MILCOM'12

TRUSTED COMMUNICATIONS...AWARENESS TO ACTION

Distributed Estimation of a Parametric Field Using Sparse Noisy Data

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Outline

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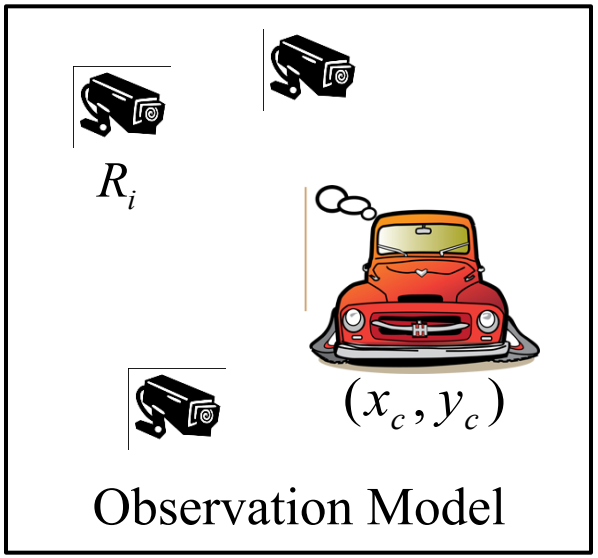
Overview and Motivation

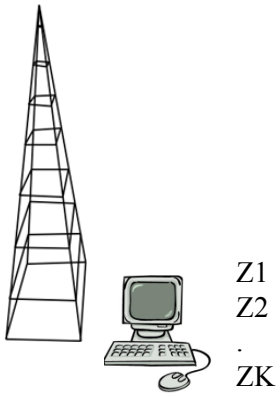
- WSNs have been used for area monitoring, surveillance, target recognition and other inference problems since 1980s [1].
- All designs and solutions are application oriented.
- Various constraints were incorporated [2]. Performance of WSNs under the constraints was analyzed.
- The task of distributed estimators was focused on estimating an unknown signal in the presence of channel noise [3].
- We consider a more general estimation problem, where an object is characterized by a physical field, and formulate the problem of distributed field estimation from noisy measurements in a WSN.

[1] C. Y. Chong, S. P. Kumar, "Sensor Networks: Evolution, Opportunities, and Challenges" Proceeding of the IEEE, vol. 91, no. 8, pp. 1247-1256, 2003.
[2] A. Ribeiro, G. B. Giannakis, "Bandwidth-Constrained Distributed Estimation for Wireless Sensor Networks - Part I:Gaussian Case," IEEE Trans. on Signal Processing, vol. 54, no. 3, pp. 1131-1143, 2006.
[3] J. Li, and G. AlRegib, "Distributed Estimation in Energy-Contrained Wireless Sensor Networks," IEEE Trans. on Signal Processing, vol. 57, no. 10, pp. 3746-3758, 2009.

Assumptions



Transmission Channel



The object generates fumes that are modeled as a Gaussian shaped field.

* K sensors randomly placed over area A.
* $\mathrm{R}_i = G(x_i, y_i) + W_i$, where $W_i \sim N(0, \sigma^2)$.
* $Q(.)$ *is an* M *Level quantizer*.
* $Z_i = Q(R_i) + N_i$, where $N_i \sim N(0, \eta^2)$.

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Problem Statement

Given noisy quantized sensor observations at the Fusion Center, the goal is to estimate the location of the target and the distribution of its physical field.

Proposed Solution:

- Signals received at the FC are independent but not i.i.d.
- Since the unknown parameters are deterministic, we take the maximum likelihood (ML) approach.
- Let $l(\mathbf{Z}:\boldsymbol{\theta})$ be the log-likelihood function of the observations at the Fusion Center. Then the ML estimates solve:

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}\in\Theta} l(\mathbf{Z}:\boldsymbol{\theta}).$$

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Proposed Solution

- The log-likelihood function of $Z_1, Z_2, ..., Z_K$ is:

$$l(\mathbf{z}) = \sum_{k=1}^{K} \log\left(\sum_{j=1}^{M} p_k(v_j) \exp\left(-\frac{(z_k - v_j)^2}{2\eta^2} \right) \right) - \frac{K}{2} \log(2\pi\eta^2),$$

where $p_k(v_j) = \int_{\tau_j}^{\tau_{j+1}} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(t - G_k)^2}{2\sigma^2} \right) dt,$

and $v_1, ..., v_M$ are reproduction points of the quantizer Q(.).

