Physical-layer Network Coding using FSK Modulation under Frequency Offset

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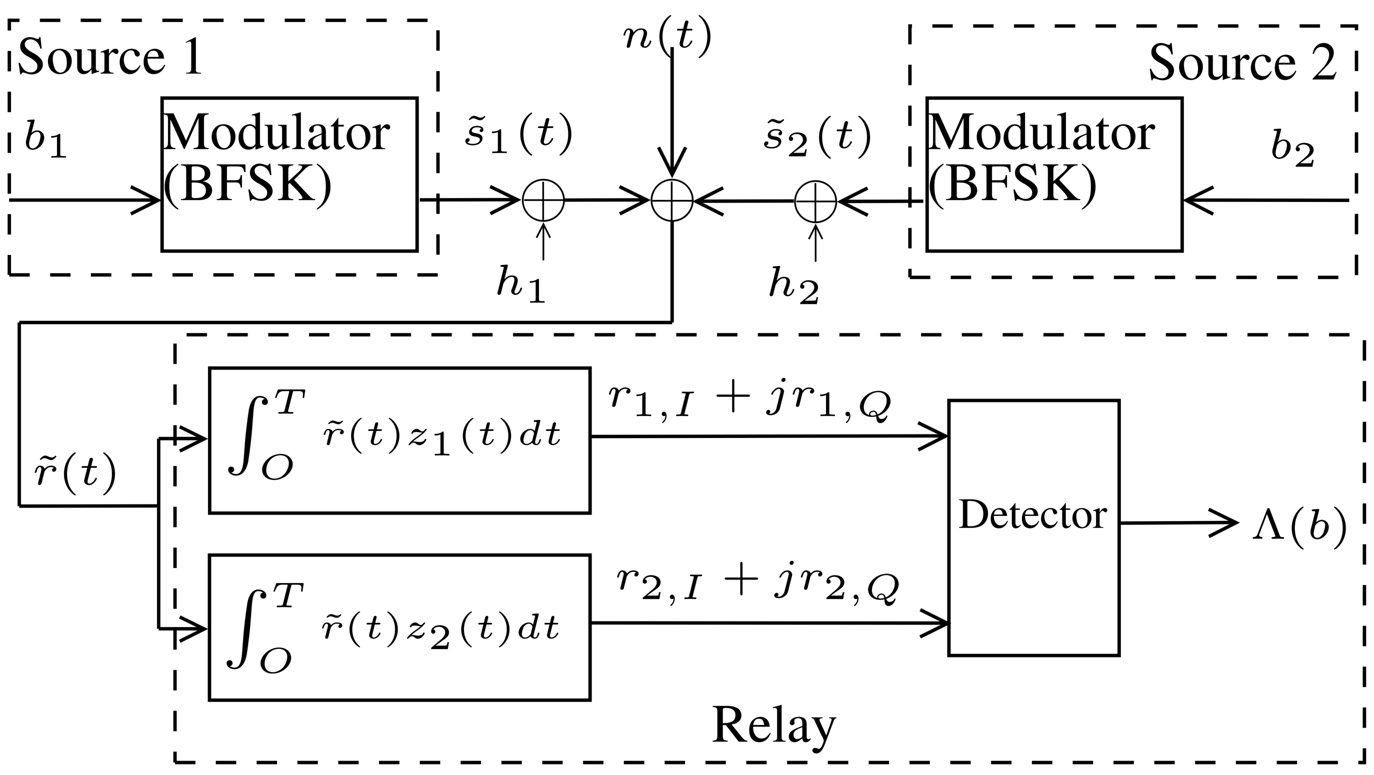
Introduction

- Physical-layer network coding (PNC) is a capacity increasing protocol which exploits simultaneous transmission by multiple nodes in the same time and band.
- This work applies PNC to the two-way relay channel under constant envelope, noncoherent, orthogonal modulation and error correction coding.
- The specific focus is to investigate the error-rate penalty of carrier frequency offset on the MAC transmission phase.
- Certain contemporary radio platforms such as USRP software radios contain high-tolerance oscillators which generate imperfect carrier frequencies.

