A New Analysis of the DS-CDMA Cellular Uplink Under Spatial Constraints

D. Torrieri [1]   M. C. Valenti [2]   **S. Talarico** [2]

[1]U.S. Army Research Laboratory
Adelphi, MD

[2]West Virginia University
Morgantown, WV

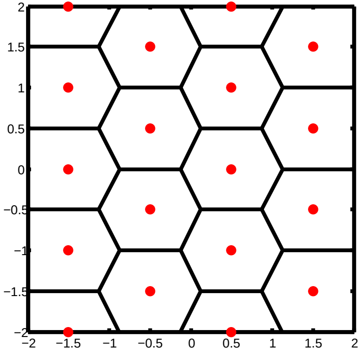
June 11, 2013

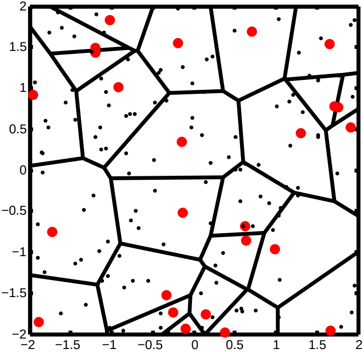
Outline

Outline

Introduction

- A cellular network is currently modeled by:





Classic approach (regular grid):

- The analysis often focuses on the worst case-locations (cell edge).

Using stochastic geometry:

- Assumes infinite network;
- A random point process with no constraint on the minimum separation is used to deploy the base stations.

Actual Vs Simulated Base-Station Locations

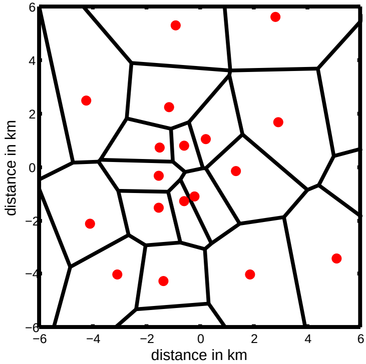


Figure: *Actual base-station locations from a current cellular deployment in a small city with a hilly terrain.*

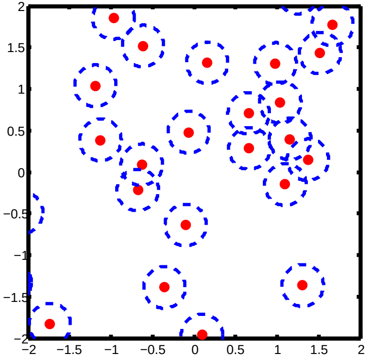


Figure: *Simulated base-station locations when the minimum base-station separation is* $r_{bs} = 0.25$.

Actual Vs Simulated Base-Station Locations

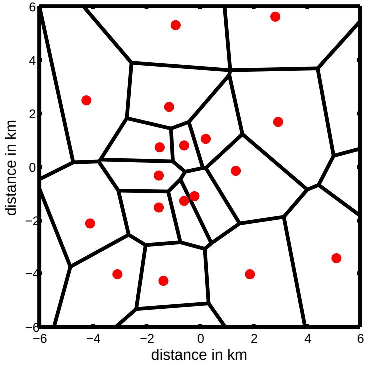


Figure: Actual base-station locations from a current cellular deployment in a small city with a hilly terrain.

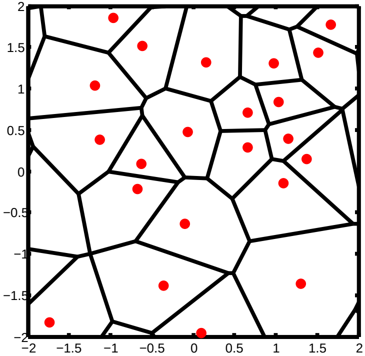


Figure: Simulated base-station locations when the minimum base-station separation is $r_{bs} = 0.25$. Cell boundaries are indicated.