- The necessary condition to find the maximum is:

$$\nabla_\theta l(Z:\theta)\big|_{\hat{\theta}_{ML}} = 0.$$

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Iterative Solution

- Incomplete data: Z_k
- Complete data: (R_k, N_k),

  where $R_k \sim N(G(x_k, y_k : \theta), \sigma^2)$, and $N_k \sim N(0, \eta^2)$.
- Mapping: $Z_k = q(R_k) + n_k$

  where $k = 1, \ldots, K$.
- The complete data log-likelihood:

$$l_{cd} = -\frac{1}{2\sigma^2} \sum_{i=1}^{K} (R_i - G(x_i, y_i))^2 + \text{terms not function of } \theta.$$

A. P. Dempster, N. M. Laird, and D. B. Rubin, "Maximum likelihood from incomplete data via the em algorithm," J. of the Royal Stat. Soc. Series B, vol. 39, no. 1, pp. 1-38, 1977.

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E- and M- steps

- Expectation Step:

$$Q^{(k+1)} = E\left[-\frac{1}{2\sigma^2} \sum_{k=1}^{K} \left(R_k - G_k\right)^2 \middle| z, \hat{\theta}^{(k)} \right].$$

- Maximization Step:

$$\frac{dQ^{(k+1)}}{d\theta_t} = E\left[-\frac{1}{2\sigma^2} \sum_{i=1}^{K} \left(R_i - G_i\right)^2 \frac{dG_i}{d\theta_t} \middle| z, \hat{\theta}^{(k)} \right]\Bigg|_{\hat{\theta}^{(k+1)}} = 0, \quad t = 1, \ldots, L.$$

$$\sum_{i=1}^{K} \frac{dG_i^{(k+1)}}{d\theta_t} A\left(G_i^{(k)}\right) - \sum_{i=1}^{K} G_i^{(k+1)} \frac{dG_i^{(k+1)}}{d\theta_t} B\left(G_i^{(k)}\right) = 0, \quad t = 1, 2, \ldots, L.$$

where $A\left(G_i^{(k)}\right)$ and $B\left(G_i^{k}\right)$ are nonlinear in θ^{k}.

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Experimental Set Up

- Assume the area A is of size 8-by-8;
- K sensors are randomly distributed over A;
- M quantization levels;
- SNR in observation channel is defined as:

$$SNR_O = \frac{\iint_A G^2(x, y : \theta) dx dy}{A\sigma^2}.$$

- SNR in transmission channel is defined as:

$$SNR_C = \frac{\iint_A E\left[q^2(R(x, y))\right] dx dy}{A\eta^2}.$$

Performance Measures

Target Localization

* Square Error $(SE) = \sqrt{\left\|\theta - \hat{\theta}\right\|^2}$
* Mean Square Error $(MSE) = E[SE]$

Shape Reconstruction

* Integrated Square Error $(ISE) = \iint_A \left| G(x, y : \hat{\theta}) - G(x, y : \theta) \right|^2 dx dy$
* Integrated Mean Square Error $(IMSE) = E[ISE]$

