Power Allocation for Distributed BLUE Estimation with Full and Limited Feedback of CSI

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Military Communications Conference (Milcom'13)
Track 1: Waveforms and Signal Processing

November 18, 2013

Outline

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What Is Distributed Estimation in WSNs?

Spatially distributed sensors

- Observe their surrounding environment.
- Process their local observations.
- Send their processed data to a fusion center (FC).
- FC performs the ultimate global estimation.

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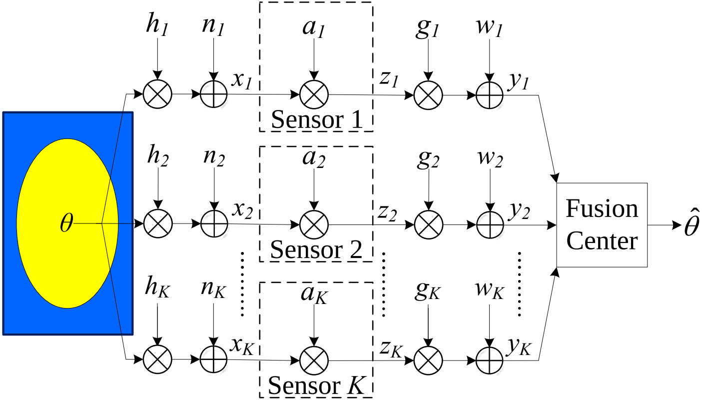
Goal of This Paper

Adaptive power allocation to local sensors for *reliable* estimation of an unknown random parameter at the fusion center of a WSN from spatially distributed linear noisy observations by local sensors and communicated through parallel fading channels.

Outline

System Model Description



Unknown random parameter θ with zero mean and unit power, otherwise unknown.

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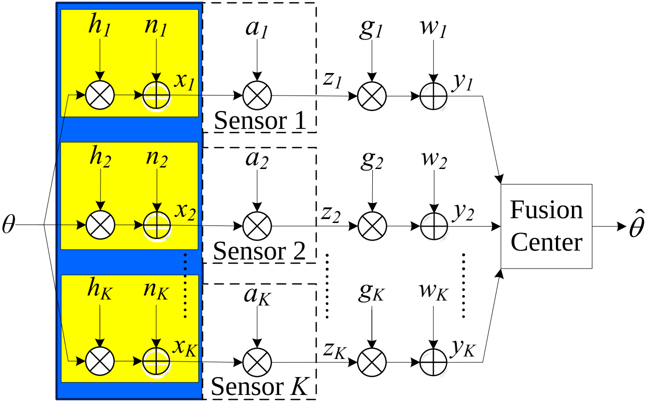

- Local linear noisy observation at each sensor

$$x_i = h_i \theta + n_i$$

- h_i: Fixed local observation gain known at the sensor and FC.

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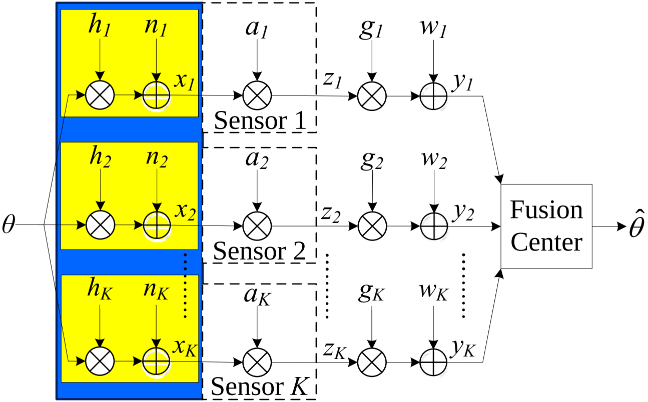

- Local linear noisy observation at each sensor

$$x_i = h_i\theta + n_i$$

- n_i: Spatially i.i.d. additive observation noise with zero mean and known variance σ_o^2, otherwise unknown.

