Analysis of Multi-Cell Downlink Cooperation with a Constrained Spatial Model

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Introduction and Problem Statement

Network Model

Conditional Outage Probability

Spatial Model

Performance Analysis

Conclusion
Introduction and Problem Statement

Outline

1. Introduction and Problem Statement
2. Network Model
3. Conditional Outage Probability
4. Spatial Model
5. Performance Analysis
6. Conclusion
What Is Multi-Cell Cooperation?

- A promising approach for mitigating inter-cell interference is *multi-cell cooperation* (MCC), which involves joint processing of signal transmitted and/or received by multiple base stations.

- The performance of a cooperative downlink is analyzed, when the signals are diversity combined at the receiver:
  - outage probability conditioned over the topology is derived in closed-form;
  - two power allocation policies are compared;
  - insight regarding how the cell edge should be defined is provided.
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The Network comprises:

- \( M \) cellular base stations \( \{X_1, ..., X_M\} \);
- \( K \) mobiles \( \{Y_1, ..., Y_K\} \);
- Each mobile is served by one or more base stations.

Finite network area.

DS-CDMA is considered and intra-cell sequences are assumed to be synchronous.

The base stations transmit with a common power \( P_0 \) such that

\[
\frac{1}{1 - f_p} \sum_{j: Y_j \in \mathcal{Y}_i} P_{i,j} = P_0
\]

where

- \( P_{i,j} \) is the average transmitted power by base station \( X_i \) to mobile \( Y_j \);
- \( f_p \) is the fraction of the base-station power reserved for pilot signals;
- \( \mathcal{Y}_i \) is the set of mobiles connected to the base station \( X_i \).
Despread Instantaneous Power

The despread instantaneous power of $X_i$ at mobile $Y_j$ is

$$
\rho_{i,j} = \begin{cases} 
    P_i,j g_{i,j} f(\tilde{d}_{i,j}) & \text{from serving base stations, if } i \in G_j \\
    \left(\frac{h}{G}\right) P_i,j g_{i,j} f(\tilde{d}_{i,j}) & \text{from interfering base stations, if } i \notin G_j 
\end{cases}
$$

where

- $G_j$ is the set of the indexes of the base stations serving $Y_j$;
- $g_{i,j}$ is the power gain due to Nakagami fading;
- $f(\cdot)$ is a path-loss function:
  $$
  f(d) = \left(\frac{d}{d_0}\right)^{-\alpha}
  $$
  - $\alpha$ is the path loss exponent;
  - $d \geq d_0$;
- $\tilde{d}_{i,j}$ is the effective distance, which is the distance perturbed by the shadowing:
  $$
  \tilde{d}_{i,j} = 10^{-\xi_{i,j}/(10\alpha)} \|X_i - Y_j\| 
  $$
  - $\xi_{i,j}$ is a shadowing factor and $\xi_{i,j} \sim N(0, \sigma_s^2)$;
- $h$ is the chip factor;
- $G$ is the common spreading factor.
Cell Association

Let $r_{\text{int}}$ represent the radius of the cell *interior* and any mobile that is within an effective distance $r_{\text{int}}$ of a base station will be served by just that base station;
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- Mobiles beyond $r_{\text{int}}$ will attempt to connect to base stations out to some maximum distance $r_{\text{max}}$, where $r_{\text{int}} \leq r_{\text{max}}$. 
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- Let $r_{\text{int}}$ represent the radius of the cell *interior* and any mobile that is within an effective distance $r_{\text{int}}$ of a base station will be served by just that base station;
- Mobiles beyond $r_{\text{int}}$ will attempt to connect to base stations out to some maximum distance $r_{\text{max}}$, where $r_{\text{int}} \leq r_{\text{max}}$.

- Because there are only $G$ orthogonal spreading sequences per cell, the number of mobiles connected to $X_i$ is limited to $K_i \leq G$, where $K_i = |\mathcal{Y}_i|$.
- If there are $K_i > G$ mobiles in the set $\mathcal{G}_j$, for the $K_i - G$ mobiles which are located at higher effective distance service is denied.
Power Allocation

- Base station $X_i$ allocates the power transmitted to mobile $Y_j$ according to the fractional power-control policy

\[
P_{i,j} = P_0 (1 - f_p) \left[ \frac{1 - \zeta}{K_i} + \zeta \tilde{d}_{i,j}^\alpha \left( \sum_{j: Y_j \in \mathcal{Y}_i} \tilde{d}_{i,j}^\alpha \right)^{-1} \right]
\]

where $0 \leq \zeta \leq 1$ is the fractional power-control factor.