System Model

- Network Topology: Two-way Relay Channel
- Protocol: Physical-Layer Network Coding (MAC phase - Sources to Relay)
- Channel Model: Frequency-flat slowly-varying Rayleigh fading
- Modulation: Orthogonal Binary Frequency-shift Keying
- Channel code: Turbo coded and Uncoded



Block Diagram of MAC Transmission Phase

Vector Channel Model under Oscillator Offset

- We show that oscillator offset may be modeled as a linear, multiplicative effect.
- The channel statistics at the output of the relay demodulator are

$$\mathbf{r} = \mathbf{h_1 O_1 s_1 + h_2 O_2 s_2 + n}$$

- The matrices $\mathbf{O_1}$ and $\mathbf{O_2}$ quantifying offset are defined as

$$\mathbf{O_i} = \begin{bmatrix} \mathbf{O_i[0,1]} & \mathbf{O_i[1,1]} \\ \mathbf{O_i[0,2]} & \mathbf{O_i[1,2]} \end{bmatrix} \quad \mathbf{i \in \{1,2\}}$$

- The elements of $\mathbf{O_1}$ and $\mathbf{O_2}$ are

$$\mathbf{O_i[b_i, m]} = \left[\frac{\sin \mathbf{A_{i,m}}}{\mathbf{A_{i,m}}} - \mathbf{j} \frac{\cos \mathbf{A_{i,m}} - 1}{\mathbf{A_{i,m}}} \right]$$

where $\mathbf{b_i}$ denotes the bit transmitted by source $\mathbf{i}$ and $\mathbf{m}$ denotes the demodulator output corresponding to FSK tone $\mathbf{m}$, and the term $\mathbf{A_{i,m}}$ is

$$\mathbf{A_{i,m}} = 2\pi\{[\mathbf{b_i} - (\mathbf{m} - 1)] + \mathbf{d_i}/\mathbf{\Delta_f}\}$$

where $\mathbf{\Delta_f}$ is the spacing between FSK tones, and $\mathbf{d_i}$ is the frequency offset.

Relay Detection Rule

- Goal: derive optimal detection rule for network-coded bit at relay defined by the log-likelihood ratio of the network-coded bit

$$\Lambda(\mathbf{b}) = \log \frac{\mathbf{P(b=1)}}{\mathbf{P(b=0)}} = \log \frac{\mathbf{P(b_1 \oplus b_2 = 1)}}{\mathbf{P(b_1 \oplus b_2 = 0)}}$$

