A New Analysis of the DS-CDMA Cellular Downlink Under Spatial Constraints

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Outline

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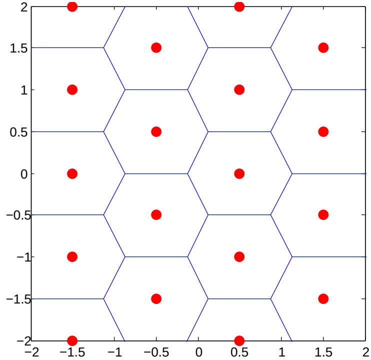
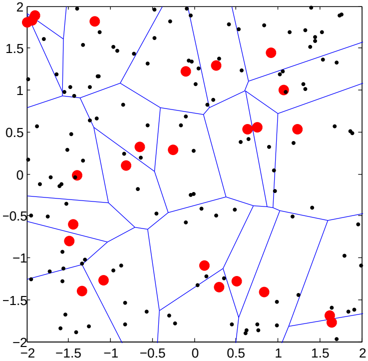
Introduction

- A cellular network is currently modeled by:



Classic approach (regular grid):

- The analysis often focuses on the worst case-locations (cell edge).



Using stochastic geometry:

- Assumes infinite network;
- A random point process with no constraint on the minimum separation is used to deploy the base stations.

Actual Vs Simulated Base-Station Locations

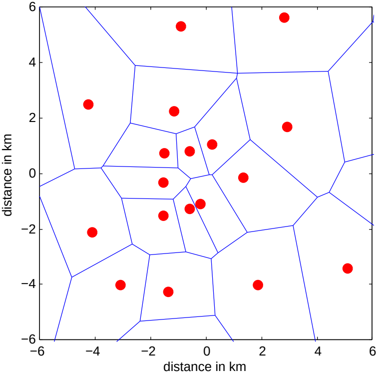


Figure: *Actual base-station locations from a current cellular deployment in a small city with a hilly terrain.*

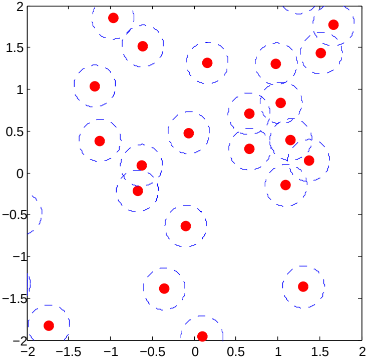


Figure: *Simulated base-station locations when the minimum base-station separation is* $r_{bs} = 0.25$.

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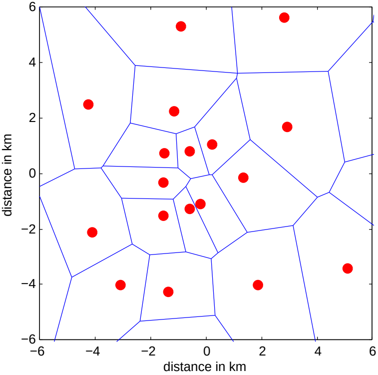

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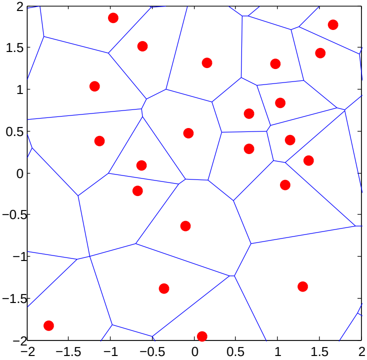

Figure: *Simulated base-station locations when the minimum base-station separation is* $r_{bs} = 0.25$. *Cell boundaries are indicated.*

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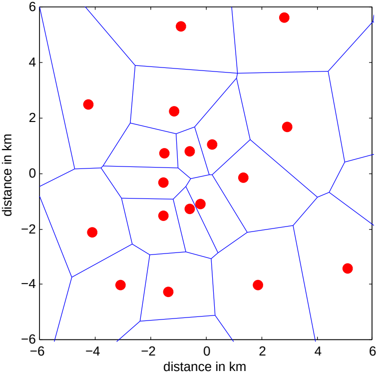