Two extreme situations are considered:
- $\zeta = 0$, which corresponds to an equal transmit power (ETP) policy;
- $\zeta = 1$, which corresponds to an equal received power (ERP) policy.
If the receiver of mobile $Y_j$ is able to resolve the signal received from each base station $X_i$, where $i \in G_j$, then it may perform maximal-ratio combining (MRC) of the paths and the resulting instantaneous signal-to-interference-and-noise ratio (SINR) at mobile $Y_j$ is given by:

$$\gamma_j = \frac{\sum_{i \in G_j} g_{i,j} \Omega_{i,j}}{\Gamma^{-1} + \frac{h}{G} \sum_{i \in G_j} g_{i,j} \Omega_{i,j}}$$

where

- $\Gamma = d_0^\alpha N_j P_0 / N$ is the signal-to-noise ratio (SNR) at a mobile located at unit distance when fading and shadowing are absent, where $N$ is the noise power;
- $\Omega_{i,j} = \frac{P_{i,j}}{N_j P_0} 10^{\xi_{i,j}/10} ||X_i - Y_j||^{-\alpha}$ is the normalized power of $X_i$ at receiver $Y_j$ before despreading.
Conditional Outage Probability

- An outage occurs when the SINR is below a threshold $\beta$.
- $\beta$ depends on the choice of modulation and coding.
- The outage probability for the mobile $Y_j$ conditioned over the network is

$$\epsilon_j = P \left[ \gamma_j \leq \beta_j \mid \Omega_j \right].$$

(5)

The conditional outage probability is found in closed form:

$$\epsilon_j = \sum_{k \in G_j} \sum_{n=1}^{m_{k,j}} \Xi_{N_j} \left( k, n, \{m_{q,j}\}_{\forall q \in G_j}, \left\{ \frac{\Omega_{q,j}}{\beta_j m_{q,j}} \right\}_{\forall q \in G_j} \right) \left\{ 1 - \exp \left( -\frac{\beta_j m_{k,j}}{\Omega_{k,j} \Gamma} \right) \times \sum_{\mu=0}^{n-1} \left( \frac{\beta_j m_{k,j}}{\Omega_{k,j} \Gamma} \right)^\mu \sum_{t=0}^\mu \frac{\Gamma^t}{(\mu - t)!} \sum_{\ell_i \geq 0} \prod_{i \notin G_j} G_{\ell_i} (i, j) \right\},$$

(6)

where

$$G_{\ell_i} (i, j) = \frac{\Gamma(\ell_i + m_{i,j})}{\ell_i! \Gamma(m_{i,j})} \left( \frac{h\Omega_{i,j}}{Gm_{i,j}} \right)^{\ell_i} \left( \frac{\beta_j m_{k,j}}{\Omega_{k,j} \Gamma} + 1 \right)^{-(m_{i,j} + \ell_i)}$$

and the function $\Xi_{N_j}(\cdot, \cdot, \cdot, \cdot)$ is defined in [13].

Example

Figure: Conditional outage probability $\epsilon_j$ as a function of SNR $\Gamma$. Analytical curves are solid while dots represent simulated values. The network topology is shown in the inset. The mobile is represented by the star at the center of the circular area, and the 50 base stations are shown as large filled circles.

Settings:
- $M = 50$ base stations;
- Circular network with radius $r_{\text{net}} = 2$;
- $\alpha = 3$;
- $f_p = 0.1$;
- $h = 2/3$;
- $G = 16$;
- $\zeta = 0$. 
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Cluster Uniform Model

Rather than using the grid model or simple random process to characterize the spatial distribution of both base stations and mobiles, the cluster uniform model is adopted:

- Each base station is characterized by an exclusion zone of radius $r_{bs}$;
Rather than using the grid model or simple random process to characterize the spatial distribution of both base stations and mobiles, the *cluster uniform model* is adopted:

- Each base station is characterized by an *exclusion zone* of radius $r_{bs}$;
- Each mobile is characterized by an *exclusion zone* of radius $r_{m}$, with $r_{m} \leq r_{bs}$. 
Distance-Dependent Fading Model

For a given topology, in (6) non-identical Nakagami-m parameters can be chosen to characterize the fading from the base station $X_k$ to the mobile $Y_j$ and a *distance-depending fading* model can be adopted:
For a given topology, in (6) non-identical Nakagami-m parameters can be chosen to characterize the fading from the base station $X_k$ to the mobile $Y_j$ and a *distance-depending fading* model can be adopted:

$$r_{ff}/2 \leq m_{k,j} \leq 3$$

where $r_f$ is the *line-of-sight radius*.

