Analysis of Multi-Cell Downlink Cooperation with a Constrained Spatial Model

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Outline

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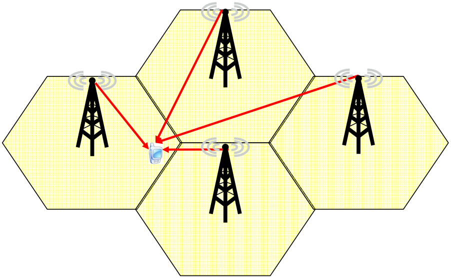
What Is Multi-Cell Cooperation?

- A promising approach for mitigating inter-cell interference is *multi-cell cooperation* (MCC), which involves joint processing of signal transmitted and/or received by multiple base stations.



- The performance of a cooperative downlink is analized, when the signals are diversity combined at the receiver:
  - outage probability conditioned over the topology is derived in closed-form;
  - two power allocation policies are compared;
  - insight regarding how the cell edge should be defined is provided.

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Network Model

- The Network comprises:
  - M cellular base stations $\{X_1, ..., X_M\}$;
  - K mobiles $\{Y_1, ..., Y_K\}$;
  - Each mobile is served by one or more base stations.
- Finite network area.
- DS-CDMA is considered and intra-cell sequences are assumed to be synchronous.
- The base stations transmit with a common power P_0 such that

$$\frac{1}{1-f_p} \sum_{j:Y_j \in \mathcal{Y}_i} P_{i,j} = P_0 \tag{1}$$

where

- $P_{i,j}$ is the average transmitted power by base station X_i to mobile Y_j;
- f_p is the fraction of the base-station power reserved for pilot signals;
- $\mathcal{Y}_i$ is the set of mobiles connected to the base station X_i.

Despread Instantaneous Power

The despread instantaneous power of X_i at mobile Y_j is

$$\rho_{i,j} = \begin{cases} P_{i,j} g_{i,j} f\left(\tilde{d}_{i,j}\right) & \text{from serving base stations, if } i \in \mathcal{G}_j \\ \left(\frac{h}{G}\right) P_{i,j} g_{i,j} f\left(\tilde{d}_{i,j}\right) & \text{from interfering base stations, if } i \notin \mathcal{G}_j \end{cases}$$

where

- $\mathcal{G}_j$ is the set of the indexes of the base stations serving Y_j;
- $g_{i,j}$ is the power gain due to Nakagami fading;
- $f(\cdot)$ is a path-loss function:

  $$f(d) = \left(\frac{d}{d_0}\right)^{-\alpha}$$

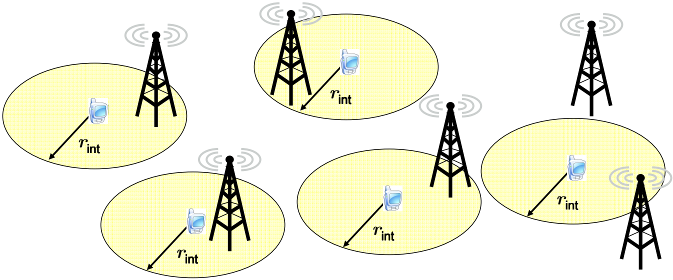
  - α is the path loss exponent;
  - $d \geq d_0$;
- $\tilde{d}_{i,j}$ is the *effective distance*, which is the distance perturbed by the shadowing:

  $$\tilde{d}_{i,j} = 10^{-\xi_{i,j}/(10\alpha)} ||X_i - Y_j|| \quad (2)$$

  - $\xi_{i,j}$ is a *shadowing factor* and $\xi_{i,j} \sim N\left(0, \sigma_s^2\right)$;
- h is the chip factor;
- G is the common spreading factor.

Cell Association

- Let r_{int} represent the radius of the cell *interior* and any mobile that is within an effective distance r_{int} of a base station will be served by just that base station;



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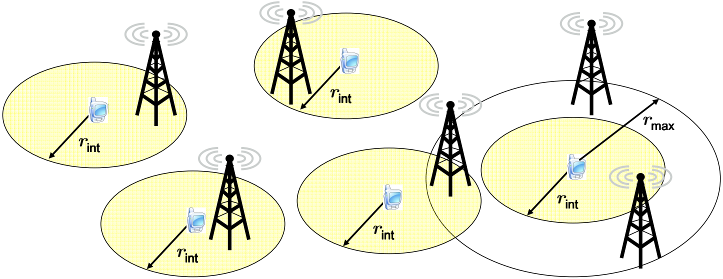
- Let r_{int} represent the radius of the cell *interior* and any mobile that is within an effective distance r_{int} of a base station will be served by just that base station;
- Mobiles beyond r_{int} will attempt to connect to base stations out to some maximum distance r_{max}, where $r_{\text{int}} \leq r_{\text{max}}$.



