Power Allocation for Distributed BLUE Estimation with Full and Limited Feedback of CSI

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1. Introduction and Problem Statement
2. System Model Description
3. Adaptive Optimal Power-Allocation Scheme
4. Limited Feedback for Adaptive Power Allocation
5. Numerical Results
6. Conclusions
Outline

1. Introduction and Problem Statement
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What Is Distributed Estimation in WSNs?

Spatially distributed sensors
- Observe their surrounding environment.
- Process their local observations.
- Send their processed data to a fusion center (FC).
- FC performs the ultimate global estimation.
Introduction and Problem Statement

What Is Distributed Estimation in WSNs?

Spatially distributed sensors
- Observe their surrounding environment.
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Goal of This Paper

Adaptive power allocation to local sensors for reliable estimation of an unknown random parameter at the fusion center of a WSN from spatially distributed linear noisy observations by local sensors and communicated through parallel fading channels.
Outline

1. Introduction and Problem Statement
2. System Model Description
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6. Conclusions
Unknown random parameter $\theta$ with zero mean and unit power, otherwise unknown.
Local linear noisy observation at each sensor

\[ x_i = h_i \theta + n_i \]

- \( h_i \): Fixed local observation gain known at the sensor and FC.
Local linear noisy observation at each sensor

\[ x_i = h_i \theta + n_i \]

\( n_i \): Spatially i.i.d. additive observation noise with zero mean and known variance \( \sigma_o^2 \), otherwise unknown.
Local linear noisy observation at each sensor

\[ x_i = h_i \theta + n_i \]

Observation SNR at sensor \( i \):

\[ \beta_i \overset{\text{def}}{=} \frac{|h_i|^2}{\sigma_0^2} \]
Amplify-and-forward (AF) local processing scheme

- No inter-sensor communication and/or collaboration.
- Each sensor acts as a relay and transmits amplified version of its raw noisy analog local observation to the fusion center.
Amplify-and-forward (AF) local processing scheme

\[ z_i = a_i x_i = a_i h_i \theta + a_i n_i \]
Amplify-and-forward (AF) local processing scheme

\[ z_i = a_i x_i = a_i h_i \theta + a_i n_i \]

- Instantaneous transmit power

\[ P_i = a_i^2 \left( |h_i|^2 + \sigma_o^2 \right) = a_i^2 \sigma_o^2 (1 + \beta_i) \]
Orthogonal coherent channels corrupted by independent fading and additive Gaussian noise:

\[ y_i = g_i z_i + w_i \quad \text{with} \quad w_i \sim \mathcal{N}(0, \sigma_c^2) \]

Channel gains are completely known at the fusion center.
Orthogonal coherent channels corrupted by independent fading and additive Gaussian noise:

\[ y_i = g_i z_i + w_i \]

\[ w_i \sim \mathcal{N}(0, \sigma_c^2) \]

- Channel SNR at sensor \( i \):

\[ \gamma_i \overset{\text{def}}{=} \frac{|g_i|^2}{\sigma_c^2} \]
The FC finds the *best linear unbiased estimator* (BLUE) for $\theta$. 
BLUE Estimation of $\theta$ at the FC

Given $a \triangleq [a_1, a_2, \ldots, a_K]^T$ and a realization of the fading channel gains $g \triangleq [g_1, g_2, \ldots, g_K]^T$, the BLUE estimator of $\theta$ at the FC is

$$
\hat{\theta} = \left( \sum_{i=1}^{K} \frac{h_i^2 a_i^2 g_i^2}{a_i^2 g_i^2 \sigma_o^2 + \sigma_c^2} \right)^{-1} \sum_{i=1}^{K} \frac{h_i a_i g_i y_i}{a_i^2 g_i^2 \sigma_o^2 + \sigma_c^2}
$$

The estimator variance is

$$
\text{Var} \left( \hat{\theta} | a, g \right) = \left( \sum_{i=1}^{K} \frac{h_i^2 a_i^2 g_i^2}{a_i^2 g_i^2 \sigma_o^2 + \sigma_c^2} \right)^{-1}
$$

$$
= \left( \sum_{i=1}^{K} \frac{\beta_i \gamma_i a_i^2 \sigma_o^2}{1 + \gamma_i a_i^2 \sigma_o^2} \right)^{-1}
$$
Problem Statement for Our Optimal Power-Allocation Scheme