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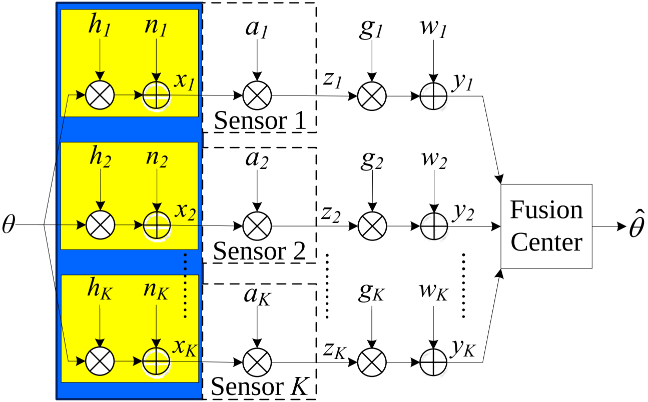

- Local linear noisy observation at each sensor

$$x_i = h_i\theta + n_i$$

- Observation SNR at sensor i: $\beta_i \overset{\text{def}}{=} \frac{|h_i|^2}{\sigma_o^2}$

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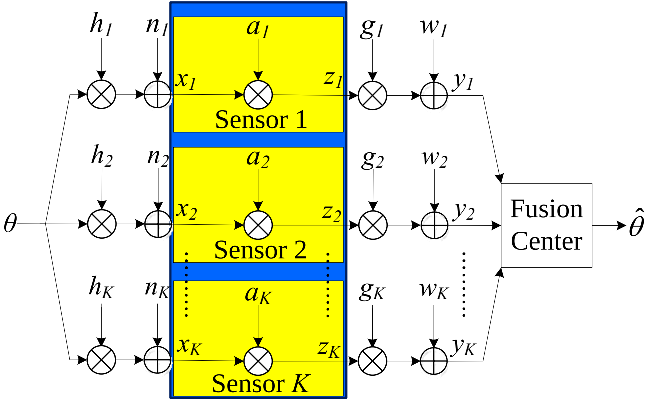

Amplify-and-forward (AF) local processing scheme

- No inter-sensor communication and/or collaboration.
- Each sensor acts as a relay and transmits amplified version of its raw noisy analog local observation to the fusion center.

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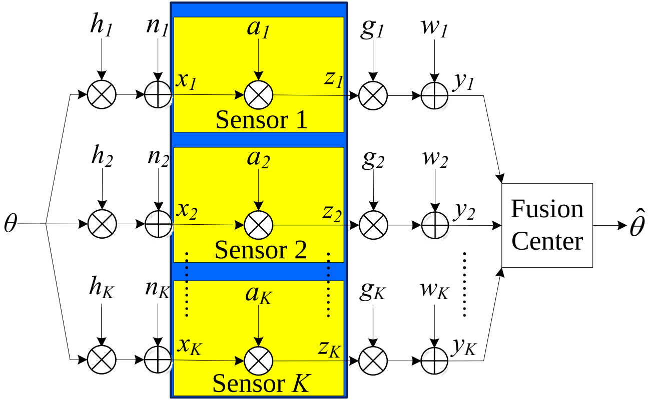

Amplify-and-forward (AF) local processing scheme

$$z_i = a_i x_i = a_i h_i \theta + a_i n_i$$

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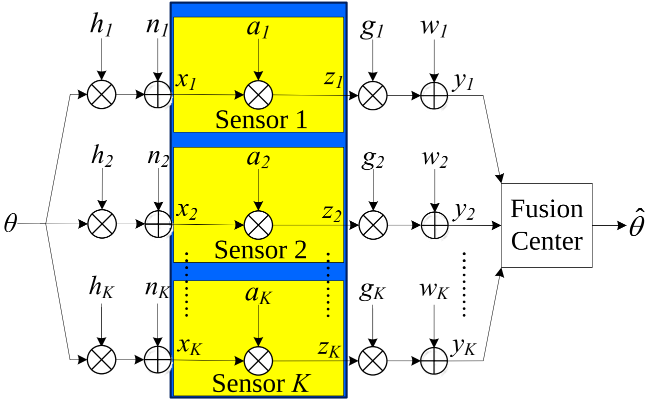