Actual Vs Simulated Base-Station Locations

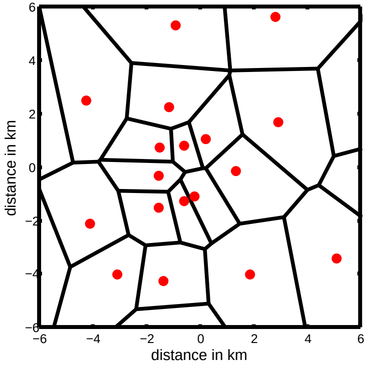


Figure: *Actual base-station locations from a current cellular deployment in a small city with a hilly terrain.*

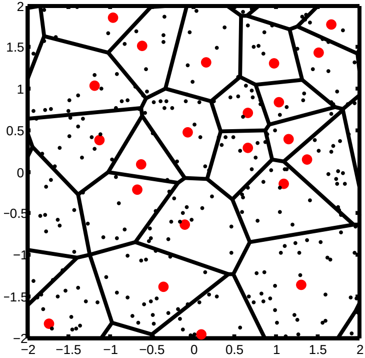


Figure: *Simulated base-station locations when the minimum base-station separation is* $r_{bs} = 0.25$. *Cell boundaries are indicated, and the average cell load is* 16 *mobiles.*

Actual Vs Simulated Base-Station Locations

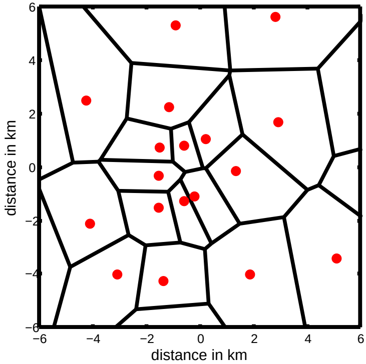


Figure: *Actual base-station locations from a current cellular deployment in a small city with a hilly terrain.*

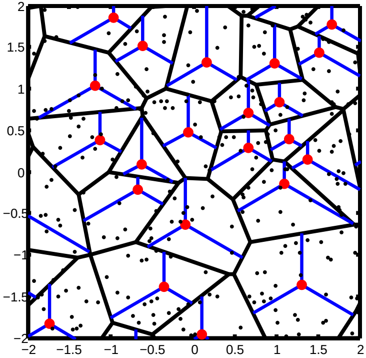


Figure: *Simulated base-station locations when the minimum base-station separation is $r_{bs} = 0.25$. Cell and sector boundaries are indicated, and the average cell load is 16 mobiles.*

Outline

Network Model

- The Network comprises:
  - C cellular base stations $\{X_1, ..., X_M\}$ with an *exclusion zone* of radius r_{bs};
  - $3C$ sectors $\{S_1, ..., S_{3C}\}$, assuming there are three ideal sector antennas per base station, each covering $2\pi/3$ radians.
  - M mobiles $\{Y_1, ..., Y_K\}$ with an *exclusion zone* of radius r_m.
- Finite circular network with area $A_{net} = \pi r_{net}^2$.
- DS-CDMA is considered.
- Both intracell and intercell interference within the coverage angle of the sector are considered.
- Let $\mathcal{A}_j$ denote the set of mobiles *covered* by sector antenna S_j. A mobile $X_i \in \mathcal{A}_j$ will be *associated* with S_j if the mobile's signal is received at S_j with a higher average power than at any other sector antenna in the network.
- Let $\mathcal{X}_j \subset \mathcal{A}_j$ denote the set of mobiles associated with sector antenna S_j.
- Let $X_r \in \mathcal{X}_j$ denote a reference mobile that transmits a desired signal to S_j.

