Limited-Feedback-Based Channel-Aware Power Allocation for Linear Distributed Estimation

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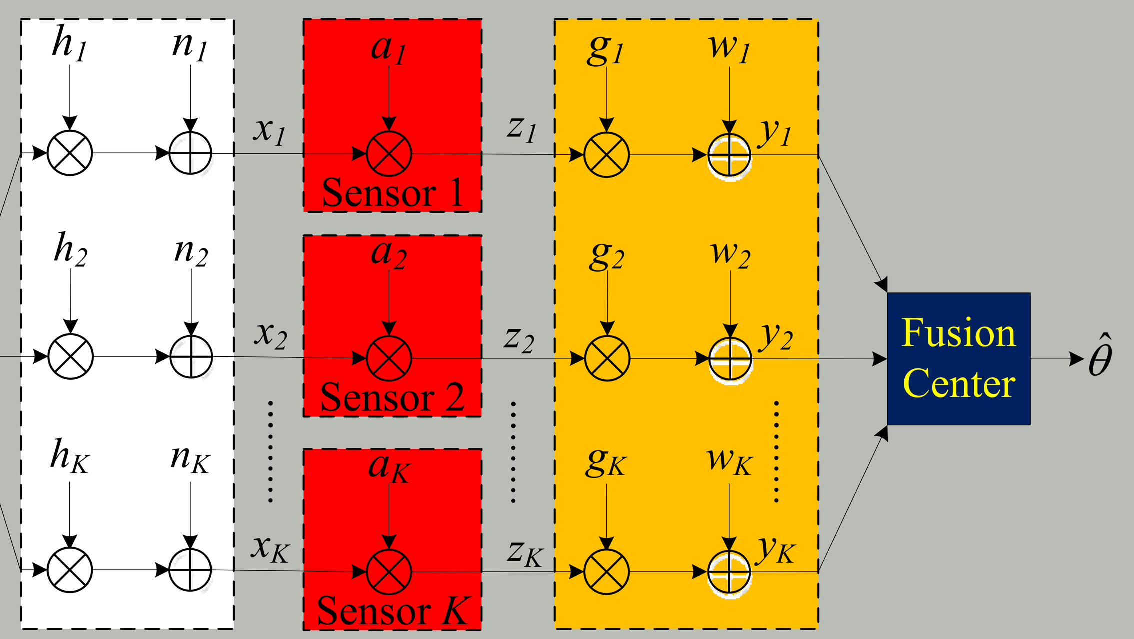
Introduction

- One of the most important applications of wireless sensor networks (WSNs) is distributed estimation.
- When distributed sensors use an amplify-and-forward scheme to process their local noisy observations, the study of the allocation of transmit power to the sensors becomes important.
- Most optimal power-allocation schemes in the literature require the feedback of the exact instantaneous channel state information (CSI) from the FC to local sensors, which is not practical.
- We use limited feedback to alleviate this requirement.

System Model



- Unknown random parameter θ with zero mean and unit power.
- Linear observation model: $x_i = h_i\theta + n_i$
  - n_i is i.i.d. additive observation noise with zero mean and known variance σ_o^2.
  - Define observation SNR as $\beta_i \stackrel{\text{def}}{=} \frac{|h_i|^2}{\sigma_o^2}$.
- Linear amplify-and-forward local processing: $z_i = a_i x_i$
  - Instantaneous transmit power of a sensor:

$$P_i = a_i^2\left(|h_i|^2 + \sigma_o^2\right) = a_i^2\sigma_o^2\left(1+\beta_i\right)$$

- Orthogonal coherent fading channel: $y_i = g_i z_i + w_i$
  - w_i is i.i.d. additive white Gaussian noise with zero mean and known variance σ_c^2.
  - Define channel SNR as $\gamma_i \stackrel{\text{def}}{=} \frac{|g_i|^2}{\sigma_c^2}$.
- The FC finds the best linear unbiased estimator (BLUE) of θ:

$$\hat{\theta} = \left(\sum_{i=1}^{K}\frac{h_i^2a_i^2g_i^2}{a_i^2g_i^2\sigma_o^2+\sigma_c^2}\right)^{-1}\sum_{i=1}^{K}\frac{h_ia_ig_iy_i}{a_i^2g_i^2\sigma_o^2+\sigma_c^2}$$

- The variance of the BLUE estimator is

$$\mathrm{Var}\left(\hat{\theta}\middle|\mathbf{a},\mathbf{g}\right) = \left(\sum_{i=1}^{K}\frac{h_i^2a_i^2g_i^2}{a_i^2g_i^2\sigma_o^2+\sigma_c^2}\right)^{-1} = \left(\sum_{i=1}^{K}\frac{\beta_i\gamma_ia_i^2\sigma_o^2}{1+\gamma_ia_i^2\sigma_o^2}\right)^{-1}$$

  - Define $\mathbf{a} \stackrel{\text{def}}{=} [a_1, a_2, \ldots, a_K]^T$ and $\mathbf{g} \stackrel{\text{def}}{=} [g_1, g_2, \ldots, g_K]^T$.