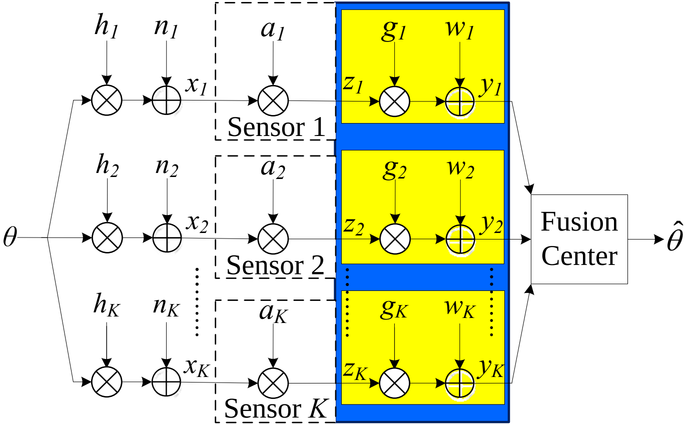
Amplify-and-forward (AF) local processing scheme

$$z_i = a_i x_i = a_i h_i \theta + a_i n_i$$

- Instantaneous transmit power

$$P_i = a_i^2 \left(|h_i|^2 + \sigma_0^2 \right) = a_i^2 \sigma_0^2 (1 + \beta_i)$$

System Model Description



Orthogonal coherent channels corrupted by independent fading and additive Gaussian noise: $y_i = g_i z_i + w_i \qquad w_i \sim \mathcal{N}\left(0, \sigma_{\mathsf{c}}^2\right)$

Channel gains are completely *known* at the fusion center.

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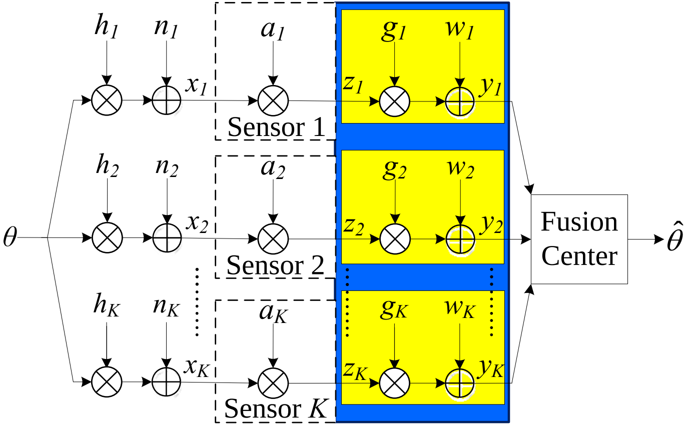

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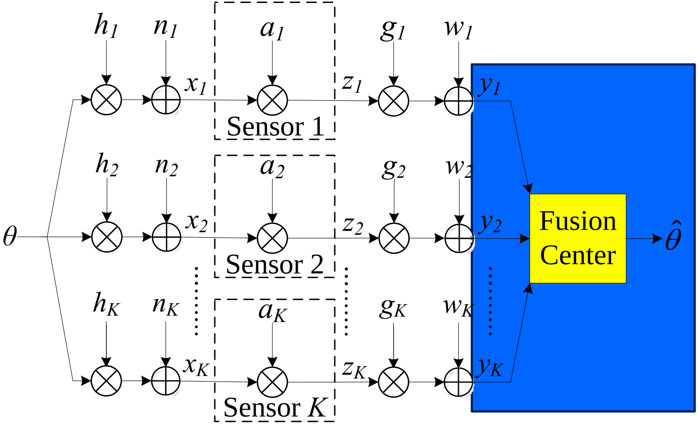

The FC finds the *best linear unbiased estimator* (BLUE) for θ.

BLUE Estimation of θ at the FC

- Given $\mathbf{a} \triangleq [a_1, a_2, \ldots, a_K]^T$ and a realization of the fading channel gains $\mathbf{g} \triangleq [g_1, g_2, \ldots, g_K]^T$, the BLUE estimator of θ at the FC is

$$\widehat{\theta} = \left(\sum_{i=1}^{K} \frac{h_i^2 a_i^2 g_i^2}{a_i^2 g_i^2 \sigma_{\mathrm{o}}^2 + \sigma_{\mathrm{c}}^2} \right)^{-1} \sum_{i=1}^{K} \frac{h_i a_i g_i y_i}{a_i^2 g_i^2 \sigma_{\mathrm{o}}^2 + \sigma_{\mathrm{c}}^2}$$