Despread Instantaneous Power

The despread instantaneous power of X_i received at S_j is

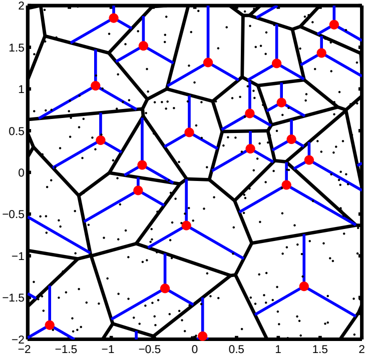
$$\rho_{i,j} = \begin{cases} P_r g_{r,j} 10^{\xi_{r,j}/10} f\left(||S_j - X_r||\right) & \text{from the reference mobile } X_r \\ \left(\frac{h}{G}\right) P_i g_{i,j} 10^{\xi_{i,j}/10} f\left(||S_j - X_i||\right) & \text{from the other mobiles } X_i \text{ covered by } S_j \\ 0 & \text{from all other mobiles, } i : X_i \notin \mathcal{A}_j \end{cases}$$

where

- P_i is the power transmitted by X_i;
- $g_{i,j}$ is the power gain due to Nakagami fading;
- $\xi_{i,j}$ is a *shadowing factor* and $\xi_{i,j} \sim N\left(0, \sigma_s^2\right)$;
- $f(\cdot)$ is a path-loss function:

$$f(d) = \left(\frac{d}{d_0}\right)^{-\alpha}$$

  - α is the path loss exponent;
  - $d \geq d_0$;
- h is the chip factor;
- G is the common spreading factor.



Despread Instantaneous Power

The despread instantaneous power of X_i received at S_j is

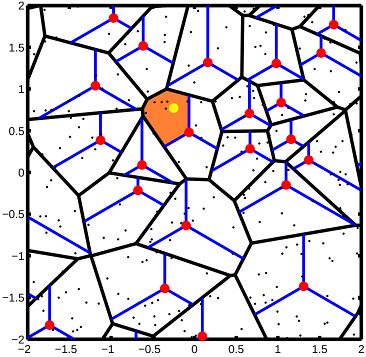
$$\rho_{i,j} = \begin{cases} P_r g_{r,j} 10^{\xi_{r,j}/10} f\left(||S_j - X_r||\right) & \text{from the reference mobile } X_r \\ \left(\frac{h}{G}\right) P_i g_{i,j} 10^{\xi_{i,j}/10} f\left(||S_j - X_i||\right) & \text{from the other mobiles } X_i \text{ covered by } S_j \\ 0 & \text{from all other mobiles, } i : X_i \notin \mathcal{A}_j \end{cases}$$

where

- P_i is the power transmitted by X_i;
- $g_{i,j}$ is the power gain due to Nakagami fading;
- $\xi_{i,j}$ is a *shadowing factor* and $\xi_{i,j} \sim N\left(0, \sigma_s^2\right)$;
- $f(\cdot)$ is a path-loss function:

$$f(d) = \left(\frac{d}{d_0}\right)^{-\alpha}$$

  - α is the path loss exponent;
  - $d \geq d_0$;
- h is the chip factor;
- G is the common spreading factor.



Despread Instantaneous Power

The despread instantaneous power of X_i received at S_j is

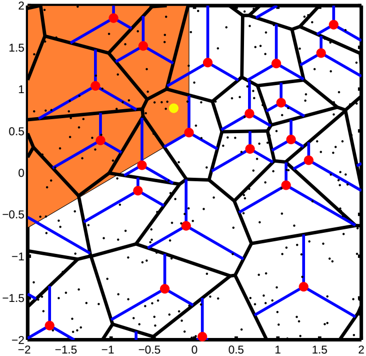
$$\rho_{i,j} = \begin{cases} P_r g_{r,j} 10^{\xi_{r,j}/10} f\left(||S_j - X_r||\right) & \text{from the reference mobile } X_r \\ \left(\frac{h}{G}\right) P_i g_{i,j} 10^{\xi_{i,j}/10} f\left(||S_j - X_i||\right) & \text{from the other mobiles } X_i \text{ covered by } S_j \\ 0 & \text{from all other mobiles, } i : X_i \notin \mathcal{A}_j \end{cases}$$

where

- P_i is the power transmitted by X_i;
- $g_{i,j}$ is the power gain due to Nakagami fading;
- $\xi_{i,j}$ is a *shadowing factor* and $\xi_{i,j} \sim N\left(0, \sigma_s^2\right)$;
- $f(\cdot)$ is a path-loss function:

$$f(d) = \left(\frac{d}{d_0}\right)^{-\alpha}$$

  - α is the path loss exponent;
  - $d \geq d_0$;
- h is the chip factor;
- G is the common spreading factor.



