MILCOM 2014
Affordable Mission Success: Meeting the Challenge

Unicast Barrage Relay Networks: Outage Analysis and Optimization

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Outline

1. Introduction
2. Network Model
3. Markov Process
4. Iterative Method
5. Network Optimization
6. Conclusion

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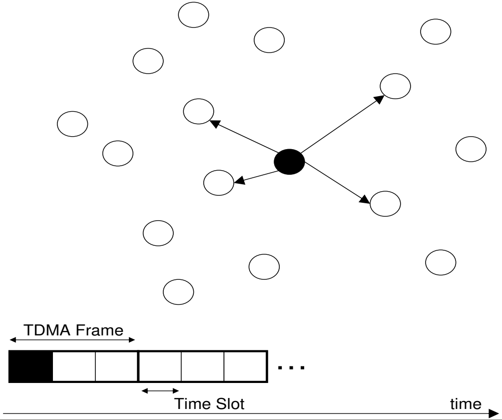
Barrage Relays Networks (BRNs)

- BRN is a cooperative MANET designed for use at the tactical edge.
- BRNs use time division multiple access and cooperative communications.
- Packets propagate outward from the source via a *decode-and-forward* approach as follows:



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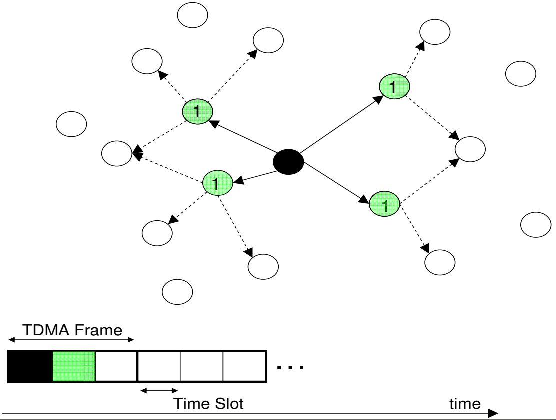

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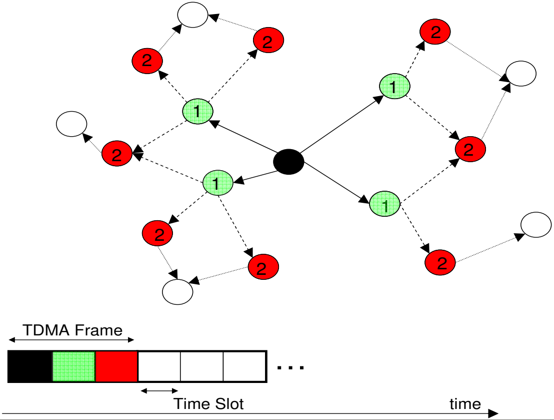

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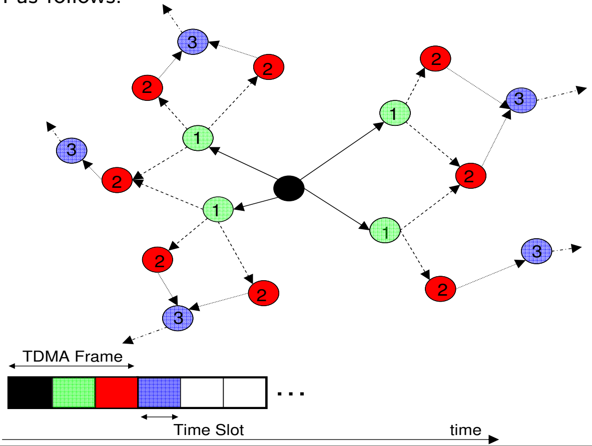
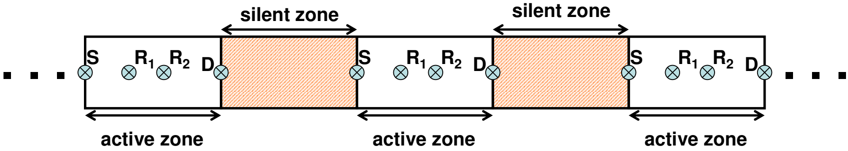


Controlled Barrage Regions (CBRs)



- CBR is the union of multiple cooperative paths within some subregion or zone of the overall network.
- CBRs can be established by specifying a set of *buffer nodes* (S and D) around a set of N cooperating *interior nodes* $(R_1,\ldots,R_N)$, enabling *spatial reuse*.
- The nodes operate in a half-duplex mode: during a particular radio frame a CBR can be *active* or *silent*.
- Since each node transmits each packet at most once, the maximum number of transmissions per frame is $F = N + 1$.

Objective

Outage analysis

- The link outage probabilities are computed in closed form.
- The dynamics of how a Barrage relay network evolves over time is modeled as a Markov process.
- An iterative method is used in order to account for co-channel interference (CCI).

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Optimization

Maximize the transport capacity by finding the optimal:
- code rate;
- number of relays;
- placement of the relays;
- size of the network.