Find the optimal local amplification gains or equivalently, the optimal power-allocation scheme that minimizes the $L^2$-norm of the vector of local transmit powers $\mathbf{P} \triangleq [P_1, P_2, \ldots, P_K]^T$, given a maximum estimation distortion as defined by the BLUE estimator variance.

$$\minimize_{\{P_i\}_{i=1}^K} \left( \sum_{i=1}^{K} P_i^2 \right)^{\frac{1}{2}}$$

subject to $\text{Var} \left( \hat{\theta} \big| \mathbf{a}, \mathbf{g} \right) \leq D_0$
Problem Statement for Our Optimal Power-Allocation Scheme

Find the optimal local amplification gains or equivalently, the optimal power-allocation scheme that minimizes the $L^2$-norm of the vector of local transmit powers $\mathbf{P} \triangleq [P_1, P_2, \ldots, P_K]^T$, given a maximum estimation distortion as defined by the BLUE estimator variance.

$$\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{K} \left[ a_i^2 \sigma_o^2 (1 + \beta_i) \right]^2 \\
\text{subject to} & \quad \sum_{i=1}^{K} \frac{\beta_i \gamma_i a_i^2 \sigma_o^2}{1 + \gamma_i a_i^2 \sigma_o^2} \geq \frac{1}{D_0}
\end{align*}$$
The unique solution for the given constrained convex optimization problem could be found as follows:

\[ a_i^2 = \begin{cases} 
\frac{1}{\gamma_i \sigma_o^2} \left( \frac{3 \sqrt{\frac{\lambda_0}{\beta_i \delta_i^2 T_i}}} {1 - \frac{2}{3} \sqrt{\frac{\beta_i \delta_i^2}{\lambda_0 T_i^2}}} - 1 \right), & i \leq K_1 \\
0, & i > K_1 
\end{cases} \]

\[ T_i = 1 + \sqrt{1 + \frac{8 \beta_i \delta_i^2}{27 \lambda_0}} \]
\[ \delta_i \overset{\text{def}}{=} \frac{1 + \beta_i}{\beta_i \gamma_i} \quad \beta_i \overset{\text{def}}{=} \frac{|h_i|^2}{\sigma_o^2} \quad \text{and} \quad \gamma_i \overset{\text{def}}{=} \frac{|g_i|^2}{\sigma_c^2} \]

- The sensors are sorted so that \( \delta_1 \leq \delta_2 \leq \cdots \leq \delta_K \).
- The observation SNR \( \beta_i \) or channel SNR \( \gamma_i \) decreases.
- Only the first \( K_1 \) sensors with the least values of \( \delta_i \) will have a positive value for \( a_i \), and \( a_i = 0 \) for all \( i > K_1 \).
The unique solution for the given constrained convex optimization problem could be found as follows:

\[
\alpha_i^2 = \begin{cases} 
\frac{1}{\gamma_i \sigma_o^2} \left( \frac{\lambda_0}{\beta_i \delta_i^2 T_i} - 1 \right), & i \leq K_1 \\
0, & i > K_1
\end{cases}
\]

\[
T_i = 1 + \sqrt{1 + \frac{8 \beta_i \delta_i^2}{27 \lambda_0}}
\]

\[
\delta_i \overset{\text{def}}{=} \frac{1 + \beta_i}{\beta_i \gamma_i}, \quad \beta_i \overset{\text{def}}{=} \frac{|h_i|^2}{\sigma_o^2} \quad \text{and} \quad \gamma_i \overset{\text{def}}{=} \frac{|g_i|^2}{\sigma_c^2}
\]

The values of the number of active sensors \(K_1\) for which \(\alpha_i > 0\), and \(\lambda_0\) are unique and can be found using the following equation:

\[
\sum_{i=1}^{K_1} \beta_i \left( 1 - \frac{2}{3} \sqrt[3]{\frac{\beta_i \delta_i^2}{\lambda_0 T_i^2}} \right) = \sum_{i=1}^{K_1} \beta_i - \frac{1}{D_0}
\]
Adaptive Optimal Power-Allocation Scheme