Despread Instantaneous Power

The despread instantaneous power of X_i received at S_j is

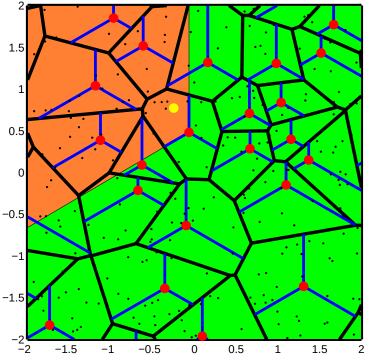
$$\rho_{i,j} = \begin{cases} P_r g_{r,j} 10^{\xi_{r,j}/10} f\left(||S_j - X_r||\right) & \text{from the reference mobile } X_r \\ \left(\frac{h}{G}\right) P_i g_{i,j} 10^{\xi_{i,j}/10} f\left(||S_j - X_i||\right) & \text{from the other mobiles } X_i \text{ covered by } S_j \\ 0 & \text{from all other mobiles, } i : X_i \notin \mathcal{A}_j \end{cases}$$

where

- P_i is the power transmitted by X_i;
- $g_{i,j}$ is the power gain due to Nakagami fading;
- $\xi_{i,j}$ is a *shadowing factor* and $\xi_{i,j} \sim N\left(0, \sigma_s^2\right)$;
- $f(\cdot)$ is a path-loss function:

$$f(d) = \left(\frac{d}{d_0}\right)^{-\alpha}$$

  - α is the path loss exponent;
  - $d \geq d_0$;
- h is the chip factor;
- G is the common spreading factor.



SINR

The performance at the sector S_j when the desired signal is from $X_r \in \mathcal{X}_j$ is characterized by the signal-to-interference and noise ratio (SINR), given by:

$$\gamma_{r,j} = \frac{g_{r,j}\Omega_{r,j}}{\Gamma^{-1} + \frac{h}{G}\sum_{\substack{i=1\\i\neq r}}^{M} g_{i,j}\Omega_{i,j}} \quad (1)$$

where

- Γ is the signal-to-noise ratio (SNR) at a mobile located at unit distance when fading and shadowing are absent;
- $\Omega_{i,j} = \frac{P_i}{P_r}10^{\xi_{i,j}/10}||S_j - X_i||^{-\alpha}$ is the normalized power of X_i received by S_j before despreading.

Outline

Definition

- An *outage* occurs when the SINR is below a threshold β.
    - β depends on the choice of modulation and coding.
- The *outage probability* of a desired signal from $X_r \in \mathcal{X}_j$ at the sector antenna S_j conditioned over the network is

$$\epsilon_r = P\left[\gamma_{r,j} \leq \beta_r \middle| \mathbf{\Omega}_j\right]. \quad (2)$$

- Substituting (1) into (2), from [8]:

$$\epsilon_r = 1 - e^{-\frac{\beta_0}{\Gamma}} \sum_{n=0}^{m_{r,j}-1} \left(\frac{\beta_0}{\Gamma}\right)^n \sum_{k=0}^{n} \frac{\Gamma^k}{(n-k)!} \sum_{\substack{\ell_i \geq 0 \\ \sum_{i=0}^{M} \ell_i = k}} \left(\prod_{\substack{i=1 \\ i \neq r}}^{M} G_{\ell_i}(\Psi_i) \right) \quad (3)$$

where $\beta_0 = \beta m_{r,j}/\Omega_0$,

$$G_\ell(\Psi_i) = \frac{\Gamma(\ell + m_{i,j})}{\ell!\Gamma(m_{i,j})} \left(\frac{\Omega_{i,j}}{m_{i,j}}\right)^\ell \left(\frac{\beta_0 h \Omega_{i,j}}{G m_{i,j}} + 1\right)^{-m_{i,j}-\ell}. \quad (4)$$