- The received signal at the relay may be written in vector form as

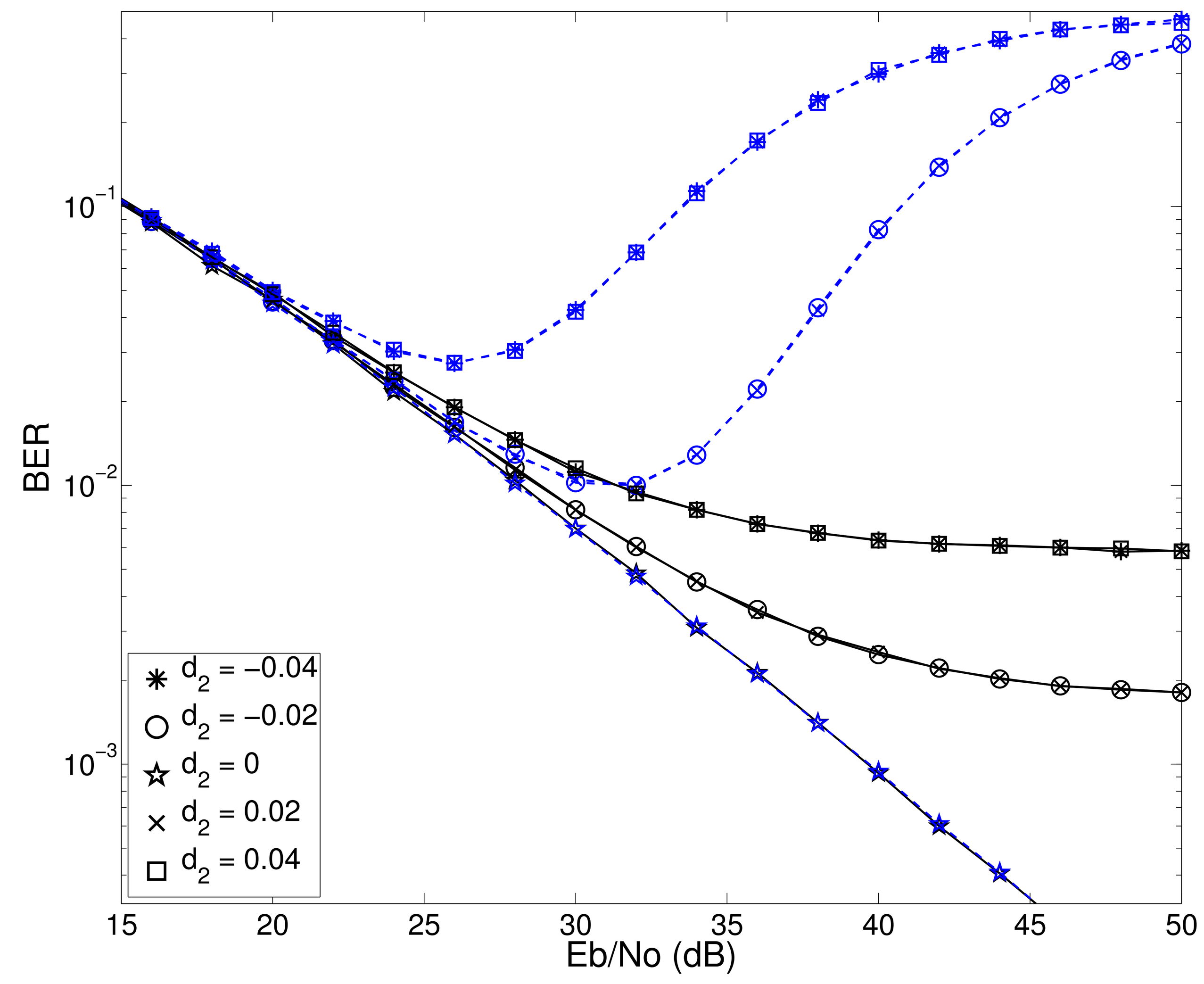
$$\mathbf{r} = \begin{bmatrix} \mathbf{h_1 O_1[b_1, 1] + h_2 O_2[b_2, 1] + n_1} \\ \mathbf{h_1 O_1[b_1, 2] + h_2 O_2[b_2, 2] + n_2} \end{bmatrix}$$

- The demodulator outputs are the sum of zero-mean i.i.d. Gaussian RV's.
- The outputs are thus complex jointly Gaussian RV's completely described by the covariance matrix $\mathbf{K}$.

$$\mathbf{p(r|S_i)} = \frac{1}{(2\pi)^2|\mathbf{K}|} \exp\left(-\mathbf{r^H K^{-1} r}\right)$$

where $\mathbf{S_i}$ is the symbol pair transmitted by the end nodes.

Simulated Error Rate Performance - no Error Correction Code



Blue, dashed lines: detector does not compensate for offset.
Black, solid lines: detector compensates for offset. $\mathbf{d_1 = 0}$

- Detection rule that does not incorporate offset reaches a minimum error rate and then degrades in performance.
- Rule incorporating offset outperforms the rule which does not, however, the error rate encounters an error floor.
- SNR required to reach error rate $\mathbf{10^{-2}}$ **:**