- The estimator variance is

$$\begin{aligned} \mathrm{Var}\left(\widehat{\theta} \middle| \mathbf{a}, \mathbf{g}\right) &= \left(\sum_{i=1}^{K} \frac{h_i^2 a_i^2 g_i^2}{a_i^2 g_i^2 \sigma_{\mathrm{o}}^2 + \sigma_{\mathrm{c}}^2} \right)^{-1} \\ &= \left(\sum_{i=1}^{K} \frac{\beta_i \gamma_i a_i^2 \sigma_{\mathrm{o}}^2}{1 + \gamma_i a_i^2 \sigma_{\mathrm{o}}^2} \right)^{-1} \end{aligned}$$

Outline

Problem Formulation

Problem Statement for Our Optimal Power-Allocation Scheme

Find the optimal local amplification gains or equivalently, the optimal power-allocation scheme that minimizes the L^2-norm of the vector of local transmit powers $\mathbf{P} \triangleq [P_1, P_2, \ldots, P_K]^T$, given a maximum estimation distortion as defined by the BLUE estimator variance.

$$\begin{aligned} \underset{\{P_i\}_{i=1}^{K}}{\text{minimize}} \quad & \left(\sum_{i=1}^{K} P_i^2 \right)^{\frac{1}{2}} \\ \text{subject to} \quad & \text{Var}\left(\widehat{\theta} \middle| \mathbf{a}, \mathbf{g}\right) \leq D_0 \end{aligned}$$

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$$\begin{aligned} \underset{\{a_i\}_{i=1}^{K}}{\text{minimize}} \quad & \sum_{i=1}^{K} \left[{a_i}^2 \sigma_0^2 \left(1+\beta_i\right)\right]^2 \\ \text{subject to} \quad & \sum_{i=1}^{K} \frac{\beta_i \gamma_i a_i^2 \sigma_0^2}{1+\gamma_i a_i^2 \sigma_0^2} \geq \frac{1}{D_0} \end{aligned}$$

Optimal Local Amplification Gains

The unique solution for the given constrained *convex* optimization problem could be found as follows:

$$a_i^2 = \begin{cases} \frac{1}{\gamma_i \sigma_o^2} \left(\frac{\sqrt[3]{\frac{\lambda_0}{\beta_i \delta_i^2 T_i}}}{1 - \frac{2}{3}\sqrt[3]{\frac{\beta_i \delta_i^2}{\lambda_0 T_i^2}}} - 1 \right), & i \leq K_1 \\ 0, & i > K_1 \end{cases}$$

$$T_i = 1 + \sqrt{1 + \frac{8\beta_i \delta_i^2}{27\lambda_0}} \qquad \delta_i \overset{\text{def}}{=} \frac{1+\beta_i}{\beta_i \gamma_i} \qquad \beta_i \overset{\text{def}}{=} \frac{|h_i|^2}{\sigma_o^2} \text{ and } \gamma_i \overset{\text{def}}{=} \frac{|g_i|^2}{\sigma_c^2}$$

- The sensors are sorted so that $\delta_1 \leq \delta_2 \leq \cdots \leq \delta_K$.
  - The observation SNR β_i or channel SNR γ_i decreases.
- *Only* the first K_1 sensors with the least values of δ_i will have a positive value for a_i, and $a_i = 0$ for all $i > K_1$.

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- The values of the number of active sensors K_1 for which $a_i > 0$, and λ_0 are *unique* and can be found using the following equation:

$$\sum_{i=1}^{K_1} \beta_i \sqrt[3]{\frac{\beta_i \delta_i^2 T_i}{\lambda_0}} \left(1 - \frac{2}{3} \sqrt[3]{\frac{\beta_i \delta_i^2}{\lambda_0 T_i^2}} \right) = \sum_{i=1}^{K_1} \beta_i - \frac{1}{D_0}$$

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- **Solution:** Water-filling-based iterative process to solve this equation:

$$\sum_{i=1}^{K_1} \beta_i \sqrt[3]{\frac{\beta_i \delta_i^2 T_i}{\lambda_0}} \left(1 - \frac{2}{3} \sqrt[3]{\frac{\beta_i \delta_i^2}{\lambda_0 T_i^2}} \right) = \sum_{i=1}^{K_1} \beta_i - \frac{1}{D_0}$$

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1. Introduction and Problem Statement
2. System Model Description
3. Adaptive Optimal Power-Allocation Scheme
4. Limited Feedback for Adaptive Power Allocation
5. Numerical Results
6. Conclusions

What Does a Sensor Need to Find a_i?

$$a_i^2 = \begin{cases} \frac{1}{\gamma_i \sigma_o^2} \left(\frac{\sqrt[3]{\frac{\lambda_0}{\beta_i \delta_i^2 T_i}}}{1 - \frac{2}{3} \sqrt[3]{\frac{\beta_i \delta_i^2}{\lambda_0 T_i^2}}} - 1 \right), & i \leq K_1 \\ 0, & i > K_1 \end{cases}$$

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- **Assumption:** Complete *forward CSI* is available at local sensors.
  - *Not* practical in most applications, especially in large-scale WSNs.
  - Feedback information is typically transmitted through finite-rate *digital* feedback links.