Optimal Local Amplification Gains

The unique solution for the given constrained convex optimization problem could be found as follows:

$$a_i^2 = \begin{cases} 
\frac{1}{\gamma_i \sigma_o^2} \left( \frac{3 \sqrt{\frac{\lambda_0}{\beta_i \delta_i^2 T_i}}}{1-\frac{2}{3} \sqrt{\frac{\beta_i \delta_i^2}{\lambda_0 T_i^2}}} - 1 \right), & i \leq K_1 \\
0, & i > K_1 
\end{cases}$$

$$T_i = 1 + \sqrt{1 + \frac{8 \beta_i \delta_i^2}{27 \lambda_0}}$$

$$\delta_i \overset{\text{def}}{=} \frac{1 + \beta_i}{\beta_i \gamma_i}, \quad \beta_i \overset{\text{def}}{=} \frac{|h_i|^2}{\sigma_o^2} \quad \text{and} \quad \gamma_i \overset{\text{def}}{=} \frac{|g_i|^2}{\sigma_c^2}$$

**Solution:** Water-filling-based iterative process to solve this equation:

$$\sum_{i=1}^{K_1} \beta_i \sqrt[3]{\frac{\beta_i \delta_i^2 T_i}{\lambda_0}} \left( 1 - \frac{2}{3} \sqrt[3]{\frac{\beta_i \delta_i^2}{\lambda_0 T_i^2}} \right) = \sum_{i=1}^{K_1} \beta_i - \frac{1}{D_0}$$
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Limited Feedback for Adaptive Power Allocation

Why Do We Need Limited Feedback?

What Does a Sensor Need to Find $a_i$?

$$a_i^2 = \begin{cases} \frac{1}{\gamma_i \sigma_o^2} \left( \frac{\sqrt[3]{\lambda_0}}{\beta_i \delta_i^2 T_i} \right) - 1, & i \leq K_1 \\ 0, & i > K_1 \end{cases}$$

$$\delta_i \triangleq \frac{1 + \beta_i}{\beta_i \gamma_i} \quad \beta_i \overset{\text{def}}{=} \frac{|h_i|^2}{\sigma_o^2} \quad \text{and} \quad \gamma_i \overset{\text{def}}{=} \frac{|g_i|^2}{\sigma_c^2} \quad K_1 \text{ and } \lambda_0$$

**Assumption:** Complete *forward* CSI is available at local sensors.

- *Not* practical in most applications, especially in large-scale WSNs.
- Feedback information is typically transmitted through finite-rate *digital* feedback links.
What Does a Sensor Need to Find $a_i$?

$$a_i^2 = \begin{cases} \frac{1}{\gamma_i \sigma_0^2} \left( \frac{3 \frac{\lambda_0}{\beta_i \delta_i^2 T_i}}{1 - \frac{2}{3} \sqrt[3]{\frac{\beta_i \delta_i^2}{\lambda_0 T_i^2}}} - 1 \right), & i \leq K_1 \\ 0, & i > K_1 \end{cases}$$

$$\delta_i \triangleq \frac{1 + \beta_i}{\beta_i \gamma_i}, \quad \beta_i \overset{\text{def}}{=} \frac{|h_i|^2}{\sigma_0^2} \quad \text{and} \quad \gamma_i \overset{\text{def}}{=} \frac{|g_i|^2}{\sigma_c^2}$$

- Type of required full-feedback from the FC to local sensors:
  1. Instantaneous amplification gain $a_i$.
  2. Fading coefficient of the channel between each sensor and the FC $g_i$, $K_1$ and $\lambda_0$.

    - $\lambda_0$ and an extra one-bit command instructing the sensor to transmit or stay silent.
    - Entire vector of $g$ sent by the FC over a broadcast channel.
Limited Feedback for Adaptive Power Allocation

Why Do We Need Limited Feedback?

What Does a Sensor Need to Find $a_i$?

\[ a_i^2 = \begin{cases} 
\frac{1}{\gamma_i \sigma_0^2} \left( \frac{3 \sqrt{\frac{\lambda_0}{\beta_i \delta_i^2 T_i}}}{1 - \frac{2}{3} \sqrt{\frac{\beta_i \delta_i^2}{\lambda_0 T_i^2}}} - 1 \right), & i \leq K_1 \\
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\[ \delta_i \triangleq \frac{1 + \beta_i}{\beta_i \gamma_i} \quad \beta_i \overset{\text{def}}{=} \frac{|h_i|^2}{\sigma_0^2} \quad \text{and} \quad \gamma_i \overset{\text{def}}{=} \frac{|g_i|^2}{\sigma_c^2} \quad K_1 \text{ and } \lambda_0 \]

**Solution:** Limited feedback of instantaneous amplification gain $a_i$

- For each channel realization $g$, the FC first finds the *exact* optimal power-allocation scheme using perfect *backward* CSI.
- The FC sends back the *index* of the quantized version of the optimized power-allocation vector to all sensors.
A *codebook* of the local amplification gains is required.

