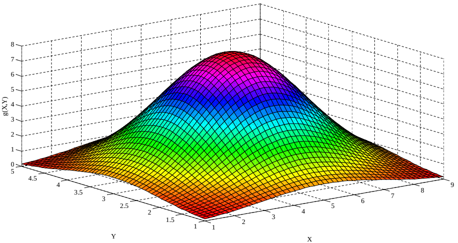


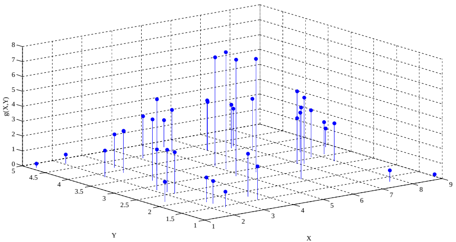


Simple Conceptual Example

Basic Question To Be Answered

How can we reliably estimate the parameters associated with function $g(x,y)$, effectively reconstructing it, using its sparse noisy samples?





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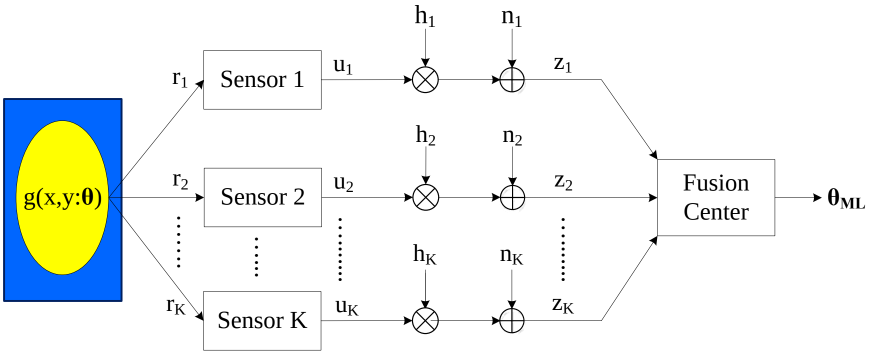
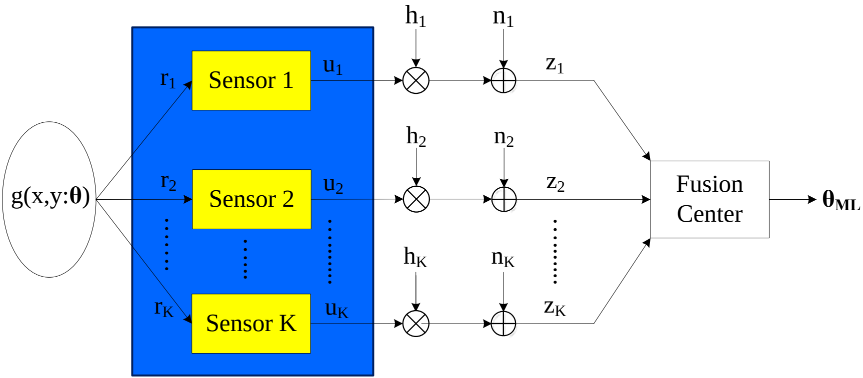
System Model Description



Two-dimensional function $g(x,y)$ is completely known except for a set of unknown deterministic parameters $\theta = [\theta_1, \theta_2, \dots, \theta_p]^T$.

System Model Description

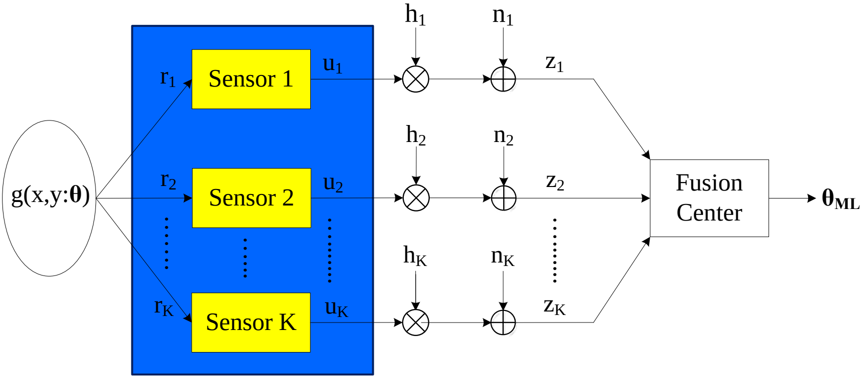


- AWGN local noisy sampling

$$r_i = g(x_i, y_i) + w_i \qquad w_i \sim N\left(0, \sigma_{O_i}^2\right) \qquad \text{SNR}_O \stackrel{\text{def}}{=} \frac{1}{2\sigma_O^2}$$

- Special case: $r_i = \mathbf{a_i}^T \theta + w_i$.