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- Type of required full-feedback from the FC to local sensors:
  1. Instantaneous amplification gain a_i.
  2. Fading coefficient of the channel between each sensor and the FC g_i, K_1 and λ_0.
     - λ_0 and an extra one-bit command instructing the sensor to transmit or stay silent.
     - Entire vector of $\mathbf{g}$ sent by the FC over a broadcast channel.

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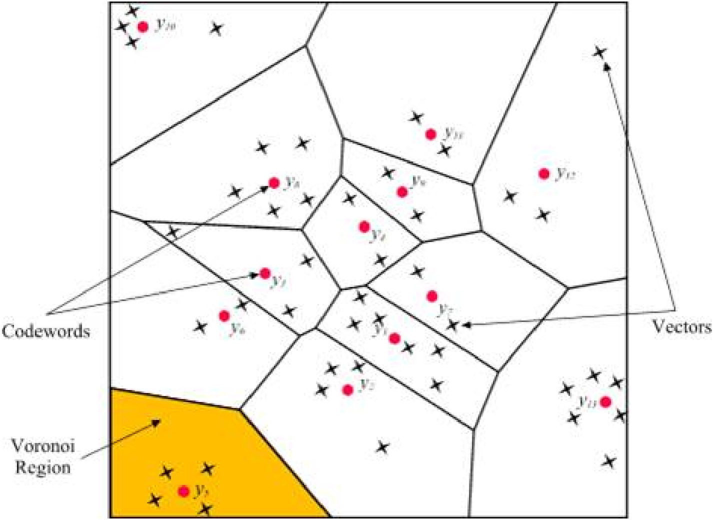
- **Solution:** Limited feedback of instantaneous amplification gain a_i
  - For each channel realization $\mathbf{g}$, the FC first finds the *exact* optimal power-allocation scheme using perfect *backward* CSI.
  - The FC sends back the *index* of the quantized version of the optimized power-allocation vector to all sensors.

Main Idea of Limited-Feedback Strategy

A *codebook* of the local amplification gains is required.

- The optimal codebook is designed *offline* by quantizing the space of the optimized power-allocation vectors using the *generalized Lloyd algorithm* with modified distortion metrics.
- L: The number of *total* feedback bits from the FC to sensors.
- The space of the optimal local power-allocation vectors is quantized into 2^L disjoint regions.
- A codeword is chosen in each quantization region.
- Codeword length is K. Its ith entry is a *real-valued* quantized version of the optimal local amplification gain for sensor i.

Overview on Generalized Lloyd Quantization



Inputs and Output of Generalized Lloyd Algorithm

- Inputs to Lloyd algorithm
  - K: The number of spatially distributed sensors.
  - L: The number of *total* feedback bits from the FC to sensors.
    - *Not* the number of bits fed back to each sensor.
  - Fading model of the channel between local sensors and the FC.
  - M: The number of *training vectors* in space $\mathcal{A}$.
  - ε: Distortion threshold to stop the codebook-design iterations.

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- Output of Lloyd algorithm
  - $\mathbf{C}^{\mathsf{OPT}}$: Codebook matrix of the optimal local amplification gains.
  - $\mathbf{C} = [\mathbf{a}_1\ \mathbf{a}_2\ \cdots\ \mathbf{a}_{2^L}]^T$ is a $2^L \times K$ matrix.
  - $[\mathbf{C}]_{\ell,i}$ denotes the element in row ℓ and column i as the optimal gain of sensor i in codeword ℓ.
  - Each $\mathbf{a}_\ell$, $\ell = 1, 2, \ldots, 2^L$ is associated with a realization of the fading coefficients of the channels between local sensors and the FC.