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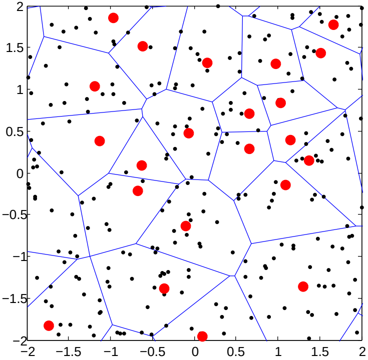

Figure: *Simulated base-station locations when the minimum base-station separation is* $r_{bs} = 0.25$. *Cell boundaries are indicated, and the average cell load is* 16 *mobiles.*

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Network model

- The Network comprises:
  - M cellular base stations $\{X_1, ..., X_M\}$ with an *exclusion zone* of radius r_{bs};
  - K mobiles $\{Y_1, ..., Y_K\}$ with an *exclusion zone* of radius r_m.
- Finite circular network arena with area $A_{net} = \pi r_{net}^2$.
- DS-CDMA is considered and intra-cell sequences are assumed to be synchronous.
- The base stations transmit with a common power P_0 such that

$$\frac{1}{1-f_p} \sum_{j:Y_j \in \mathcal{Y}_i} P_{i,j} = P_0 \tag{1}$$

where

  - $P_{i,j}$ is the average transmitted power by base station X_i to mobile Y_j;
  - f_p is the fraction of the base-station power reserved for pilot signals;
  - $\mathcal{Y}_i$ is the set of mobiles connected to the base station X_i.

Despread Instantaneous Power

The despread instantaneous power of X_i at mobile Y_j is

$$\rho_{i,j} = \begin{cases} P_{i,j} g_{i,j} 10^{\xi_{i,j}/10} f\left(||X_i - Y_j||\right) & \text{from serving base station, if } \mathrm{g}(j) = i \\ \left(\frac{h}{G}\right) P_{i,j} g_{i,j} 10^{\xi_{i,j}/10} f\left(||X_i - Y_j||\right) & \text{from interfering base stations, if } \mathrm{g}(j) \neq i \end{cases}$$

where

- $g_{i,j}$ is the power gain due to Nakagami fading;
- $\xi_{i,j}$ is a *shadowing factor* and $\xi_{i,j} \sim N\left(0, \sigma_s^2\right)$;
- $f(\cdot)$ is a path-loss function:
$$f(d) = \left(\frac{d}{d_0}\right)^{-\alpha}$$
  - α is the path loss exponent;
  - $d \geq d_0$;
- $\mathrm{g}(j)$ is a function that returns the index of the base station serving Y_j;
- h is the chip factor;
- G is the common spreading factor.

SINR

The performance at the mobile is characterized by the signal-to-interference and noise ratio (SINR), given by:

$$\gamma_j = \frac{g_{\mathrm{g}(j),j}\Omega_{\mathrm{g}(j),j}}{\Gamma^{-1} + \frac{h}{G}\sum_{\substack{i=1 \\ i\neq \mathrm{g}(j)}}^{M} g_{i,j}\Omega_{i,j}} \quad (2)$$

where

- Γ is the signal-to-noise ratio (SNR) at a mobile located at unit distance when fading and shadowing are absent;
- $\Omega_{i,j} = \frac{P_{i,j}}{P_0} 10^{\xi_{i,j}/10} ||X_i - Y_j||^{-\alpha}$ is the normalized power of X_i at receiver Y_j before despreading.