- The optimal codebook is designed *offline* by quantizing the space of the optimized power-allocation vectors using the *generalized Lloyd algorithm* with modified distortion metrics.

- $L$: The number of *total* feedback bits from the FC to sensors.

- The space of the optimal local power-allocation vectors is quantized into $2^L$ disjoint regions.

- A codeword is chosen in each quantization region.

- Codeword length is $K$. Its $i$th entry is a *real-valued* quantized version of the optimal local amplification gain for sensor $i$. 
Overview on Generalized Lloyd Quantization
Inputs and Output of Generalized Lloyd Algorithm

Inputs to Lloyd algorithm

- **$K$**: The number of spatially distributed sensors.
- **$L$**: The number of total feedback bits from the FC to sensors.  
  - *Not* the number of bits fed back to each sensor.
- Fading model of the channel between local sensors and the FC.
- **$M$**: The number of *training vectors* in space $\mathcal{A}$.
- **$\epsilon$**: Distortion threshold to stop the codebook-design iterations.
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  - $M$: The number of training vectors in space $\mathcal{A}$.
  - $\varepsilon$: Distortion threshold to stop the codebook-design iterations.

- **Output of Lloyd algorithm**
  - $C^{\text{OPT}}$: Codebook matrix of the optimal local amplification gains.
  - $C = [a_1 \ a_2 \ \cdots \ a_{2^L}]^T$ is a $2^L \times K$ matrix.
  - $[C]_{\ell,i}$ denotes the element in row $\ell$ and column $i$ as the optimal gain of sensor $i$ in codeword $\ell$.
  - Each $a_\ell$, $\ell = 1, 2, \ldots, 2^L$ is associated with a realization of the fading coefficients of the channels between local sensors and the FC.
Initialization of Generalized Lloyd Algorithm

- Generate $G_s$ as a set of $M$ length-$K$ vectors of channel-fading realizations based on the given fading model of the channels between local sensors and the FC.
- $M \gg 2^L$. 
Limited Feedback for Adaptive Power Allocation

Generalized Lloyd Algorithm

Initialization of Generalized Lloyd Algorithm

- Generate $G_s$ as a set of $M$ length-$K$ vectors of channel-fading realizations based on the given fading model of the channels between local sensors and the FC.
  - $M \gg 2^L$.

- Find $A_s$ as the set of optimal local power-allocation vectors associated with the channel fading vectors in $G_s$.
  - $A_s$ is the set of training vectors, and $A_s \subseteq A$. 
Initialization of Generalized Lloyd Algorithm

- Generate $\mathcal{G}_s$ as a set of $M$ length-$K$ vectors of channel-fading realizations based on the given fading model of the channels between local sensors and the FC.
  - $M \gg 2^L$.

- Find $\mathcal{A}_s$ as the set of optimal local power-allocation vectors associated with the channel fading vectors in $\mathcal{G}_s$.
  - $\mathcal{A}_s$ is the set of training vectors, and $\mathcal{A}_s \subseteq \mathcal{A}$.

- Randomly select $2^L$ optimal power-allocation vectors from the set $\mathcal{A}_s$ as the initial set of codewords.
  - Denote each codeword by $a^0_\ell$, $\ell = 1, 2, \ldots, 2^L$. 
Initialization of Generalized Lloyd Algorithm

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- Randomly select $2^L$ optimal power-allocation vectors from the set $A_s$ as the initial set of codewords.
  - Denote each codeword by $a_\ell^0$, $\ell = 1, 2, \ldots, 2^L$.

- Set the initial codebook as $C^0 \leftarrow [a_1^0 \ a_2^0 \ \cdots \ a_{2^L}^0]^T$.

- Find the average distortion of codebook.
  - Set NewCost $\leftarrow D_B(C^0)$ and $j \leftarrow 0$. 