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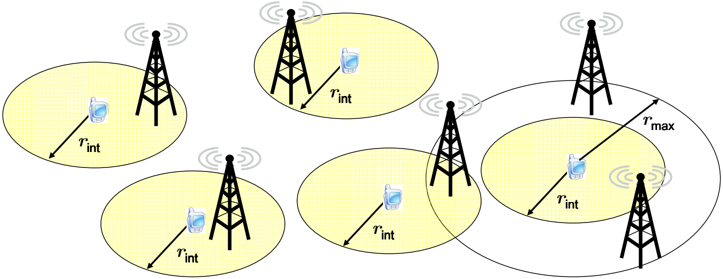

- Because there are only G orthogonal spreading sequences per cell, the number of mobiles connected to X_i is limited to $K_i \leq G$, where $K_i = |\mathcal{Y}_i|$.
- If there are $K_i > G$ mobiles in the set $\mathcal{G}_j$, for the $K_i - G$ mobiles which are located at higher effective distance service is denied.

Power Allocation

- Base station X_i allocates the power transmitted to mobile Y_j according to the fractional power-control policy

$$P_{i,j} = P_0 \left(1 - f_p\right) \left[\frac{1-\zeta}{K_i} + \zeta \tilde{d}_{i,j}^{\alpha} \left(\sum_{j:Y_j \in \mathcal{Y}_i} \tilde{d}_{i,j}^{\alpha} \right)^{-1} \right] \quad (3)$$

  where $0 \leq \zeta \leq 1$ is the *fractional power-control factor*.

  Two extreme situations are considered:

  - $\zeta = 0$, which corresponds to an *equal transmit power* (ETP) policy;
  - $\zeta = 1$, which corresponds to an *equal received power* (ERP) policy.

SINR

If the receiver of mobile Y_j is able to resolve the signal received from each base station X_i, where $i \in \mathcal{G}_j$, then it may perform *maximal-ratio combining* (MRC) of the paths and the resulting instantaneous signal-to-interference-and-noise ratio (SINR) at mobile Y_j is given by:

$$\gamma_j = \frac{\sum_{i \in \mathcal{G}_j} g_{i,j} \Omega_{i,j}}{\Gamma^{-1} + \frac{h}{G} \sum_{i \notin \mathcal{G}_j} g_{i,j} \Omega_{i,j}} \tag{4}$$

where

- $\Gamma = d_0^{\alpha} N_j P_0 / \mathcal{N}$ is the signal-to-noise ratio (SNR) at a mobile located at unit distance when fading and shadowing are absent, where $\mathcal{N}$ is the noise power;
- $\Omega_{i,j} = \frac{P_{i,j}}{N_j P_0} 10^{\xi_{i,j}/10} ||X_i - Y_j||^{-\alpha}$ is the normalized power of X_i at receiver Y_j before despreading.

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1. Introduction and Problem Statement
2. Network Model
3. **Conditional Outage Probability**
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Conditional Outage Probability

- An *outage* occurs when the SINR is below a threshold β.
    - β depends on the choice of modulation and coding.
- The *outage probability* for the mobile Y_j conditioned over the network is

$$\epsilon_j \quad = \quad P\left[\gamma_j \leq \beta_j \middle| \boldsymbol{\Omega}_j\right]. \tag{5}$$