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Definition

- An *outage* occurs when the SINR is below a threshold β.
  - β depends on the choice of modulation and coding.
- The *outage probability* for the mobile Y_j conditioned over the network is

$$\epsilon_j = P\left[\gamma_j \leq \beta_j \middle| \mathbf{\Omega}_j\right]. \tag{3}$$

- Substituting (2) into (3), from [7]:

$$\epsilon_j = 1 - e^{-\frac{\beta_0}{\Gamma}} \sum_{n=0}^{m_{\mathrm{g}(j),j}-1} \left(\frac{\beta_0}{\Gamma}\right)^n \sum_{k=0}^{n} \frac{\Gamma^k}{(n-k)!} \sum_{\substack{\ell_i \geq 0 \\ \sum_{i=0}^{M} \ell_i = k}} \left(\prod_{\substack{i=1 \\ i \neq \mathrm{g}(j)}}^{M} G_{\ell_i}(\Psi_i) \right) \tag{4}$$

where $\beta_0 = \beta m_{\mathrm{g}(j),j}/\Omega_0$,

$$G_\ell(\Psi_i) = \frac{\Gamma(\ell + m_{i,j})}{\ell!\Gamma(m_{i,j})} \left(\frac{\Omega_{i,j}}{m_{i,j}}\right)^\ell \left(\frac{\beta_0 h \Omega_{i,j}}{G m_{i,j}} + 1\right)^{-m_{i,j}-\ell}. \tag{5}$$

[7] D. Torrieri and M.C. Valenti, "The outage probability of a finite ad hoc network in Nakagami fading", *IEEE Trans. Commun.*, Nov. 2012.

Outline

Policies

1. Rate control:
    - Base station $X_i, i = \mathrm{g}(j)$ transmits to mobile Y_j with power

$$P_{i,j} = \frac{P_0(1 - f_p)}{K_i},$$

where $K_i = |\mathcal{Y}_i|$;

    - The threshold β_j of mobile Y_j is selected such that the conditional outage probability of mobile Y_j satisfies the constraint $\epsilon_j = \hat{\epsilon}$.

2. Power control:
    - The threshold β_j is common for all the mobiles inside a cell;
    - All the mobiles have conditional outage probability equal to $\epsilon_j = \hat{\epsilon}$;
    - The power allocated by the serving base station to each user inside a cell is adapted such that the constraint on the outage is met.

- For both policies, a mobile in an overloaded cell is denied service, and its rate is set to $R_j = 0$.

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Transmission Capacity

- Let $R_j = C(\beta_j)$ represent the relationship between R_j and β_j. For modern cellular systems, it is reasonable to use:

$$C(\beta_j) \quad = \quad \log_2(1 + \beta_j)$$

- The performance metric used is the *transmission capacity*, defined as

$$\tau \quad = \quad \lambda \, (1 - \hat{\epsilon}) \, E[R]$$

where

- $\lambda = K/A_{net}$ is the density of transmissions in the network;
- $E[R]$ is computed using a Monte Carlo approach as follows:
  1. Draw a realization of the network;
  2. Compute the path loss from each base station to each mobile;
  3. Determine the set of mobiles associated with each base station;
  4. At each base station, apply the power allocation policy;
  5. By setting $\epsilon_j = \hat{\epsilon}$, invert (4) to determine β_j for each mobile in the cell;
  6. By applying the function $R_j = C(\beta_j)$, find the rate for the mobile;
  7. Repeat this process for a large number of networks.

Example: Power Vs Rate Control

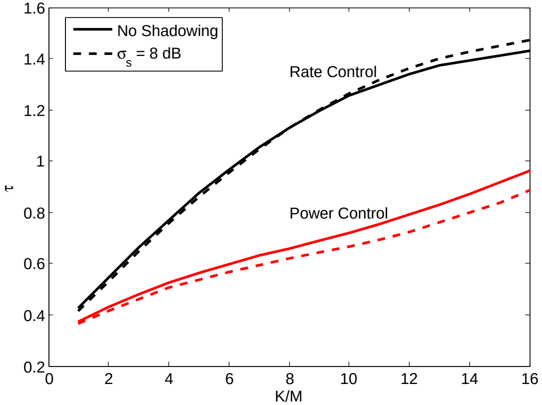

Figure: *Transmission capacity as a function of K/M with rate control and power control.*

Example:

- $M = 50$ base stations;
- Circular arena with $r_{net} = 2$;
- $r_{bs} = 0.25$;
- $r_m = 0.01$;
- $\alpha = 3$;
- $\Gamma = 10$ dB;
- $h = 2/3$;
- $G = 16$;
- $\hat{\epsilon} = 0.1$;
- $m_{i,j} = 3$ for $i = \text{g}(j)$, while $m_{i,j} = 1$ for $i \neq \text{g}(j)$;
- $\sigma_s = 8$ dB.