* Occurrence of outliers $P_{outliers}(\tau) = P[SE > \tau]$

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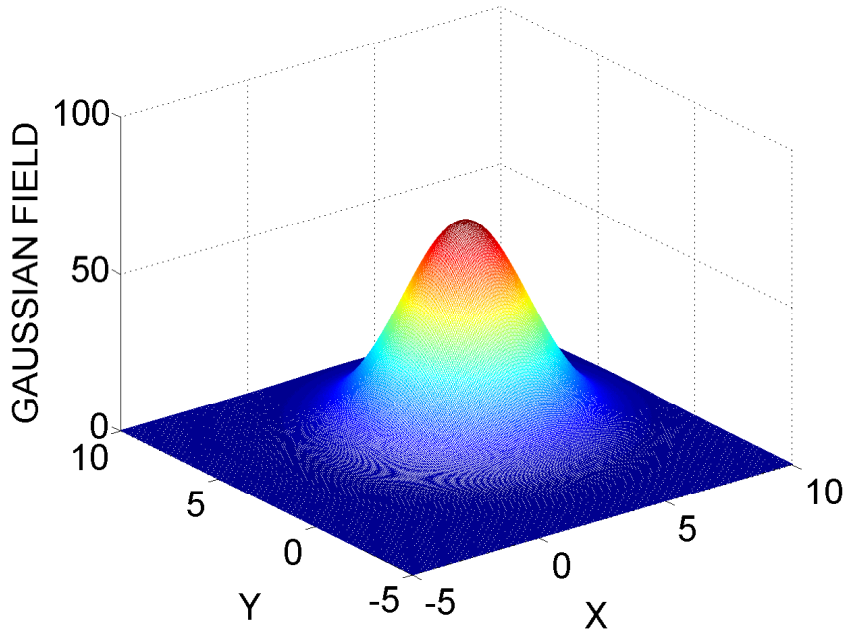
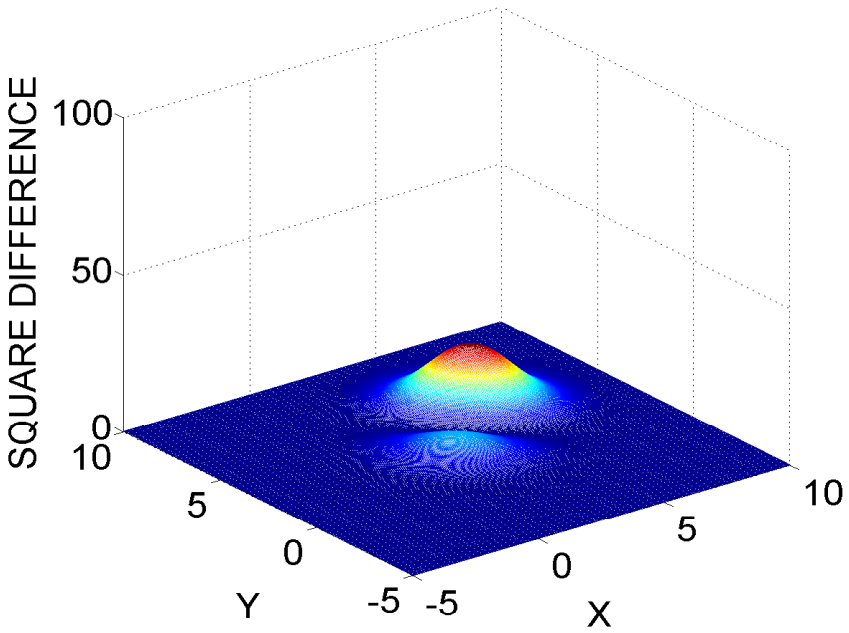
Numerical Results

The simulated Gaussian field and squared difference between the original and reconstructed fields where