Generalized Lloyd Algorithm
Let $\mathbf{P}_\ell$ and $\mathbf{P}$ be the vectors of local transmit powers, when the vector of local amplification gains is $\mathbf{a}_\ell$ and $\mathbf{a}$, respectively.

$$P_i = a_i^2 \left( |h_i|^2 + \sigma_o^2 \right) = a_i^2 \sigma_o^2 (1 + \beta_i)$$
Let $\mathbf{P}_ℓ$ and $\mathbf{P}$ be the vectors of local transmit powers, when the vector of local amplification gains is $\mathbf{a}_ℓ$ and $\mathbf{a}$, respectively.

$$P_i = a_i^2 \left( |h_i|^2 + \sigma_o^2 \right) = a_i^2 \sigma_o^2 (1 + \beta_i)$$

Optimization cost of a power-allocation vector

$$J(\mathbf{a}) = \left( \sum_{i=1}^{K} P_i^2 \right)^{\frac{1}{2}} = \left( \sum_{i=1}^{K} [a_i^2 \sigma_o^2 (1 + \beta_i)]^2 \right)^{\frac{1}{2}}$$
Let $P_\ell$ and $P$ be the vectors of local transmit powers, when the vector of local amplification gains is $a_\ell$ and $a$, respectively.

Optimization cost of a power-allocation vector

$$J(a) = \left( \sum_{i=1}^{K} P_i^2 \right)^\frac{1}{2} = \left( \sum_{i=1}^{K} \left[ a_i^2 \sigma_o^2 (1 + \beta_i) \right]^2 \right)^\frac{1}{2}$$

Distance between codeword $a_\ell$ and the optimal power-allocation vector $a$

$$D_W(a_\ell, a) \triangleq |J(a_\ell) - J(a)|$$
Let $\mathbf{P}_\ell$ and $\mathbf{P}$ be the vectors of local transmit powers, when the vector of local amplification gains is $\mathbf{a}_\ell$ and $\mathbf{a}$, respectively.

Optimization cost of a power-allocation vector

$$J(\mathbf{a}) = \left( \sum_{i=1}^{K} P_i^2 \right)^{\frac{1}{2}} = \left( \sum_{i=1}^{K} [a_i^2 \sigma_o^2 (1 + \beta_i)]^2 \right)^{\frac{1}{2}}$$

Distance between codeword $\mathbf{a}_\ell$ and the optimal power-allocation vector $\mathbf{a}$

$$D_W(\mathbf{a}_\ell, \mathbf{a}) \triangleq |J(\mathbf{a}_\ell) - J(\mathbf{a})|$$

Average distortion for codebook $\mathbf{C}$

$$D_B(\mathbf{C}) \triangleq \mathbb{E}_\mathbf{a} \left[ \min_{\ell \in \{1, 2, ..., 2^L\}} D_W(\mathbf{a}_\ell, \mathbf{a}) \right]$$
Repeat the following steps:

- \( j \leftarrow j + 1 \) and \( \text{OldCost} \leftarrow \text{NewCost} \).
Repeat the following steps:

- \( j \leftarrow j + 1 \) and OldCost \( \leftarrow \) NewCost.

- Given codebook \( C^{j-1} \), optimally partition the set \( \mathcal{A}_s \) into \( 2^L \) disjoint subsets and denote the resulted optimal partitions by \( \mathcal{A}^{j-1}_\ell, \ell = 1, 2, \ldots, 2^L \).

  - **Nearest-Neighbor Condition** finds the optimal Voronoi cells of the vector space to be quantized, given a fixed codebook.
  - Each point \( a \in \mathcal{A}_s \) is assigned to partition \( \ell \) represented by codeword \( a_\ell \in C^{j-1} \) if and only if its distance to codeword \( a_\ell \), with respect to the defined distance function, is less than its distance to any other codeword in the codebook.
  - Given a codebook \( C^{j-1} \), the space \( \mathcal{A}_s \) is divided into \( 2^L \) disjoint quantization regions (or Voronoi cells) with the \( \ell \)th region defined as

    \[
    \mathcal{A}_\ell = \{ a \in \mathcal{A}_s : D_W (a_\ell, a) \leq D_W (a_k, a), \forall k \neq \ell \}
    \]
Repeat the following steps:

- $j \leftarrow j + 1$ and OldCost $\leftarrow$ NewCost.

- Given codebook $C^{j-1}$, optimally partition the set $A_s$ into $2^L$ disjoint subsets and denote the resulted optimal partitions by $A^{j-1}_\ell$.

- For every $A^{j-1}_\ell$, $\ell = 1, 2, \ldots, 2^L$:
  - Find the optimal codeword associated with partition $A^{j-1}_\ell$. Denote this new optimal codeword as $a^j_\ell$. 