Initialization of Generalized Lloyd Algorithm

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- *Randomly* select 2^L optimal power-allocation vectors from the set $\mathcal{A}_s$ as the initial set of codewords.
  - Denote each codeword by $\mathbf{a}_\ell^0$, $\ell = 1, 2, \ldots, 2^L$.

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  - Denote each codeword by $\mathbf{a}_\ell^0$, $\ell = 1, 2, \ldots, 2^L$.
- Set the initial codebook as $\mathbf{C}^0 \longleftarrow \left[\mathbf{a}_1^0 \;\; \mathbf{a}_2^0 \;\; \cdots \;\; \mathbf{a}_{2^L}^0\right]^T$.
- Find the *average distortion of codebook*.
  - Set NewCost $\longleftarrow D_{\mathsf{B}}\left(\mathbf{C}^0\right)$ and $j \longleftarrow 0$.

Modified Distortions for Generalized Lloyd Algorithm

- Let $\mathbf{P}_\ell$ and $\mathbf{P}$ be the vectors of local transmit powers, when the vector of local amplification gains is $\mathbf{a}_\ell$ and $\mathbf{a}$, respectively.

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- Optimization cost of a power-allocation vector

$$J(\mathbf{a}) = \left(\sum_{i=1}^{K} P_i^2 \right)^{\frac{1}{2}} = \left(\sum_{i=1}^{K} \left[a_i^2 \sigma_o^2 (1 + \beta_i) \right]^2 \right)^{\frac{1}{2}}$$

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- Distance between codeword $\mathbf{a}_\ell$ and the optimal power-allocation vector $\mathbf{a}$

$$D_{\mathrm{W}}(\mathbf{a}_\ell, \mathbf{a}) \triangleq |J(\mathbf{a}_\ell) - J(\mathbf{a})|$$

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- Average distortion for codebook $\mathbf{C}$

$$D_{\mathsf{B}}(\mathbf{C}) \triangleq \mathbb{E}_{\mathbf{a}}\left[\min_{\ell \in \{1,2,\ldots,2^L\}} D_{\mathsf{W}}(\mathbf{a}_\ell, \mathbf{a})\right]$$

Detailed Opertaion of Generalized Lloyd Algorithm

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- $j \longleftarrow j+1$ and OldCost $\longleftarrow$ NewCost.
- Given codebook $\mathbf{C}^{j-1}$, optimally partition the set $\mathcal{A}_s$ into 2^L disjoint subsets and denote the resulted optimal partitions by $\mathcal{A}_\ell^{j-1}$, $\ell = 1, 2, \ldots, 2^L$.
  - **Nearest-Neighbor Condition** finds the optimal *Voronoi cells* of the vector space to be quantized, given a fixed codebook.
  - Each point $\mathbf{a} \in \mathcal{A}_s$ is assigned to partition ℓ represented by codeword $\mathbf{a}_\ell \in \mathbf{C}^{j-1}$ if and only if its distance to codeword $\mathbf{a}_\ell$, with respect to the defined distance function, is less than its distance to any other codeword in the codebook.
  - Given a codebook $\mathbf{C}^{j-1}$, the space $\mathcal{A}_s$ is divided into 2^L disjoint quantization regions (or Voronoi cells) with the ℓth region defined as

$$\mathcal{A}_\ell = \{\mathbf{a} \in \mathcal{A}_s : D_{\mathsf{W}}(\mathbf{a}_\ell, \mathbf{a}) \leq D_{\mathsf{W}}(\mathbf{a}_k, \mathbf{a}), \forall k \neq \ell\}$$

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- Given codebook $\mathbf{C}^{j-1}$, optimally partition the set $\mathcal{A}_s$ into 2^L disjoint subsets and denote the resulted optimal partitions by $\mathcal{A}_\ell^{j-1}$.
- For every $\mathcal{A}_\ell^{j-1}$, $\ell = 1, 2, \ldots, 2^L$:
  - Find the optimal codeword associated with partition $\mathcal{A}_\ell^{j-1}$. Denote this new optimal codeword as $\mathbf{a}_\ell^j$.