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Problem Statement for Our Case Study

- Cui et al. [5] have derived optimal local amplification gains to minimize the BLUE-estimator variance, given a cumulative transmission-power constraint P_{Total}:

$$a_i = \begin{cases}\sqrt{\frac{1}{\gamma_i\sigma_o^2}\left(\sqrt{\delta_i}\,\rho(K_1)-1\right)}, & i \le K_1\\ 0, & i > K_1\end{cases} \qquad \rho(n) \stackrel{\text{def}}{=} \frac{P_{\text{Total}} + \sum_{i=1}^{n}\frac{\beta_i}{\delta_i}}{\sum_{i=1}^{n}\frac{\beta_i}{\sqrt{\delta_i}}}$$

  - Define $\delta_i \stackrel{\text{def}}{=} \frac{\beta_i\gamma_i}{1+\beta_i}$ and sort the sensors so that $\delta_1 \ge \delta_2 \ge \cdots \ge \delta_K$.
  - K_1: *Unique* largest integer that $\sqrt{\delta_{K_1}}\,\rho(K_1) > 1$ and $\sqrt{\delta_{K_1+1}}\,\rho(K_1+1) \le 1$.
- The above optimal power-allocation scheme is based on the assumption that the complete uplink CSI is available at local sensors, which is not practical in most applications, especially in large-scale WSNs.

Proposed Solution: Limited Feedback of CSI

- An optimal codebook is designed offline by quantizing the space of the optimized power-allocation vectors using the generalized Lloyd algorithm with modified distortion functions.
- For each channel realization, the FC:
  - Finds the optimal power-allocation scheme using the perfect backward CSI.
  - Feeds back the index of the closest codeword in the optimal codebook to the optimal power-allocation vector.

Designing Optimal Codebook Using Lloyd Algorithm

- Define the conditional codeword distortion for any codeword $\mathbf{a}_\ell \in \mathcal{C}$, when it is used instead of $\mathbf{a}^{\text{OPT}}$:

$$D_{\text{W}}\left(\mathbf{a}_\ell|\mathbf{g}\right) \stackrel{\text{def}}{=} \left|\mathrm{Var}\left(\hat{\theta}\middle|\mathbf{a}_\ell,\mathbf{g}\right) - \mathrm{Var}\left(\hat{\theta}\middle|\mathbf{a}^{\text{OPT}},\mathbf{g}\right)\right|$$

- Define the average codebook distortion for $\mathcal{C}$:

$$D_{\text{B}}\left(\mathcal{C}\right) \stackrel{\text{def}}{=} \mathbb{E}_{\mathbf{g}}\left[\min_{\ell\in\{1,2,\ldots,2^L\}} D_{\text{W}}\left(\mathbf{a}_\ell|\mathbf{g}\right)\right]$$

  - L is the number of feedback bits broadcast by the FC.
- **Nearest–Neighbor Condition**
  - Find $|\mathcal{C}| = 2^L$ optimal Voronoi cells of the vector space to be quantized $\mathcal{A}$, given a fixed codebook $\mathcal{C}$.
  - Each point $\mathbf{a} \in \mathcal{A}$ is assigned to partition ℓ represented by codeword $\mathbf{a}_\ell \in \mathcal{C}$ if and only if its distance to codeword $\mathbf{a}_\ell$, with respect to the conditional codeword distortion function, is less than its distance to any other codeword.
- **Centroid Condition**
  - Find the optimal codebook, given a specific partitioning of the vector space to be quantized $\mathcal{A}$.
  - The optimal codeword associated with each Voronoi cell $\mathcal{A}_\ell \subseteq \mathcal{A}$ is the centroid of that cell with respect to the conditional codeword-distortion function:

$$\mathbf{a}_\ell = \arg\min_{\mathbf{a}\in\mathcal{A}_\ell} \mathbb{E}_{\mathbf{g}\in\mathcal{G}_\ell}\left[D_{\text{W}}\left(\mathbf{a}|\mathbf{g}\right)\right]$$

Main Reference

- S. Cui, J.-J. Xiao, A.J. Goldsmith, Z.-Q. Luo, and H.V. Poor, "Estimation diversity and energy efficiency in distributed sensing," *IEEE Transactions on Signal Processing*, vol. 55, no. 9, pp. 4683–4695, September 2007.