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Repeat the following steps:

- \( j \leftarrow j + 1 \) and \( \text{OldCost} \leftarrow \text{NewCost} \).

- Given codebook \( C^{j-1} \), optimally partition the set \( \mathcal{A}_s \) into \( 2^L \) disjoint subsets and denote the resulted optimal partitions by \( \mathcal{A}_l^{j-1} \).

- Find the optimal codeword associated with partition \( \mathcal{A}_l^{j-1} \).

- **Centroid Condition** finds the optimal codebook, given a specific partitioning of the vector space.

- The optimal codeword associated with each Voronoi cell \( \mathcal{A}_l \subseteq \mathcal{A}_s \) is the centroid of that cell with respect to the defined distance function.

- Given a specific partitioning \( \{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_{2L}\} \), the optimal codeword associated with partition \( \mathcal{A}_l \) is defined as

\[
\mathbf{a}_l^* = \arg \min_{\mathbf{a}_l \in \mathcal{A}_l} \mathbb{E}_{\mathbf{a} \in \mathcal{A}_l} [D_W(\mathbf{a}_l, \mathbf{a})]
\]
Detailed Operation of Generalized Lloyd Algorithm

Repeat the following steps:

1. $j \leftarrow j + 1$ and OldCost $\leftarrow$ NewCost.

2. Given codebook $C^{j-1}$, optimally partition the set $A_s$ into $2^L$ disjoint subsets and denote the resulted optimal partitions by $A^{j-1}_\ell$.

3. For every $A^{j-1}_\ell$, $\ell = 1, 2, \ldots, 2^L$:
   - Find the optimal codeword associated with partition $A^{j-1}_\ell$. Denote this new optimal codeword as $a^j_\ell$.

4. Form the new codebook $C^j \leftarrow \begin{bmatrix} a^j_1 & a^j_2 & \cdots & a^j_{2^L} \end{bmatrix}^T$.
   - Set NewCost $\leftarrow D_B(C^j)$.
Limited Feedback for Adaptive Power Allocation

Generalized Lloyd Algorithm

Detailed Opertaion of Generalized Lloyd Algorithm

Repeat the following steps:

1. $j \leftarrow j + 1$ and OldCost $\leftarrow$ NewCost.

2. Given codebook $C^{j-1}$, optimally partition the set $A_s$ into $2^L$ disjoint subsets and denote the resulted optimal partitions by $A^{j-1}_\ell$.

3. For every $A^{j-1}_\ell$, $\ell = 1, 2, \ldots, 2^L$:
   - Find the optimal codeword associated with partition $A^{j-1}_\ell$. Denote this new optimal codeword as $a^j_\ell$.

4. Form the new codebook $C^j \leftarrow [a^j_1 \ a^j_2 \ \cdots \ a^j_{2^L}]^T$.
   - Set NewCost $\leftarrow D_B (C^j)$.

Stop if OldCost − NewCost $\leq \varepsilon$. 

Comments on Generalized Lloyd Algorithm

- The average codebook distortion monotonically decreases through the iterative usage of the Centroid Condition and the Nearest Neighbor Condition.
- The optimal codebook is designed offline by the FC once.
- The optimal codebook is stored in the FC and all sensors.
Comments on Generalized Lloyd Algorithm

Upon observing a realization of the channel vector $g$:

- The FC finds the associated optimal power-allocation vector $a^{OPT}$.
- It identifies the closest codeword in the optimal codebook $C$ to $a^{OPT}$ with respect to the defined distance metric.

$$\ell = \arg\min_{k \in \{1, 2, \ldots, 2^L\}, a_k \in C} D_W(a_k, a^{OPT})$$

- It broadcasts the index of that codeword $\ell$ to all sensors.
On observing a realization of the channel vector $g$:

- The FC finds the associated optimal power-allocation vector $a^{OPT}$.
- It identifies the closest codeword in the optimal codebook $C$ to $a^{OPT}$ with respect to the defined distance metric.

$$
\ell = \arg \min_{k \in \{1, 2, \ldots, 2^L\}, a_k \in C} D_W (a_k, a^{OPT}).
$$

- It broadcasts the index of that codeword $\ell$ to all sensors.

Upon reception of the index $\ell$, the quantized local amplification gain for each sensor $i$ is $[C]_{\ell,i}$. 