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- Find the optimal codeword associated with partition $\mathcal{A}_\ell^{j-1}$.
  - **Centroid Condition** finds the optimal codebook, given a specific partitioning of the vector space.
  - The optimal codeword associated with each Voronoi cell $\mathcal{A}_\ell \subseteq \mathcal{A}_s$ is the *centroid* of that cell with respect to the defined distance function.
  - Given a specific partitioning $\{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_{2^L}\}$, the optimal codeword associated with partition $\mathcal{A}_\ell$ is defined as

$$\mathbf{a}_\ell^\star = \arg\min_{\mathbf{a}_\ell \in \mathcal{A}_\ell} \mathbb{E}_{\mathbf{a} \in \mathcal{A}_\ell} \left[D_{\mathsf{W}} \left(\mathbf{a}_\ell, \mathbf{a} \right) \right]$$

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- For every $\mathcal{A}_\ell^{j-1}$, $\ell = 1, 2, \ldots, 2^L$:
  - Find the optimal codeword associated with partition $\mathcal{A}_\ell^{j-1}$. Denote this new optimal codeword as $\mathbf{a}_\ell^j$.
- Form the new codebook $\mathbf{C}^j \longleftarrow \begin{bmatrix} \mathbf{a}_1^j & \mathbf{a}_2^j & \cdots & \mathbf{a}_{2^L}^j \end{bmatrix}^T$.
  - Set NewCost $\longleftarrow D_{\mathsf{B}}\left(\mathbf{C}^j\right)$.

Detailed Opertaion of Generalized Lloyd Algorithm

Repeat the following steps:

- $j \longleftarrow j+1$ and OldCost $\longleftarrow$ NewCost.
- Given codebook $\mathbf{C}^{j-1}$, optimally partition the set $\mathcal{A}_s$ into 2^L disjoint subsets and denote the resulted optimal partitions by $\mathcal{A}_\ell^{j-1}$.
- For every $\mathcal{A}_\ell^{j-1}$, $\ell = 1, 2, \ldots, 2^L$:
  - Find the optimal codeword associated with partition $\mathcal{A}_\ell^{j-1}$. Denote this new optimal codeword as $\mathbf{a}_\ell^j$.
- Form the new codebook $\mathbf{C}^j \longleftarrow \left[\mathbf{a}_1^j \; \mathbf{a}_2^j \; \cdots \; \mathbf{a}_{2^L}^j\right]^T$.
  - Set NewCost $\longleftarrow D_{\mathrm{B}}\left(\mathbf{C}^j\right)$.

Stop if OldCost $-$ NewCost $\leq \varepsilon$.

Comments on Generalized Lloyd Algorithm

- The average codebook distortion monotonically decreases through the iterative usage of the Centroid Condition and the Nearest Neighbor Condition.
- The optimal codebook is designed offline by the FC once.
- The optimal codebook is stored in the FC and all sensors.

Comments on Generalized Lloyd Algorithm

- Upon observing a realization of the channel vector $\mathbf{g}$:
  - The FC finds the associated optimal power-allocation vector $\mathbf{a}^{\mathsf{OPT}}$.
  - It identifies the closest codeword in the optimal codebook $\mathbf{C}$ to $\mathbf{a}^{\mathsf{OPT}}$ with respect to the defined distance metric.

$$\ell = \underset{k \in \left\{1,2,\ldots,2^L\right\}, \mathbf{a}_k \in \mathbf{C}}{\arg\min} D_{\mathsf{W}}\left(\mathbf{a}_k, \mathbf{a}^{\mathsf{OPT}}\right).$$

  - It broadcasts the *index* of that codeword ℓ to all sensors.

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  - It broadcasts the *index* of that codeword ℓ to all sensors.
- Upon reception of the index ℓ, the quantized local amplification gain for each sensor i is $[\mathbf{C}]_{\ell,i}$.

Outline

Simulation Setup

- Local observation gains are randomly chosen from a Gaussian distribution $h_i \sim \mathcal{N}(1, 0.09)$.
- Average power of h_i across all sensors is 1.2.
- Observation noise variance is $\sigma_{\mathrm{o}}^2 = 10$ dBm.
- Channel noise variance is $\sigma_{\mathrm{c}}^2 = -90$ dBm.
- Fading model for the channels between local sensors and the FC:

$$g_i = \eta_0 \left(\frac{d_i}{d_0} \right)^{-\alpha} f_i$$

  - $\eta_0 = -30$ dB: Nominal fading gain at reference distance $d_0 = 1$ meter.
  - d_i: Distance between sensor i and the FC (in meters).
  - $\alpha = 2$: Path-loss exponent.
  - f_i: i.i.d. Rayleigh-fading random variable with unit variance.
  - The distance between sensors and the FC is uniformly distributed between 50 and 150 meters.