Example: Rate

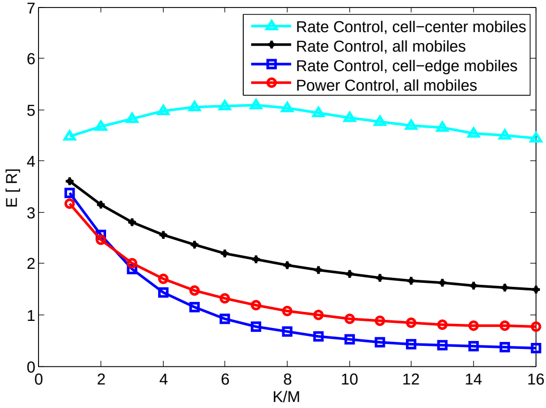

Figure: *Average rate as a function of K/M in the presence of shadowing with rate control and power control.*

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Example: Fairness

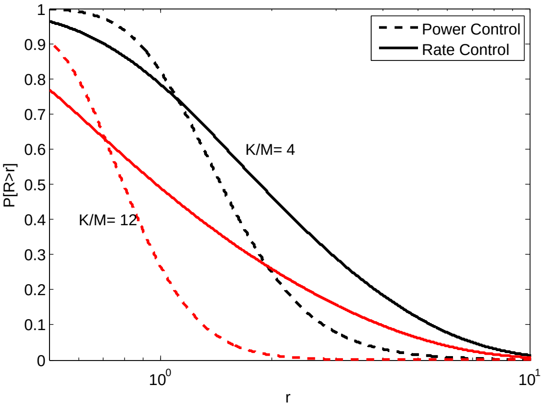

Figure: *Cumulative cdf of R with either rate control or power control for a lightly loaded system $(K/M = 4)$ and a moderately-loaded system $(K/M = 12)$.*

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Example: Minimum Distance between Base-Stations

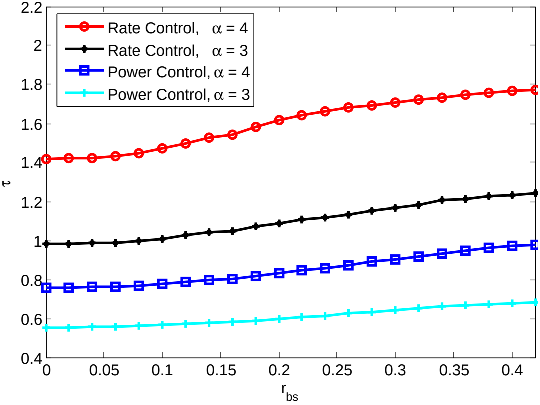

Figure: *Transmission capacity for rate and power control as function of r_{bs} for $K/M = 8$ in Mixed Fading and Shadowing ($\sigma_s = 8$ dB).*

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- $m_{i,j} = 3$ for $i = \text{g}(j)$, while $m_{i,j} = 1$ for $i \neq \text{g}(j)$;
- $\sigma_s = 8$ dB.

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Conclusions

- The new approach for modeling and analyzing the DS-CDMA cellular downlink has the following benefits:
  - the model allows constraints to be placed on the distance between base stations, the geographic footprint of the network, and the number of base stations and mobiles;
  - a flexible channel model, accounting for path loss, shadowing, and Nakagami-m fading with non-identical parameters, is considered.
- The results show that:
  - the rate control policy performs better than the power control policy in terms of transmission capacity, but a significant amount of users are provided with lower rate;
  - transmission capacity increases with r_{bs}, and this effect is more pronounced for the rate control policy and when the path loss exponent is higher.
- The approach is general enough and it can be extended:
  - to compare various access and resource allocation techniques;
  - to analyze reselection schemes;
  - to analyze the uplink and to model other types of access, such as orthogonal frequency-division multiple access (OFDMA);
  - to handle sectorized cells and coordinated multipoint strategies involving transmissions from multiple base stations.

Thank You