[38x265]Limited Feedback for Adaptive Power Allocation

Generalized Lloyd Algorithm

Comments on Generalized Lloyd Algorithm
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Local observation gains are randomly chosen from a Gaussian distribution $h_i \sim \mathcal{N}(1, 0.09)$.

Average power of $h_i$ across all sensors is 1.2.

Observation noise variance is $\sigma_0^2 = 10$ dBm.

Channel noise variance is $\sigma_c^2 = -90$ dBm.

Fading model for the channels between local sensors and the FC:

$$g_i = \eta_0 \left( \frac{d_i}{d_0} \right)^{-\alpha} f_i$$

- $\eta_0 = -30$ dB: Nominal fading gain at reference distance $d_0 = 1$ meter.
- $d_i$: Distance between sensor $i$ and the FC (in meters).
- $\alpha = 2$: Path-loss exponent.
- $f_i$: i.i.d. Rayleigh-fading random variable with unit variance.
- The distance between sensors and the FC is uniformly distributed between 50 and 150 meters.
Local observation gains are randomly chosen from a Gaussian distribution $h_i \sim \mathcal{N}(1, 0.09)$.

Average power of $h_i$ across all sensors is 1.2.

Observation noise variance is $\sigma_o^2 = 10$ dBm.

Channel noise variance is $\sigma_c^2 = -90$ dBm.

Fading model for the channels between local sensors and the FC:

$$g_i = \eta_0 \left( \frac{d_i}{d_0} \right)^{-\alpha} f_i, \quad i = 1, 2, \ldots, K,$$

The training-set size in the optimal codebook-design is $M = 5,000$.

Codebook-distortion threshold is $\varepsilon = 10^{-4}$.

The results are obtained by averaging over 10,000 Monte Carlo simulations.
Numerical Results

Simulation Setup

- Local observation gains are randomly chosen from a Gaussian distribution $h_i \sim \mathcal{N}(1, 0.09)$.
- Average power of $h_i$ across all sensors is 1.2.
- Observation noise variance is $\sigma_0^2 = 10 \text{ dBm}$.
- Channel noise variance is $\sigma_c^2 = -90 \text{ dBm}$.
- Fading model for the channels between local sensors and the FC:
  $$ g_i = \eta_0 \left( \frac{d_i}{d_0} \right)^{-\alpha} f_i, \quad i = 1, 2, \ldots, K, $$

- **Performance Metric:** The energy efficiency of a power-allocation scheme defined as
  $$ J(a) = \left( \sum_{i=1}^{K} P_i^2 \right)^{1/2} = \left( \sum_{i=1}^{K} \left[ a_i^2 \sigma_0^2 (1 + \beta_i) \right]^2 \right)^{1/2} $$
Energy Efficiency of Adaptive Power Allocation

- Equal Power Allocation, $K=10$
- Optimal Power Allocation, $K=10$
- Equal Power Allocation, $K=50$
- Optimal Power Allocation, $K=50$

Average $L^2$-Norm of Transmit Power Vector vs. Maximum Distortion Target ($D_0$)
Effect of $L$ on Energy Efficiency ($K = 50$ Sensors)

![Graph showing the effect of $L$ on energy efficiency. The graph plots the average $L^2$-norm of transmit power vector against the maximum distortion target ($D_0$). Each line represents a different value of $L$ (2, 3, 4) and the full feedback case. The x-axis represents $D_0$, ranging from 0.01 to 0.1, and the y-axis represents the $L^2$-norm, ranging from 0 to 0.1. The graph illustrates how increasing $L$ generally decreases the $L^2$-norm for a given $D_0$.](image-url)
1. Introduction and Problem Statement
2. System Model Description
3. Adaptive Optimal Power-Allocation Scheme
4. Limited Feedback for Adaptive Power Allocation
5. Numerical Results
6. Conclusions
An adaptive power-allocation scheme was proposed for distributed BLUE estimation of a random scalar parameter at the FC of a WSN.

The proposed scheme minimizes the $L^2$-norm of the vector of local transmit powers, given a maximum estimation distortion measured as the variance of the BLUE estimator.

A limited-feedback strategy was proposed to eliminate the requirement of infinite-rate feedback of the instantaneous forward CSI from the FC to local sensors.

- Designed an optimal codebook by quantizing the vector space of the optimal local amplification gains using the generalized Lloyd algorithm with modified distortion functions.
Thank You for Your Attention.

Questions and/or Comments?