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- The training-set size in the optimal codebook-design is $M = 5,000$.
- Codebook-distortion threshold is $\varepsilon = 10^{-4}$.
- The results are obtained by averaging over 10,000 Monte Carlo simulations.

Simulation Setup

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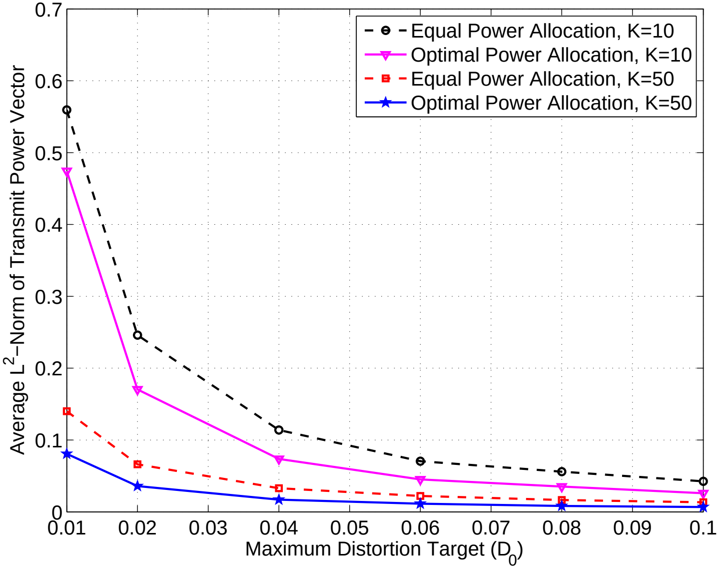
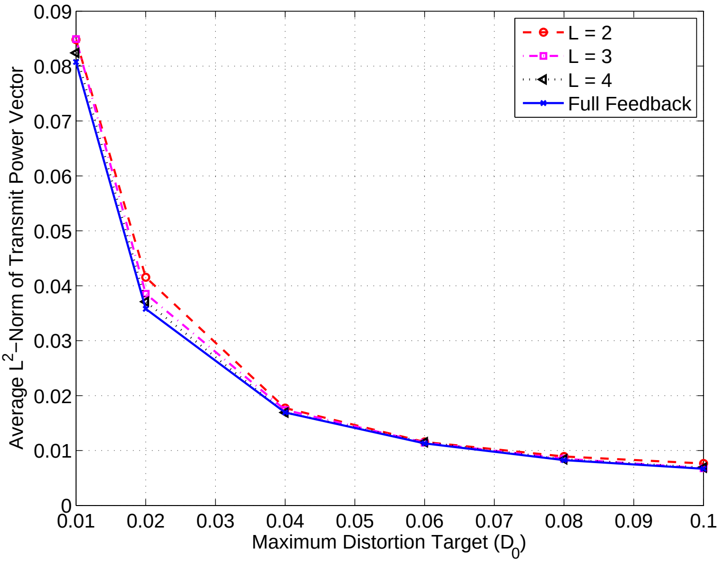
- **Performance Metric:** The energy efficiency of a power-allocation scheme defined as

$$J(\mathbf{a}) = \left(\sum_{i=1}^{K} P_i^2\right)^{\frac{1}{2}} = \left(\sum_{i=1}^{K} \left[a_i^2 \sigma_{\text{o}}^2 (1 + \beta_i)\right]^2\right)^{\frac{1}{2}}$$

Energy Efficiency of Adaptive Power Allocation



Effect of L on Energy Efficiency ($K = 50$ Sensors)



Outline

Summary

- An adaptive power-allocation scheme was proposed for distributed BLUE estimation of a random scalar parameter at the FC of a WSN.

- The proposed scheme minimizes the L^2-norm of the vector of local transmit powers, given a maximum estimation distortion measured as the variance of the BLUE estimator.

- A limited-feedback strategy was proposed to eliminate the requirement of infinite-rate feedback of the instantaneous forward CSI from the FC to local sensors.
  - Designed an optimal codebook by quantizing the vector space of the optimal local amplification gains using the generalized Lloyd algorithm with modified distortion functions.

Thank You for Your Attention.

Questions and/or Comments?