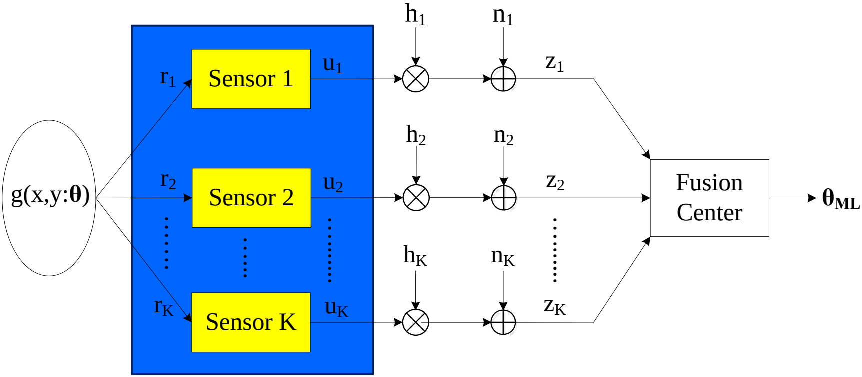
System Model Description



Two local processing schemes:

1. Analog processing scheme
   - Each sensor acts as a pure relay and transmits amplified version of its raw analog local observation to the fusion center.
   - Maximum-likelihood (ML) estimation at FC.

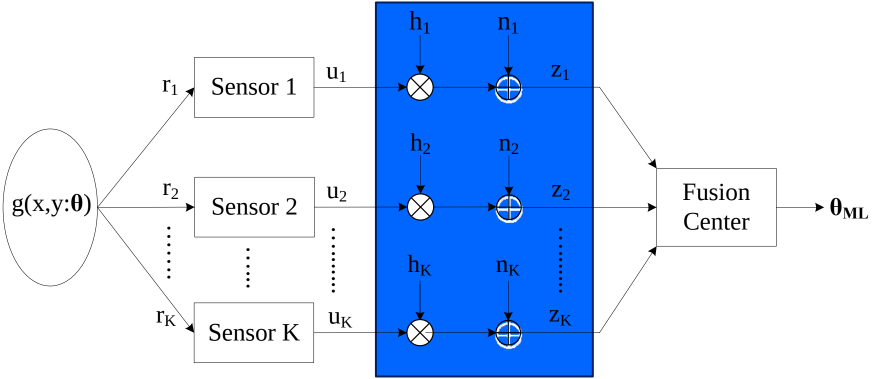
System Model Description



Two local processing schemes:

1. Digital processing scheme
   - Each sensor quantizes its local observation and sends its quantized data to the fusion center.
   - Expectation maximization (EM) algorithm at FC.

System Model Description

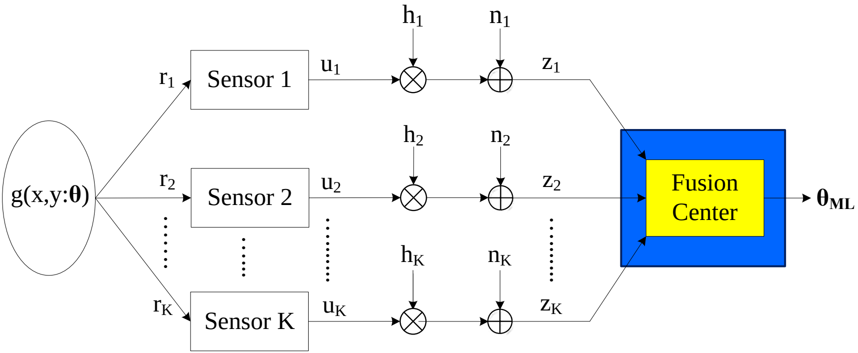


Parallel independent flat-fading channels

$$z_i = h_i u_i + n_i \qquad n_i \sim N\left(0, \sigma_{C_i}^2\right)$$

Channel gains are completely *known* at the fusion center.