$$\theta = [8,4,4]^T, \hat{\theta} = [7.90,3.88,3.88]^T$$





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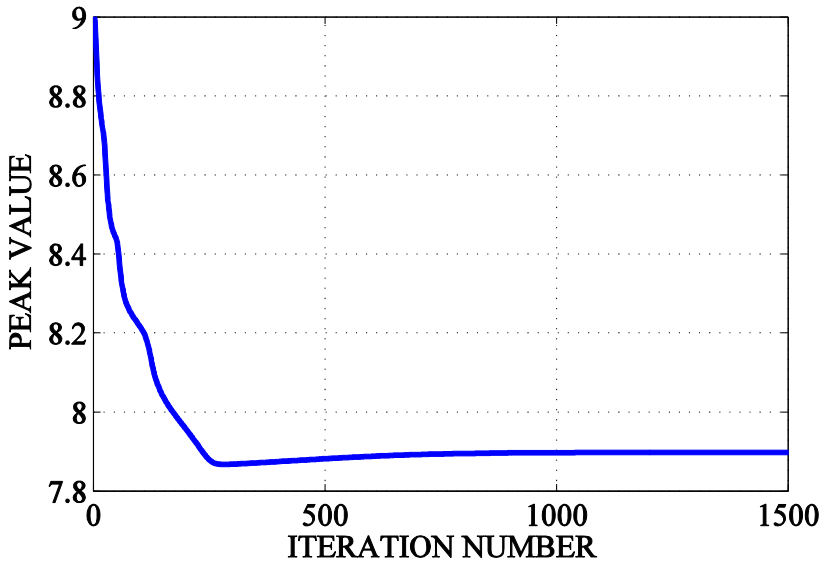
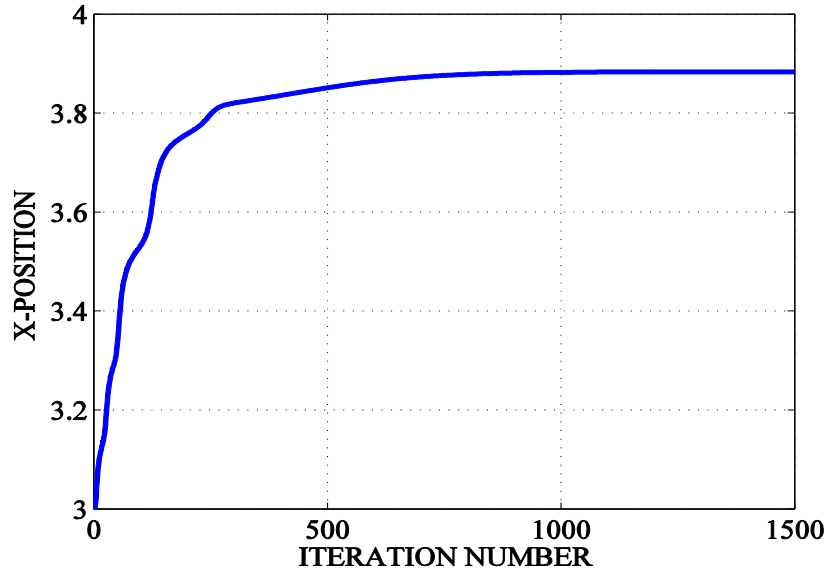
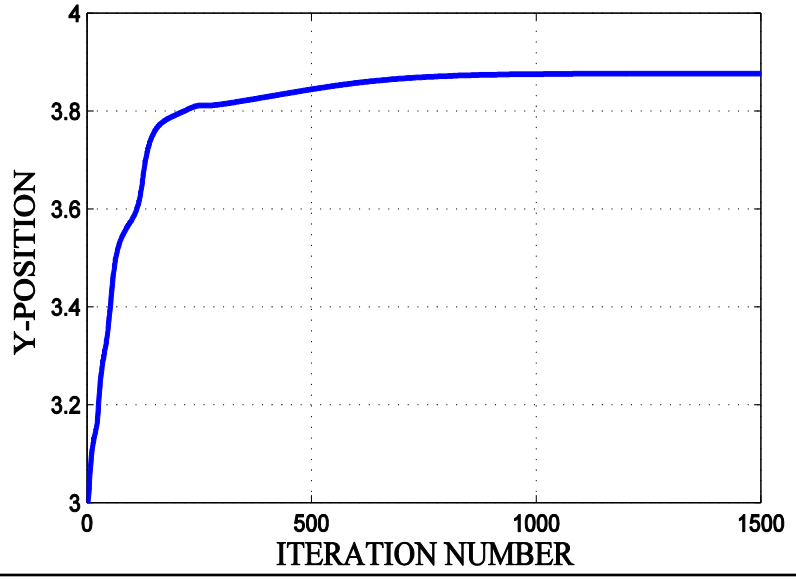
EM - convergence



- SNRo=SNRc=15dB.
- Number of sensors K=20.