Details of Our Codebook-Design Process

$\mathcal{G}_s \longleftarrow$ A set of M length-K vectors of channel-fading realizations.
$\mathcal{A}_s \longleftarrow$ The set of optimal local power-allocation vectors associated with the channel fading vectors in $\mathcal{G}_s$.

$\mathbf{a}_\ell^0 \longleftarrow$ *Randomly* select 2^L optimal power-allocation vectors from the set $\mathcal{A}_s$ ($\ell = 1, 2, \ldots, 2^L$).

$\mathcal{C}^0 \longleftarrow \left[\mathbf{a}_1^0\ \mathbf{a}_2^0\ \cdots\ \mathbf{a}_{2^L}^0\right]^T$
NewCost $\longleftarrow D_{\text{B}}\left(\mathcal{C}^0\right)$
$j \longleftarrow 0$

$j \longleftarrow j+1$
OldCost $\longleftarrow$ NewCost

Nearest-Neighbor Condition
Given codebook $\mathcal{C}^{j-1}$, optimally partition the set $\mathcal{A}_s$ into 2^L disjoint subsets $\mathcal{A}_\ell^{j-1}$, $\ell = 1, 2, \ldots, 2^L$.

$\ell \longleftarrow 0$

$\ell \longleftarrow \ell + 1$

Centroid Condition
$\mathbf{a}_\ell^j \longleftarrow$ Find the optimal codeword associated with partition $\mathcal{A}_\ell^{j-1}$.

$\ell \le 2^L$ — Yes / No

$\mathcal{C}^j \longleftarrow \left[\mathbf{a}_1^j\ \mathbf{a}_2^j\ \cdots\ \mathbf{a}_{2^L}^j\right]^T$
NewCost $\longleftarrow D_{\text{B}}\left(\mathcal{C}^j\right)$

OldCost$-$NewCost $\le \epsilon$ — No / Yes

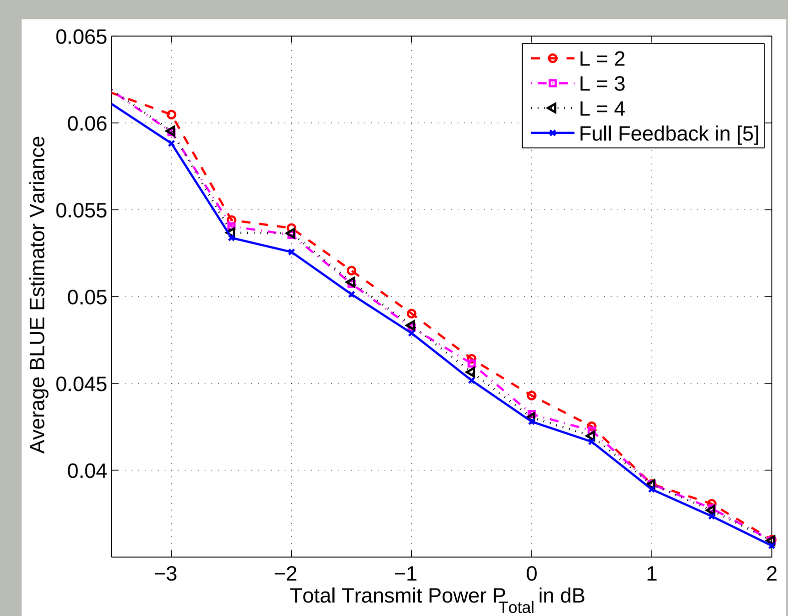
Return $\mathcal{C}^{\text{OPT}} \longleftarrow \mathcal{C}^j$

System Parameters in Numerical Simulations

Parameter	Value
K	5 and 10
L	2, 3, and 4
σ_θ^2	1
σ_o^2	10 dBm
σ_c^2	-90 dBm
$h_i \sim \mathcal{N}(1, 0.09)$ and $\mathbb{E}\left[\lvert h_i\rvert^2\right] = 1.2$	
g_i	$\eta_0\left(\frac{d_i}{d_0}\right)^{-\alpha} f_i$
d_0	1
η_0	-30 dB
α	2
d_i	Uniform between 50 and 150
f_i: i.i.d. Rayleigh fading with unit variance	
M	$5{,}000 \gg 2^L$
ϵ	10^{-6}
50,000 Monte-Carlo simulations	

Numerical Results

For $K = 5$ sensors in the network.



For $K = 10$ sensors in the network.