[8] D. Torrieri and M.C. Valenti, "The outage probability of a finite ad hoc network in Nakagami fading", *IEEE Trans. Commun.*, Nov. 2012.

Distance-Dependent Fading Model

- In (3) non-identical Nakagami-m parameters can be chosen to characterize the fading from the mobile X_i to the sector antenna S_j and a *distance-depending fading* model can be adopted:

$$m_{i,j} = \begin{cases} 3 & \text{if } ||S_j - X_i|| \leq r_{\text{bs}}/2 \\ 2 & \text{if } r_{\text{bs}}/2 < ||S_j - X_i| \leq r_{\text{bs}} \\ 1 & \text{if } ||S_j - X_i| > r_{\text{bs}} \end{cases} . \quad (5)$$

- The distance-dependent-fading model characterizes the situation where a mobile close to the base station is in the line-of-sight (LOS), while mobiles farther away tend to be non-LOS.

Outline

Resource Allocation

1. Power control:
   - The transmit power $\{P_i\}$ for all mobiles in the set $\mathcal{X}_j$ is selected such that, after compensation for shadowing and power-law attenuation, each mobile's transmission is received at sector antenna S_j with the same power P_0:

$$P_i 10^{\xi_{i,j}/10} f\left(||S_j - X_i||\right) = P_0, \quad X_i \in \mathcal{X}_j.$$

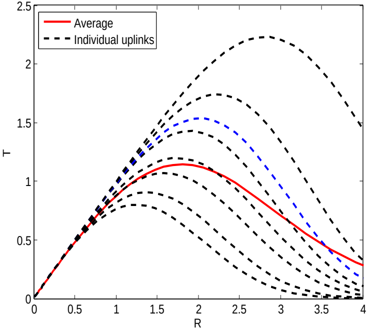
2. Rate control:
   - Let $R_j = C(\beta_j)$ represent the relationship between R_j and β_j. For modern cellular systems, it is reasonable to assume the use of a capacity-approaching code, two-dimensional signaling over an AWGN channel, and Gaussian interference, and in this case:

$$C(\beta_j) \quad = \quad \log_2(1 + \beta_j).$$

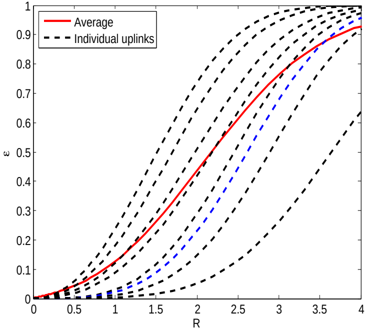
   - Let T_i indicate the throughput of the $i-th$ uplink. The throughput represents the rate of successful transmissions and is found as

$$T_i = R_i(1 - \epsilon_i). \tag{6}$$

Policies



(a) Throughput vs the rate R.

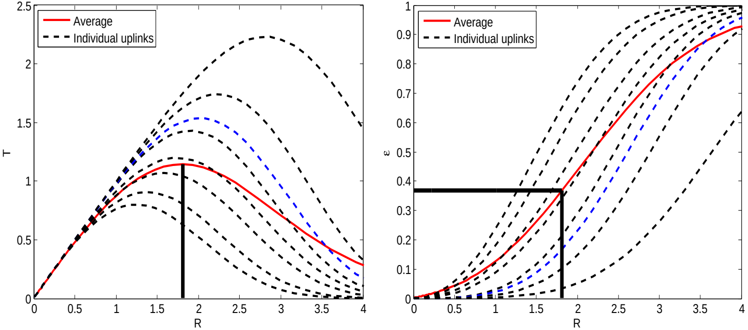


(b) Outage probability vs the rate R.