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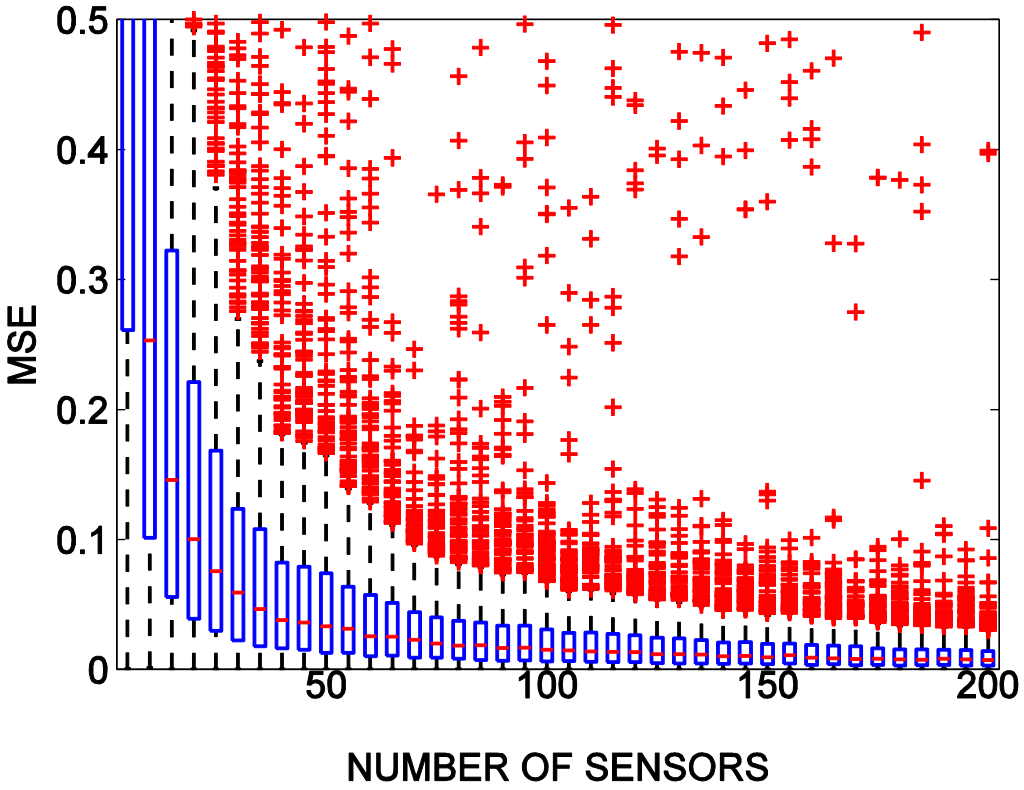
Box-plot of Square Error

- 1000 Monte Carlo realizations.
- SNRo=SNRc=15dB.

$$\text{Square Error}\left(SE\right)=\sqrt{\left\|\theta-\hat{\theta}\right\|^2}$$



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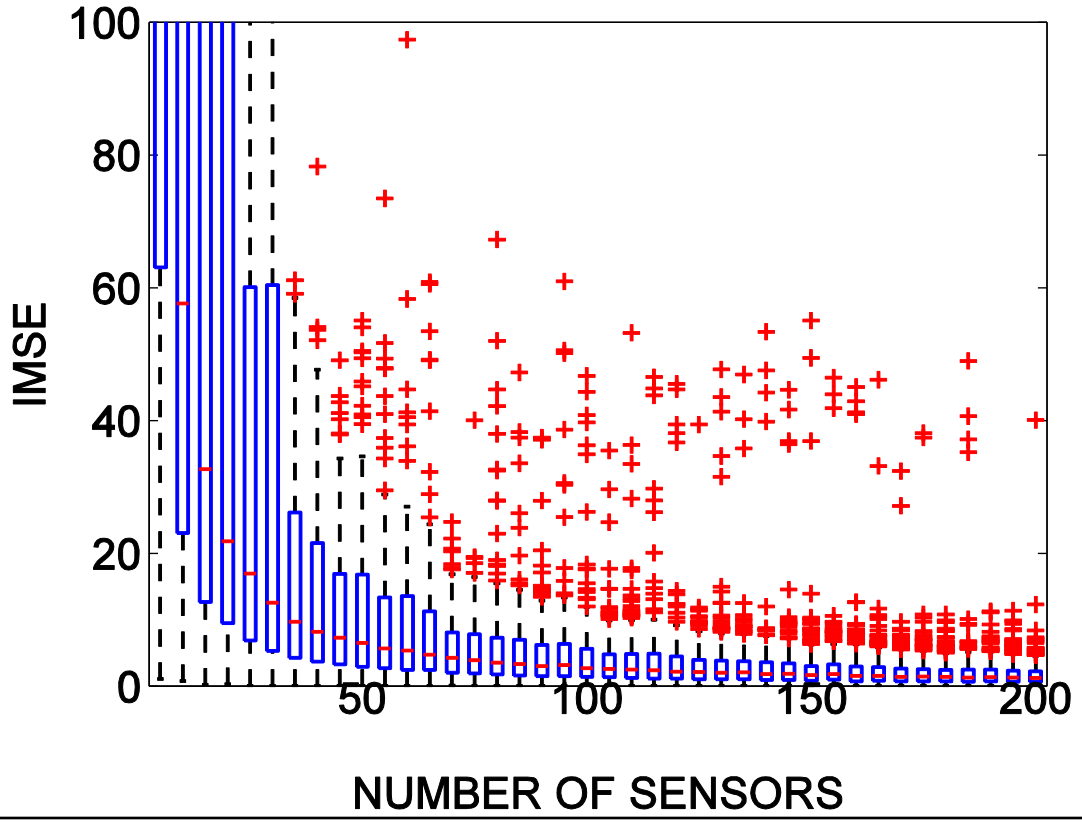
Box-plot of Integrated Square Error

