Closing the Gap to the Capacity of APSK: Constellation Shaping and Degree Distributions

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ICNC – Jan. 30, 2013
Outline

1. Introduction
2. Constellation Shaping
3. LDPC Code Optimization
4. Optimization Results
5. Conclusion
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Features of Digital Video Broadcasting - Satellite - Second Generation:

- LDPC Coding with two lengths and several rates.
- Amplitude-phase shift keying (APSK) up to $M = 32$.
- Variable and adaptive coding to support interactive services.
APS K vs. Q AM for Nonlinear Channels

- Due to the use of TWTA, satellite channels are nonlinear.
- QAM constellations become highly distorted.

- APSK maintains distinct rings despite nonlinearity.
Contributions of This Paper

- Baseline system:
  - 32-APSK.
  - $R = 3$ bits/symbol.
  - AWGN channel.

- Performance improvements:
  1. BICM-ID decoder: 0.3 dB gain.
  2. Optimized LDPC code’s degree distribution: 0.3 dB gain.
  3. Constellation shaping: 0.5 dB gain.
  4. Both code optimization and constellation shaping: 0.9 dB gain.
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The energy efficiency can be improved by transmitting lower-energy signals more frequently than higher-energy signals.

**Figure**: Uniform 32APSK vs. shaped 32APSK. Both constellations have the same energy.

**Figure**: The capacity of shaped 32APSK is about 0.3 dB better than uniform 32APSK.
Partition the constellation into two equal-sized sub-constellations.

Use a shaping bit to select between the two sub-constellations.

- The lower-energy sub-constellation is selected more frequently.
- Requires the shaping bit to be encoded so that it is not uniform.

The remaining bits select from among the $M/2$ symbols in the selected sub-constellation with equal probability.
Shaping Encoder

- Shaping encoder maps $k_s$ bits to a $n_s$ bit shaping codeword.
- Code is designed with the goal of having more zeros than ones.
- Example $(k_s = 3, n_s = 5)$ code:

<table>
<thead>
<tr>
<th>3 input data bits</th>
<th>5 output codeword bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>0 0 1</td>
<td>0 0 0 0 1</td>
</tr>
<tr>
<td>0 1 0</td>
<td>0 0 0 1 0</td>
</tr>
<tr>
<td>0 1 1</td>
<td>0 0 1 0 0</td>
</tr>
<tr>
<td>1 0 0</td>
<td>0 1 0 0 0</td>
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<td>1 0 1</td>
<td>1 0 0 0 0</td>
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<td>1 1 0</td>
<td>0 0 0 1 1</td>
</tr>
<tr>
<td>1 1 1</td>
<td>1 0 1 0 0</td>
</tr>
</tbody>
</table>

- $p_0 = 31/40$ is the probability of 0.
- $p_1 = 9/40$ is the probability of 1.
Here, the \((5, 3)\) shaping code is used as an example.

- The \([P/S]\) block segments groups of 23 bits.
- Three bits delivered to the shaping encoder.
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Key features of the DVB-S2 LDPC code:

- **Variable rate:** \( R_c = \frac{k_c}{n_c} = \{ \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{8}{9}, \frac{9}{10} \} \).
- **Two lengths:** \( n_c = 16, 200 \) (short) and \( n_c = 64, 800 \) (long).
- **Systematic encoding.**
- **Last** \( m_c = n_c - k_c \) columns of \( H \) are a *dual diagonal* submatrix, making it an *extended irregular repeat accumulate* (eIRA) code\(^1\).
- **Constant row weight; i.e., check regular.**
- **Variable column weight, with** \( D = 3 \) different values\(^2\).

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\(^2\) Not including the last column, which has a weight of 1.
The *convergence threshold* is the SNR value in which the bit error rate of an LDPC-coded system starts dropping sharply.

- The value of the threshold depends on the *degree distribution*.

EXIT charts\(^3\)

- Predict the convergence threshold.
- Can be used to identify good candidate degree distributions.
- However, because it is just a prediction, the candidate codes still need to be simulated to determine which is best.

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**Figure**: EXIT chart for the uniform system at $E_b/N_0 = 4.93$ dB.
EXIT Charts with Constellation Shaping

When shaping is used, the variable-node decoder (VND) accounts for the effects of shaping.

**Figure**: Model of decoder used for constructing EXIT charts.

**Figure**: EXIT chart for the shaped system at $E_b/N_0 = 4.53$ dB.
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Optimization Results

Optimization Procedure

Common considerations:
- Spectral efficiency set to $R = 3$ bits/symbol.
- Systematic eIRA code structure.
- Row-weights from DVB-S2 maintained.
- Either $D = 3$ or $D = 4$ distinct column weights.

Optimization steps:
- Optimize LDPC code for uniform modulation.
- Shaping with off-the-shelf DVB-S2 code.
- Jointly optimize the LDPC code and the shaping.

<table>
<thead>
<tr>
<th>$R_c$</th>
<th>$R_s$</th>
<th>$\xi_b/N_0$ in dB (BER = $10^{-5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/5 (38880/64800)</td>
<td>1</td>
<td>standard: 5.42 optimized (D=4) 5.13</td>
</tr>
<tr>
<td>2/3 (43200/64800)</td>
<td>2/4</td>
<td>standard: 4.96</td>
</tr>
<tr>
<td>9/14 (41661/64806)</td>
<td>2/3</td>
<td>optimized (D=4) 4.51</td>
</tr>
</tbody>
</table>
BER Comparison

- BICM-ID Uniform
- Uniform with optimized 3/5 LDPC code (D=3)
- Uniform with optimized 3/5 LDPC code (D=4)
- DVB-S2 2/3 LDPC and (4,2) shaping code
- Optimized 9/14 LDPC with D=3 in shaping system
- Optimized 9/14 LDPC with D=4 in shaping system

Eb/N0 in dB vs. BER plot.
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Conclusion

- Performance of LDPC-coded APSK can be improved by over 1 dB through the combination of:
  - BICM-ID instead of just BICM.
  - Constellation shaping.
  - Optimization of LDPC degree distributions.
- An extra 0.1 dB gain is achieved by using $D = 4$ distinct variable-node degrees, instead of just $D = 3$.
- Drawbacks:
  - Per-iteration complexity increase.
  - Slight increase in the PAPR.
- See journal version for more detail:
Thank You.