Example:

- $C = 50.$
- $M = 400.$
- $r_{net} = 2.$
- $r_{bs} = 0.25.$
- $r_m = 0.01.$
- $G = 16.$
- $h = 2/3.$
- $\Gamma = 10$ dB.
- $\alpha = 3.$
- $\sigma_s = 8$ dB.

Policies



(a) Throughput vs the rate R. (b) Outage probability vs the rate R.

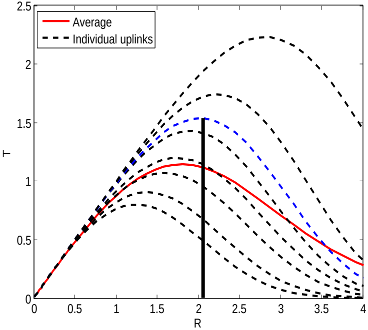
Example:

- $C = 50$.
- $M = 400$.
- $r_{net} = 2$.
- $r_{bs} = 0.25$.
- $r_m = 0.01$.
- $G = 16$.
- $h = 2/3$.
- $\Gamma = 10$ dB.
- $\alpha = 3$.
- $\sigma_s = 8$ dB.

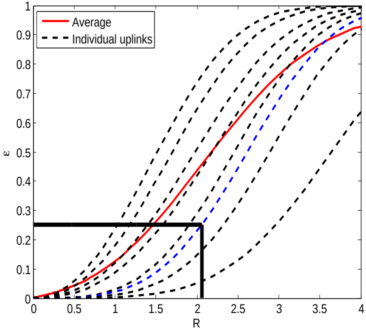
Policies:

1. **maximal-throughput fixed rate (MTFR) policy;**

Policies



(a) Throughput vs the rate R.



(b) Outage probability vs the rate R.

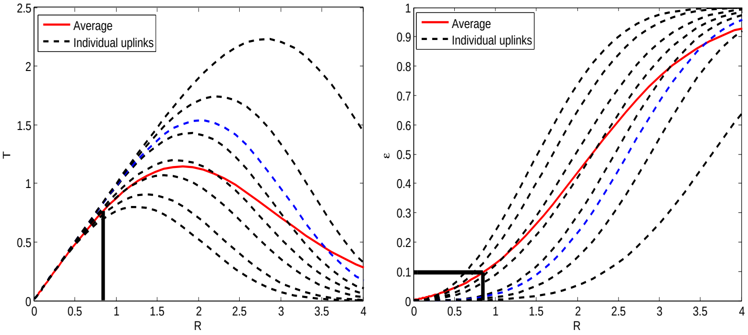
Example:

- $C = 50$.
- $M = 400$.
- $r_{net} = 2$.
- $r_{bs} = 0.25$.
- $r_m = 0.01$.
- $G = 16$.
- $h = 2/3$.
- $\Gamma = 10$ dB.
- $\alpha = 3$.
- $\sigma_s = 8$ dB.

Policies:

1. *maximal-throughput fixed rate* (MTFR) policy;
2. **maximal-throughput variable-rate (MTVR) policy;**

Policies



(a) Throughput vs the rate R.

(b) Outage probability vs the rate R.

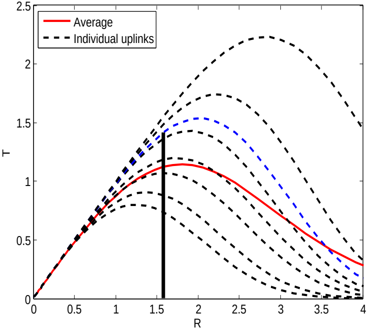
Example:

- $C = 50$.
- $M = 400$.
- $r_{net} = 2$.
- $r_{bs} = 0.25$.
- $r_m = 0.01$.
- $G = 16$.
- $h = 2/3$.
- $\Gamma = 10$ dB.
- $\alpha = 3$.
- $\sigma_s = 8$ dB.