- 1000 Monte Carlo realizations.
- SNRo=SNRc=15dB.
- Number of quantization levels M=8

$$\text{Integrated Square Error}\,(ISE) = \iint_A \left| G(x,y:\hat{\theta}) - G(x,y:\theta) \right|^2 dxdy$$



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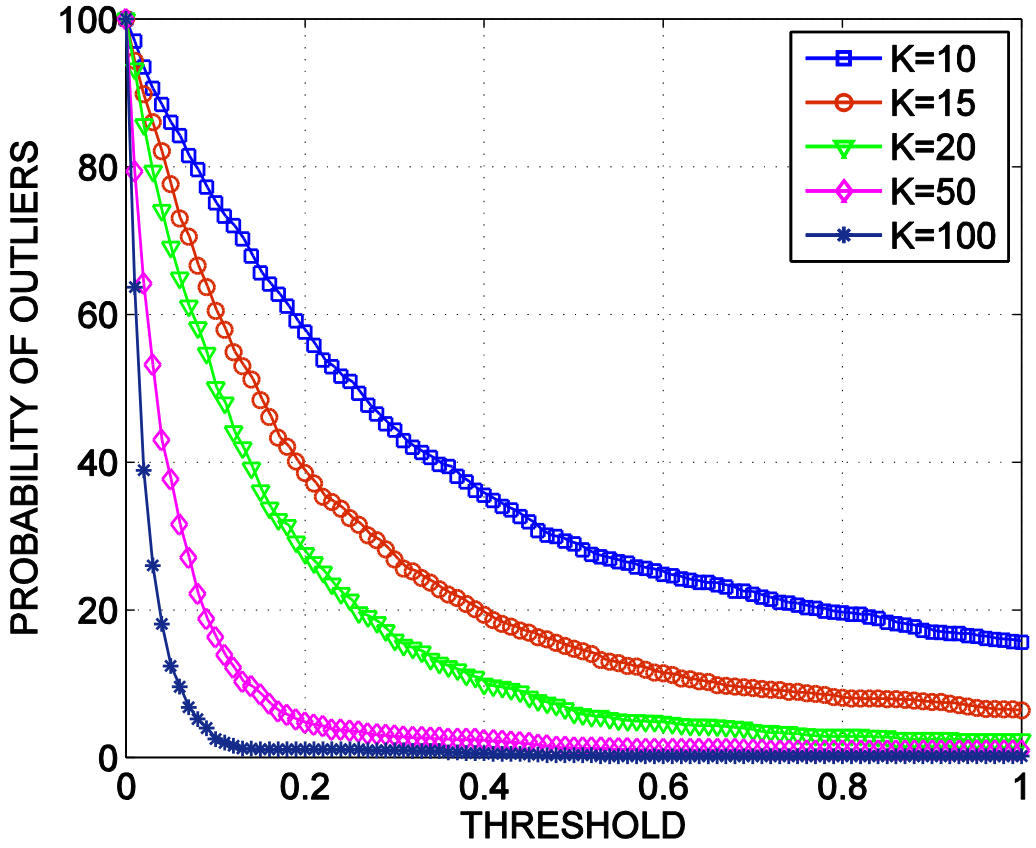
Probability of Outliers

- 1000 Monte Carlo realizations.
- SNRo=SNRc=15dB.
- Number of quantization levels M=8.

$$P_{outliers}(\tau) = P[SE > \tau],$$

τ = Threshold.