System Model Description



$\mathbf{z} = [z_1, z_2, \ldots, z_K]^T$ is combined at the fusion center to estimate θ.

Outline

Main Idea and Final Result

- Each sensor acts as a pure relay and transmits an amplified version of its local raw analog observation to FC.

$$u_i = \alpha_i r_i$$

- α_i is known at FC.

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ML Estimate of θ Based on Analog Local Processing

- Non-linear system of equations in unknown parameters:

$$\sum_{i=1}^{K} \left[\frac{1}{\sigma_i^2} \left(\frac{\partial g_i}{\partial \theta_j} \right) \left(z_i' - g_i \right) \right] \Bigg|_{\widehat{\theta}_{\mathrm{ML}}} = 0, \qquad j = 1, 2, \ldots, p.$$

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$$\boxed{\sigma_i^2 \stackrel{\mathrm{def}}{=} \sigma_{O_i}^2 + \frac{\sigma_{C_i}^2}{|h_i|^2} \qquad z_i' = h_i^{-1} z_i \qquad g_i \stackrel{\mathrm{def}}{=} g(x_i, y_i)}$$

Outline

Local Quantization Rule

- Sensor i quantizes its local observation to $b_i = \log_2 M_i$ bits.
- The index of quantized data is sent to FC.
- Set of quantization thresholds: $\mathscr{L}_i = \{\beta_i(0), \beta_i(1), \ldots, \beta_i(M_i)\}$.

$$u_i \;=\; \ell \quad \Longleftrightarrow \quad \beta_i(\ell) \leq r_i < \beta_i(\ell+1), \qquad \ell = 0, 1, \ldots, M_i - 1$$

Final Result of ML Estimate in Digital Case

$$\sum_{i=1}^{K} \frac{\sum_{\ell=0}^{M_i-1} A_{i,j}(\ell) \exp\left[-\frac{(z_i'-\ell)^2}{2\sigma_i^2}\right]}{\sum_{\ell=0}^{M_i-1} \Delta Q_i(\ell) \exp\left[-\frac{(z_i'-\ell)^2}{2\sigma_i^2}\right]}\Bigg|_{\widehat{\theta}_{\mathsf{ML}}} = 0, \qquad j = 1,2,\ldots,p.$$

$$A_{i,j}(\ell) = \frac{1}{\sqrt{2\pi\sigma_{O_i}^2}} \left(\frac{\partial g_i}{\partial \theta_j}\right) \exp\left(-\frac{g_i^2}{2\sigma_{O_i}^2}\right) B_i(\ell),$$

$$B_i(\ell) = \exp\left(\frac{2g_i\beta_i(\ell) - \beta_i^2(\ell)}{2\sigma_{O_i}^2}\right) - \exp\left(\frac{2g_i\beta_i(\ell+1) - \beta_i^2(\ell+1)}{2\sigma_{O_i}^2}\right).$$

$$\Delta Q_i(\ell) \stackrel{\text{def}}{=} Q\left(\frac{\beta_i(\ell) - g_i}{\sigma_{O_i}}\right) - Q\left(\frac{\beta_i(\ell+1) - g_i}{\sigma_{O_i}}\right)$$

Final Result of ML Estimate in Digital Case

$$\sum_{i=1}^{K} \frac{\sum_{\ell=0}^{M_i-1} A_{i,j}(\ell) \exp\left[-\frac{(z_i'-\ell)^2}{2\sigma_i^2}\right]}{\sum_{\ell=0}^{M_i-1} \Delta Q_i(\ell) \exp\left[-\frac{\left(z_i'-\ell\right)^2}{2\sigma_i^2}\right]}\Bigg|_{\widehat{\theta}_{\mathbf{ML}}} = 0, \qquad j = 1, 2, \ldots, p.$$

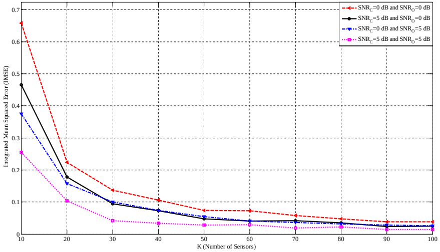
- Non-linear non-convex optimization problem.
- Efficient numerical methods are needed that converge in a reasonable time.
- We have developed a linearized expectation maximization (EM) solution to find the ML estimate of the vector of parameters iteratively.
- Details are omitted due to time constraints.