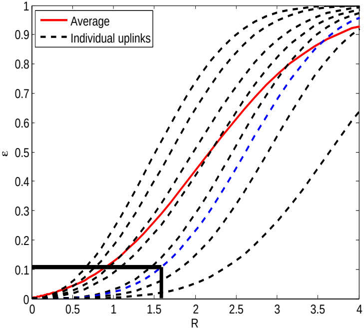
Policies:

1. *maximal-throughput fixed rate* (MTFR) policy;
2. *maximal-throughput variable-rate* (MTVR) policy;
3. **outage-constrained fixed rate (OCFR) policy ($\mathbb{E}[\epsilon] = \zeta$);**

Policies



(a) Throughput vs the rate R.



(b) Outage probability vs the rate R.

Example:

- $C = 50$.
- $M = 400$.
- $r_{net} = 2$.
- $r_{bs} = 0.25$.
- $r_m = 0.01$.
- $G = 16$.
- $h = 2/3$.
- $\Gamma = 10$ dB.
- $\alpha = 3$.
- $\sigma_s = 8$ dB.

Policies:

1. *maximal-throughput fixed rate* (MTFR) policy;
2. *maximal-throughput variable-rate* (MTVR) policy;
3. *outage-constrained fixed rate* (OCFR) policy ($\mathbb{E}[\epsilon] = \zeta$);
4. **outage-constrained variable-rate (OCVR) policy ($\epsilon_i = \zeta$).**

Outline

Transmission Capacity

- The performance metric used is the *transmission capacity*, defined as

$$\tau = \lambda \mathbb{E}[T] = \lambda \mathbb{E}\left[(1-\epsilon) R\right] \quad (7)$$

where

- $\lambda = M/A_{net}$ is the density of transmissions in the network;
- $\mathbb{E}[T]$ is computed using a Monte Carlo approach as follows:
  1. Draw a realization of the network;
  2. Compute the path loss from each base station to each mobile;
  3. Determine the set of mobiles associated with each base station;
  4. Determine the set of mobiles associated with each cell sector;
  5. Apply a denial policy if there are more than G mobiles in a cell sector;
  6. Apply at sector antenna the power-control policy and the rate-control;
  7. Determine the outage probability conditioned over the topology ϵ_j by (3);
  8. By applying the function $R_j = C(\beta_j)$, find the rate for the mobile;
  9. Compute the throughput by (6);
  10. Repeat this process for a large number of networks.

Example: Policy Comparison

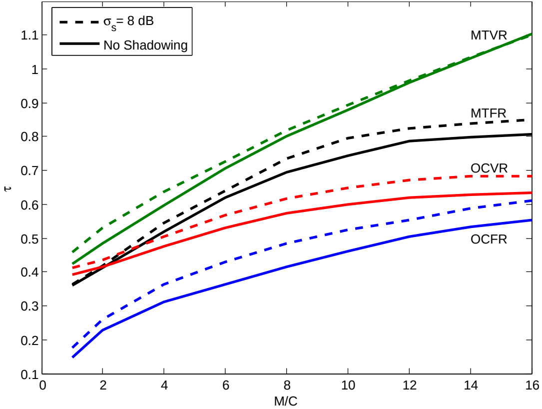


Figure: *Transmission capacity for the four network policies as function of the load M/C for distance-dependent fading and both shadowed (σ_s = 8 dB) and unshadowed cases.*

Example:

- $M = 50$ base stations;
- Circular arena with $r_{net} = 2$;
- $r_{bs} = 0.25$;
- $r_m = 0.01$;
- $\alpha = 3$;
- $\Gamma = 10$ dB;
- $h = 2/3$;
- $G = 16$;
- $\zeta = 0.1$ for OCFR and OCVR.