- The conditional outage probability is found in closed form:

$$\epsilon_j = \sum_{k \in \mathcal{G}_j} \sum_{n=1}^{m_{k,j}} \Xi_{N_j}\left(k, n, \{m_{q,j}\}_{\forall q \in \mathcal{G}_j}, \left\{\frac{\Omega_{q,j}}{\beta_j m_{q,j}}\right\}_{\forall q \in \mathcal{G}_j}\right) \left\{1 - \exp\left(-\frac{\beta_j m_{k,j}}{\Omega_{k,j}\Gamma}\right) \times \sum_{\mu=0}^{n-1} \left(\frac{\beta_j m_{k,j}}{\Omega_{k,j}\Gamma}\right)^{\mu} \sum_{t=0}^{\mu} \frac{\Gamma^t}{(\mu - t)!} \sum_{\substack{\ell_i \geq 0 \\ \sum_{i=0}^{M-N_j} \ell_i = t}} \prod_{i \notin \mathcal{G}_j} G_\ell(i,j)\right\}, \tag{6}$$

where

$$G_\ell(i,j) \quad = \quad \frac{\Gamma(\ell_i + m_{i,j})}{\ell_i!\Gamma(m_{i,j})}\left(\frac{h\Omega_{i,j}}{Gm_{i,j}}\right)^{\ell_i}\left(\frac{\beta_j m_{k,j}}{\Omega_{k,j}}\frac{h\Omega_{i,j}}{Gm_{i,j}} + 1\right)^{-(m_{i,j}+\ell_i)}$$

and the function $\Xi_{N_j}(\cdot,\cdot,\cdot,\cdot)$ is defined in [13].

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[13] G. K. Karagiannidis, N. C. Sagias, and T. A. Tsiftsis, “Closed-form statistics for the sum of squared Nakagami-m variates and its applications”, *IEEE Trans. Commun.*, vol. 45, pp. 1353-1359, Aug. 2006.

Example

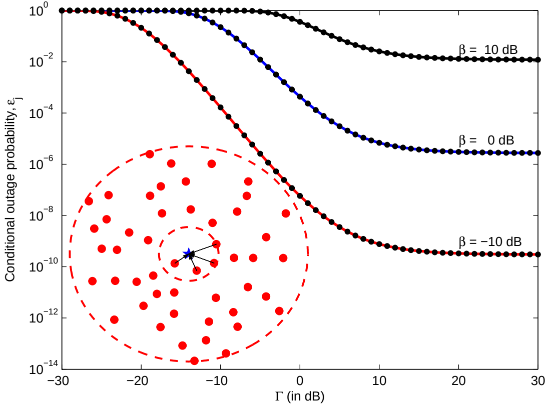


Figure: *Conditional outage probability ϵ_j as a function of SNR Γ. Analytical curves are solid while dots represent simulated values. The network topology is shown in the inset. The mobile is represented by the star at the center of the circular area, and the 50 base stations are shown as large filled circles.*

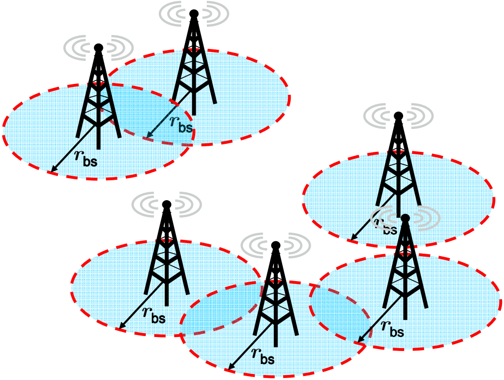
Settings:

- $M = 50$ base stations;
- Circular network with radius $r_{\text{net}} = 2$;
- $\alpha = 3$;
- $f_p = 0.1$;
- $h = 2/3$;
- $G = 16$;
- $\zeta = 0$.

Outline

Cluster Uniform Model

- Rather than using the grid model or simple random process to characterize the spatial distribution of both base stations and mobiles, the *cluster uniform model* is adopted:
  - Each base station is characterized by an *exclusion zone* of radius r_{bs};



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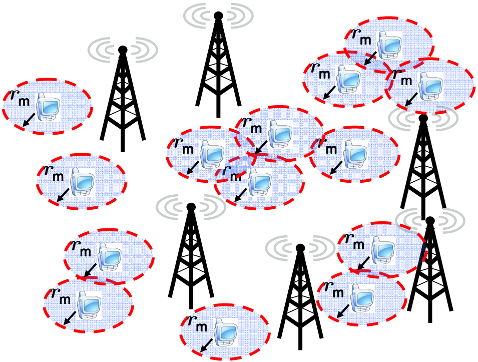
  - Each base station is characterized by an *exclusion zone* of radius r_{bs};
  - Each mobile is characterized by an *exclusion zone* of radius r_{m}, with $r_{\mathsf{m}} \leq r_{\mathsf{bs}}$.