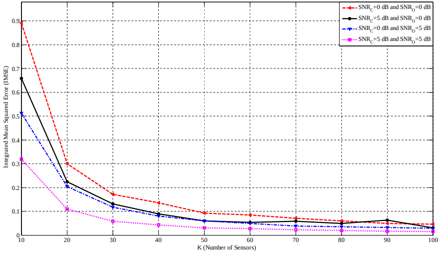
Outline

Effect of Number of Sensors K

ML estimation based on analog local processing

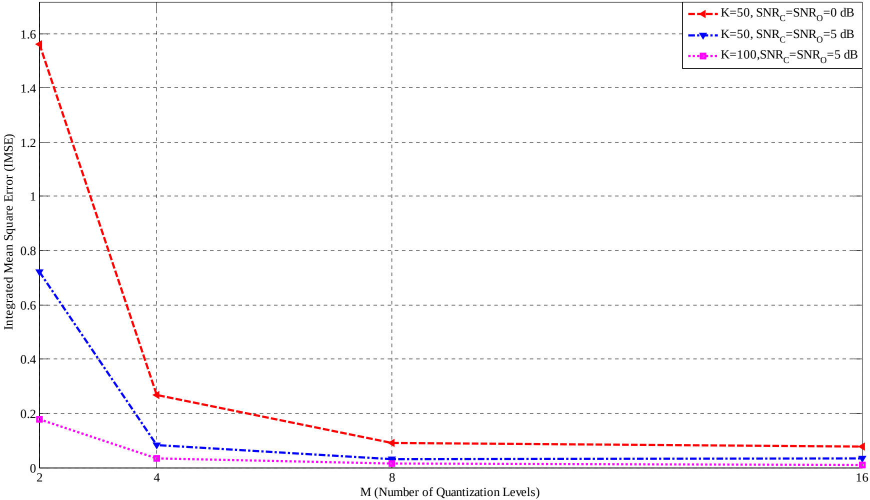
EM estimation based on digital local processing ($M = 8$)





Effect of Number of Quantization Levels (M)

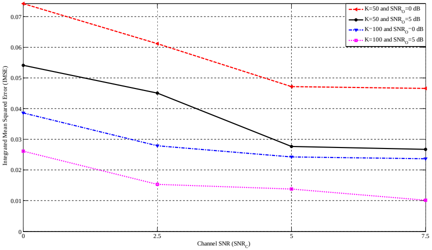
EM estimation based on digital local processing

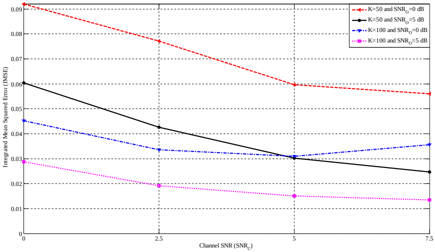


Effect of Channel SNR (SNR_C)

ML estimation based on analog local processing

EM estimation based on digital local processing

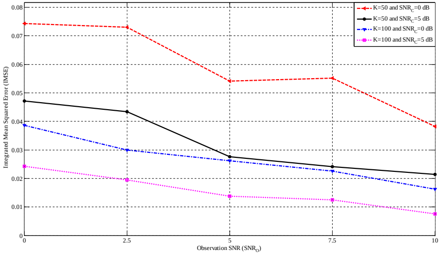


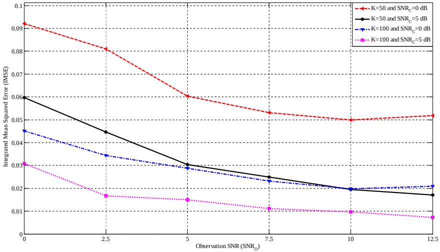


Effect of Observation SNR (SNR_O)

ML estimation based on analog local processing

EM estimation based on digital local processing





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Conclusions

- Distributed estimation of a vector of parameters based on noisy samples of the underlying function in the context of wireless sensor networks was studies.
- ML estimation techniques were developed in two cases of analog and digital local processing schemes .
- For the case of digital local processing, linearized expectation maximization was applied to iteratively find the ML estimate.

Thank You Very Much for Your Attention.

Questions?