Recall:

- MT = Maximal-throughput;
- OC = Outage-constrained;
- FR = Fixed-rate;
- VR = Variable-rate.

Example: Spreading Factor

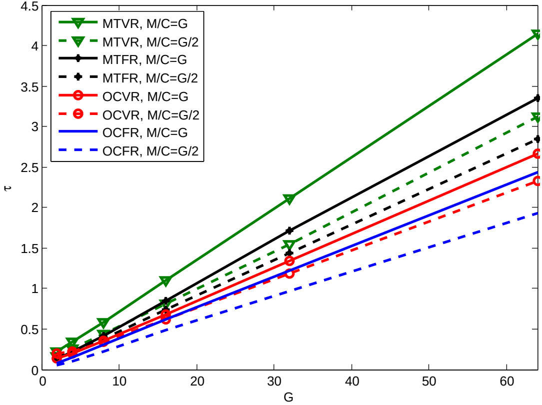


Figure: *Transmission capacity as function of spreading factor G for two values of system load, distance-dependent fading, and shadowing with $\sigma_s = 8$ dB.*

Example:

- $M = 50$ base stations;
- Circular arena with $r_{net} = 2$;
- $r_{bs} = 0.25$;
- $r_m = 0.01$;
- $\alpha = 3$;
- $\Gamma = 10$ dB;
- $h = 2/3$;
- $\zeta = 0.1$ for OCFR and OCVR;
- $\sigma_s = 8$ dB.

Recall:

- MT = Maximal-throughput;
- OC = Outage-constrained;
- FR = Fixed-rate;
- VR = Variable-rate.

Example: Minimum Distance between Base-Stations

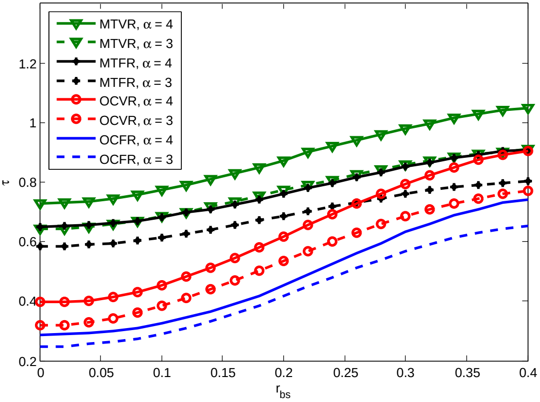


Figure: *Transmission capacity as a function of the base-station exclusion-zone radius r_{bs} for four policies and two values of path-loss exponent α.*

Example:

- $M = 50$ base stations;
- Circular arena with $r_{net} = 2$;
- $r_m = 0.01$;
- $\Gamma = 10$ dB;
- $h = 2/3$;
- $G = 16$;
- $\zeta = 0.1$ for OCVR and OCFR;
- $\sigma_s = 8$ dB.

Recall:

- MT $=$ Maximal-throughput;
- OC $=$ Outage-constrained;
- FR $=$ Fixed-rate;
- VR $=$ Variable-rate.

Outline

Conclusions

- The new approach for modeling and analyzing the DS-CDMA cellular uplink has the following benefits:
  - the model allows constraints to be placed on the distance between base stations, the geographic footprint of the network, and the number of base stations and mobiles;
  - a flexible channel model, accounting for path loss, shadowing, and Nakagami-m fading with non-identical parameters, is considered.
- The approach is general enough and it can be extended:
  - to compare various access and resource allocation techniques;
  - to analyze reselection schemes;
  - to model other types of access, such as orthogonal frequency-division multiple access (OFDMA).
- See journal version for more details:
  D. Torrieri, M.C. Valenti and S. Talarico, "An Analysis of the DS-CDMA Cellular Uplink for Arbitrary and Constrained Topologies", *IEEE Trans. Commun.*, to appear.

Thank You