Distance-Dependent Fading Model

- For a given topology, in (6) non-identical Nakagami-m parameters can be chosen to characterize the fading from the base station X_k to the mobile Y_j and a *distance-depending fading* model can be adopted:

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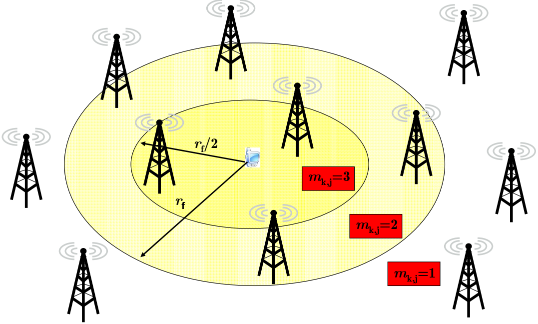

where r_f is the *line-of-sight radius*.

- The distance-dependent-fading model characterizes the situation where a mobile close to the base station is in the line-of-sight (LOS), while mobiles farther away tend to be non-LOS.

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Area Spectral Efficiency

- The threshold β_j of mobile Y_j is selected such that the conditional outage probability of mobile Y_j satisfies the constraint $\epsilon_j = \hat{\epsilon}$. For a given β_j, there is a corresponding transmission rate R_j that can be supported. Let $R_j = C(\beta_j)$ represent the relationship between R_j and β_j. For modern cellular systems, it is reasonable to use:

$$C(\beta_j) = \log_2(1+\beta_j)$$

- The performance metric used is the *area spectral efficiency*, defined as

$$\mathcal{A} = \lambda\,(1-\hat{\epsilon})\,\mathbb{E}[R]$$

where

  - $\lambda = K/A_{\text{net}}$ is the density of transmissions in the network;
  - $E[R]$ is computed using a Monte Carlo approach and R is averaged among the mobiles in the network realizations of the spatial model.

Area Spectral Efficiency vs Cell Edge Size

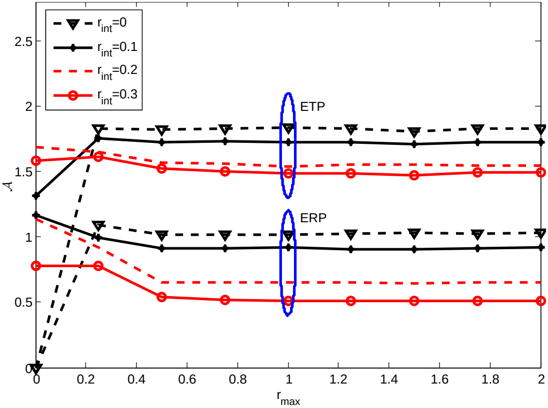

Figure: *Area spectral efficiency as a function of r_{max} for four different choice of r_{int} with equal received power (ERP) and equal transmit power (ETP) policy for a shadowed ($\sigma_s = 8$ dB) environment.*

Settings:

- $M = 50$ base stations;
- $K = 800$ mobiles;
- $r_{\text{net}} = 2$;
- $r_{\text{f}} = 0.25$;
- $r_{\text{bs}} = 0.25$;
- $r_{\text{m}} = 0.01$;
- $\alpha = 3$;
- $\Gamma = 10$ dB;
- $f_p = 0.1$;
- $h = 2/3$;
- $G = 16$;
- $\hat{\epsilon} = 0.1$.

Area Spectral Efficiency vs Average Load per Cell

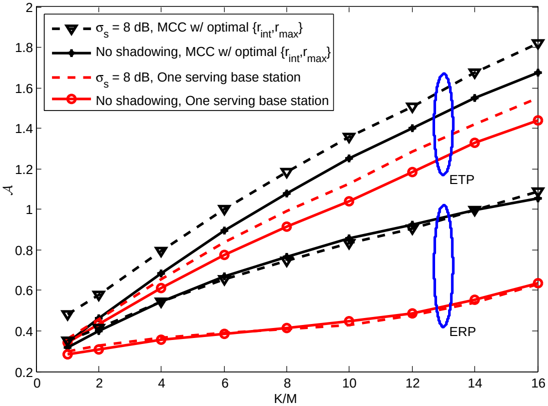

Figure: *Area spectral efficiency as a function of K/M with equal received power (ERP) and equal transmit power (ETP) policy, for both an unshadowed and a shadowed ($\sigma_s = 8$ dB) environment.*