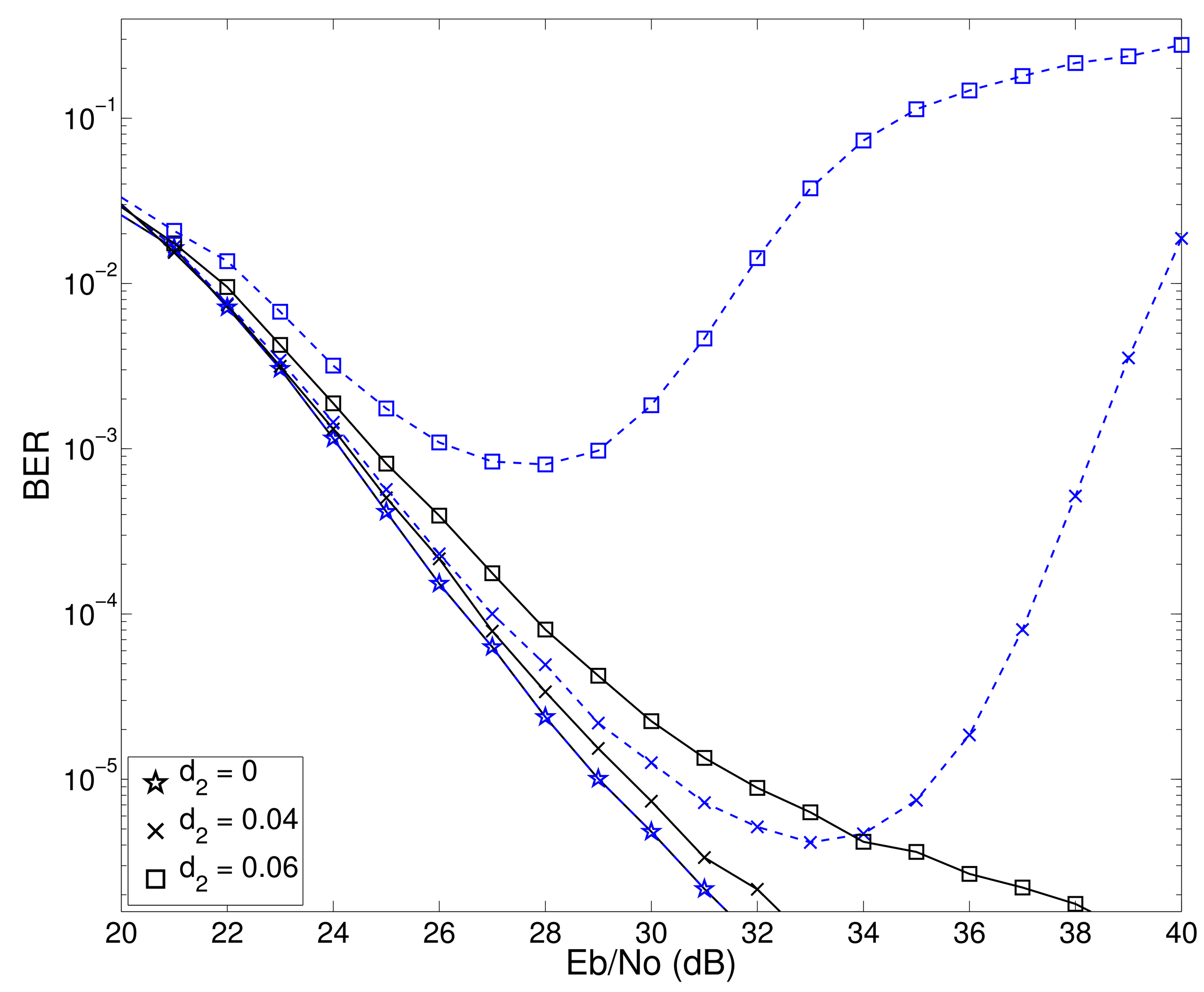
No Offset Compensation

Frequency offset ($\lvert\mathbf{d_i}\rvert$)	SNR (dB)
0.04	N/A
0.02	32
0	26

Offset Incorporated

Frequency offset	SNR
0.04	32
0.02	27
0	26

Simulation Results - Turbo Channel Coding



Blue, dashed lines: detector does not compensate for offset.
Black, solid lines: detector compensates for offset. $\mathbf{d_1 = 0}$
Channel code rate r=4500/6500

- Turbo Channel lowers error floor exhibited by rule incorporating offset beneath an error rate of $\mathbf{10^{-5}}$
- SNR required to reach error rate $\mathbf{10^{-3}}$ **:**

No Offset Compensation

Frequency offset ($\lvert\mathbf{d_i}\rvert$	SNR (dB)
0.06	28
0.04	24.5
0	24

Offset Incorporated

Frequency offset	SNR
0.06	25
0.04	24.5
0	24

Conclusion

- Result: Noncoherent demodulator and detector for the MAC phase of the TWRC which compensates for carrier frequency offset.
- Detector exhibits error rates below $\mathbf{10^{-5}}$ for offsets on the order of a few hundredths of a frequency spacing when paired with a Turbo channel code.
- Uncoded error rates on the order of $\mathbf{10^{-2}}$. Further analysis is required to discover the cause of and eliminate the error floor.