The distance-dependent-fading model characterizes the situation where a mobile close to the base station is in the line-of-sight (LOS), while mobiles farther away tend to be non-LOS.
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Area Spectral Efficiency

- The threshold $\beta_j$ of mobile $Y_j$ is selected such that the conditional outage probability of mobile $Y_j$ satisfies the constraint $\epsilon_j = \hat{\epsilon}$. For a given $\beta_j$, there is a corresponding transmission rate $R_j$ that can be supported. Let $R_j = C(\beta_j)$ represent the relationship between $R_j$ and $\beta_j$. For modern cellular systems, it is reasonable to use:

$$C(\beta_j) = \log_2(1 + \beta_j)$$

- The performance metric used is the area spectral efficiency, defined as

$$A = \lambda (1 - \hat{\epsilon}) \mathbb{E}[R]$$

where

- $\lambda = K/A_{\text{net}}$ is the density of transmissions in the network;
- $\mathbb{E}[R]$ is computed using a Monte Carlo approach and $R$ is averaged among the mobiles in the network realizations of the spatial model.
Figure: Area spectral efficiency as a function of $r_{\text{max}}$ for four different choice of $r_{\text{int}}$ with equal received power (ERP) and equal transmit power (ETP) policy for a shadowed ($\sigma_s = 8$ dB) environment.

Settings:
- $M = 50$ base stations;
- $K = 800$ mobiles;
- $r_{\text{net}} = 2$;
- $r_f = 0.25$;
- $r_{bs} = 0.25$;
- $r_{m} = 0.01$;
- $\alpha = 3$;
- $\Gamma = 10$ dB;
- $f_p = 0.1$;
- $h = 2/3$;
- $G = 16$;
- $\hat{\epsilon} = 0.1$. 
Area Spectral Efficiency vs Average Load per Cell

Figure: Area spectral efficiency as a function of $K/M$ with equal received power (ERP) and equal transmit power (ETP) policy, for both an unshadowed and a shadowed ($\sigma_s = 8$ dB) environment.

Settings:
- $M = 50$ base stations;
- $r_{net} = 2$;
- $r_f = 0.25$;
- $r_{bs} = 0.25$;
- $r_m = 0.01$;
- $\alpha = 3$;
- $\Gamma = 10$ dB;
- $f_p = 0.1$;
- $h = 2/3$;
- $G = 16$;
- $\hat{\epsilon} = 0.1$;
- $\sigma_s = 8$ dB.
Figure: Ccdf of the transmission rate $R$ for equal received power (ERP) and equal transmit power (ETP) policy for a half loaded system ($K/M = 8$) in shadowing ($\sigma_s = 8$ dB).

Settings:

- $M = 50$ base stations;
- $r_{\text{net}} = 2$;
- $r_f = 0.25$;
- $r_{\text{bs}} = 0.25$;
- $r_m = 0.01$;
- $\alpha = 3$;
- $\Gamma = 10$ dB;
- $f_p = 0.1$;
- $h = 2/3$;
- $G = 16$;
- $\hat{\epsilon} = 0.1$;
- $\sigma_s = 8$ dB.
Probability of Denial vs Cell Edge Size

**Figure:** Probability that a mobile is denied service as function of $r_{\text{int}}$ for four different choice of $r_{\text{max}}$ and for three different loads for a shadowed ($\sigma_s = 8 \text{ dB}$) environment.

**Settings:**
- $M = 50$ base stations;
- $r_{\text{net}} = 2$;
- $r_f = 0.25$;
- $r_{bs} = 0.25$;
- $r_m = 0.01$;
- $\alpha = 3$;
- $\Gamma = 10 \text{ dB}$;
- $f_p = 0.1$;
- $h = 2/3$;
- $G = 16$;
- $\hat{\epsilon} = 0.1$;
- $\sigma_s = 8 \text{ dB}$.
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Conclusions

- The results show that:
  - Multi-cell cooperation as expected improves performance;
  - ETP is more favorable in terms of the area spectral efficiency, while ERP is more fair;
  - There is an optimal set of \( \{r_{\text{int}}, r_{\text{max}}\} \) that indicates how the cell edge should be defined.

- A new closed-form expression is derived for the outage probability conditioned over the topology when the receiver is able to resolve the signal received from the cooperative base stations and coherently combines them.

- The model is general enough that can be expanded to analyze different type of access, such as orthogonal frequency-division multiple access (OFDMA), when the desired signals are diversity-combined at the receiver, i.e multicast-broadcast single-frequency-network (MBSFN) in the LTE standard.
Conclusion

Thank You