Settings:

- $M = 50$ base stations;
- $r_{\text{net}} = 2$;
- $r_{\text{f}} = 0.25$;
- $r_{\text{bs}} = 0.25$;
- $r_{\text{m}} = 0.01$;
- $\alpha = 3$;
- $\Gamma = 10$ dB;
- $f_p = 0.1$;
- $h = 2/3$;
- $G = 16$;
- $\hat{\epsilon} = 0.1$;
- $\sigma_s = 8$ dB.

Fairness

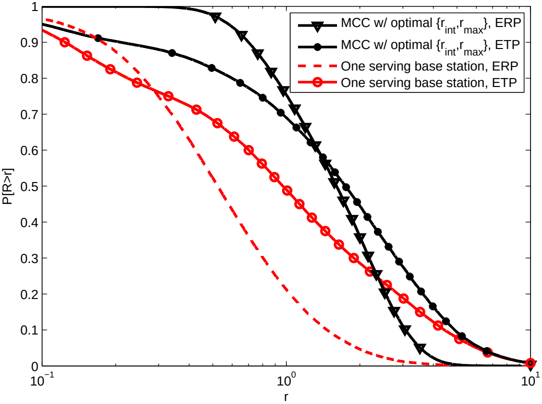

Figure: *Ccdf of the transmission rate R for equal received power (ERP) and equal transmit power (ETP) policy for a half loaded system $(K/M = 8)$ in shadowing ($\sigma_s = 8$ dB).*

Settings:

- $M = 50$ base stations;
- $r_{\text{net}} = 2$;
- $r_{\text{f}} = 0.25$;
- $r_{\text{bs}} = 0.25$;
- $r_{\text{m}} = 0.01$;
- $\alpha = 3$;
- $\Gamma = 10$ dB;
- $f_p = 0.1$;
- $h = 2/3$;
- $G = 16$;
- $\hat{\epsilon} = 0.1$;
- $\sigma_s = 8$ dB.

Probability of Denial vs Cell Edge Size

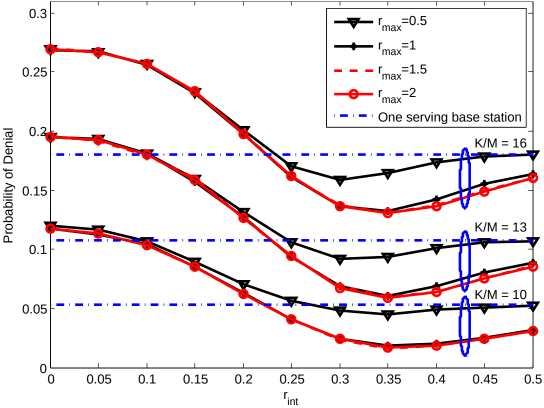

Figure: *Probability that a mobile is denied service as function of r_{int} for four different choice of r_{max} and for three different loads for a shadowed ($\sigma_s = 8$ dB) environment.*

Settings:

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- $r_{\text{net}} = 2$;
- $r_{\text{f}} = 0.25$;
- $r_{\text{bs}} = 0.25$;
- $r_{\text{m}} = 0.01$;
- $\alpha = 3$;
- $\Gamma = 10$ dB;
- $f_p = 0.1$;
- $h = 2/3$;
- $G = 16$;
- $\hat{\epsilon} = 0.1$;
- $\sigma_s = 8$ dB.

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Conclusions

- The results show that:
  - Multi-cell cooperation as expected improves performance;
  - ETP is more favorable in terms of the area spectral efficiency, while ERP is more fair;
  - There is an optimal set of $\{r_{\text{int}}, r_{\text{max}}\}$ that indicates how the cell edge should be defined.
- A new closed-form expression is derived for the outage probability conditioned over the topology when the receiver is able to resolve the signal received from the cooperative base stations and coherently combines them.
- The model is general enough that can be expanded to analyze different type of access, such as orthogonal frequency-division multiple access (OFDMA), when the desired signals are diversity-combined at the receiver, i.e multicast-broadcast single-frequency-network (MBSFN) in the LTE standard.

Thank You