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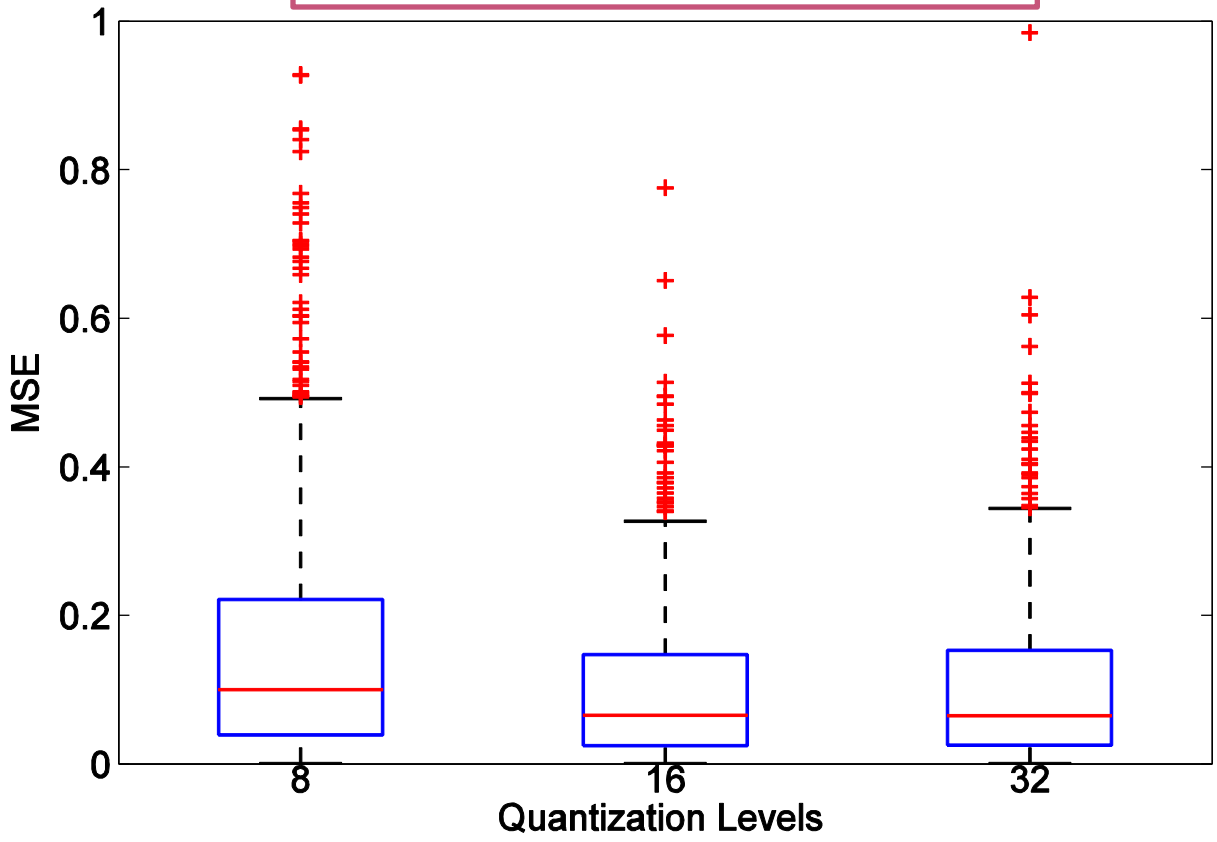
Effect of Quantization Levels

- 1000 Monte Carlo realizations.
- SNRo=SNRc=15dB.
- Number of sensors K=20.



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Summary

- An iterative linearized EM solution to distributed field estimation is presented and numerically evaluated.
- SNRo dominates SNRc in terms of its effect on the performance of the estimator.
- Increasing the number of sensors results in fewer outliers and thus in increased quality of the estimated values.
- At small number of sensors the EM algorithm produces a substantial number of outliers.
- More number of quantization levels makes the EM algorithm takes fewer iterations to converge.
- For large K, increasing the number of sensors does not have a notable effect on the performance of the algorithms.

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