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Network Model

- The BRN comprises:
  - K CBRs;
  - M mobiles radios or *nodes* $\mathcal{X} = \{X_1, ..., X_M\}$.
- The instantaneous power of X_i at receiver X_j during time slot t is

$$\rho_{i,j}^{(t)} = P_i g_{i,j}^{(t)} f(d_{i,j}) \quad (1)$$

where

  - P_i is the transmit power;
  - $g_{i,j}^{(t)}$ is the power gain due to Rayleigh fading;
  - $d_{i,j} = ||X_i - X_j||$ is the distance between X_i and X_j;
  - $f(\cdot)$ is a path-loss function:

$$f(d) = \left(\frac{d}{d_0}\right)^{-\alpha}$$

    - α is the path loss exponent;
    - $d \geq d_0$;

SINR

The signal-to-interference-and-noise ratio (SINR) for the node X_j during time slot t, when the signals are diversity combined at the receiver, is (cf., [1]):

$$\gamma_j^{(t)} = \frac{|\mathcal{X}_j^{(t)}| \sum_{k \in \mathcal{G}_j^{(t)}} g_{k,j}^{(t)} d_{k,j}^{-\alpha}}{\Gamma^{-1} + \sum_{i \notin \mathcal{G}_j^{(t)}} I_i^{(t)} g_{i,j}^{(t)} d_{i,j}^{-\alpha}} \quad (2)$$

where

- $\mathcal{X}_j^{(t)} \subset \mathcal{X}$ is the set of *barraging* nodes that transmit identical packets to X_j during the t^{th} time slot;
- $\mathcal{G}_j^{(t)}$ is the set of the indexes of the nodes in $\mathcal{X}_j^{(t)}$;
- Γ is the signal-to-noise ratio (SNR);
- $I_i^{(t)}$ is a Bernoulli variable with probability $P[I_i^{(t)} = 1] = p_i^{(t)}$;
- $p_i^{(t)}$ is the *activity probability* for the node X_i during time slot t.

[1] V. A. Aalo, and C. Chayawan, "Outage probability of cellular radio systems using maximal ratio combining in Rayleigh fading channel with multiple interferers", *IEEE Electronics Letters*, vol. 36, pp. 1314 - 1315, Jul. 2000.

Conditional Outage Probability

- An *outage* occurs when the SINR is below a threshold β.
  - β depends on the choice of modulation and coding.
- Conditioning on the network topology $\mathcal{X}$ and the set of barraging nodes $\mathcal{X}_j^{(t)}$, the *outage probability* of mobile X_j during slot t is

$$\epsilon_j^{(t)} = P\left[\gamma_j^{(t)} \leq \beta \middle| \mathcal{X}, \mathcal{X}_j^{(t)}\right]; \quad (3)$$

- The outage probability depends on the particular network realization, which has dynamics over timescales that are much slower than the fading;
- The conditional outage probability is found in closed form [2].

[2] S. Talarico, M. C. Valenti, and D. Torrieri, "Analysis of multi-cell downlink cooperation with a constrained spatial model", *Proc. IEEE Global Telecommun. Conf (GLOBECOM)*, Atlanta, GA, Dec. 2013.

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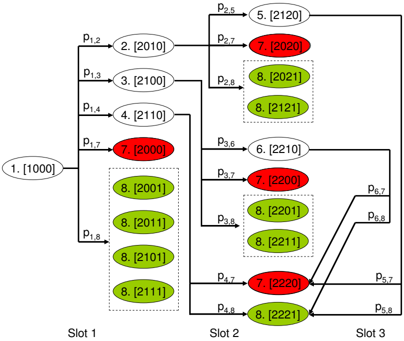
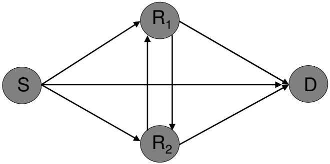
Node and CBR States

- At the boundary between any two time slots, each node can be in one of the three following *node states*:
  - Node state 0: The node has not yet successfully decoded the packet.
  - Node state 1: It has just decoded the packet received during previous slot, and it will transmit on next slot.
  - Node state 2: It has decoded the packet in an earlier slot and it will no longer transmit or receive that packet.
- The *CBR state* is the concatenation of the states of the individual nodes within the CBR.
- The *CBR state* is represented by a vector of the form $[S, R_1, ..., R_N, D]$ containing the states of the source, relays, and destination.

Markov States

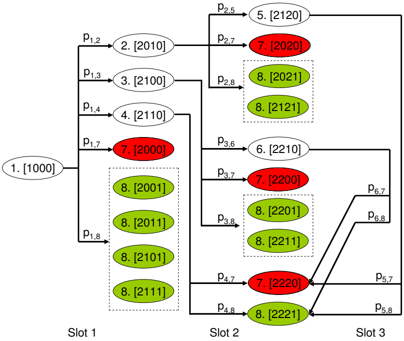
- The dynamics of how the CBR states evolve over a radio frame can be described as an *absorbing* Markov process.
- The *state space* of the process is composed of ϱ *Markov* states s_i.
- An absorbing Markov process is characterized by:
  - *transient states*;
  - *absorbing states*: CBR outage state and CBR success state.





Transition Probability

- The *transition probability* $p_{i,j}$ is the probability that the process moves from Markov state s_i to state s_j.
- The transition probability is evaluated as follows:
  - If s_j is a transient Markov state, it is the product of the individual transmission probabilities;
  - If s_j is an absorbing Markov state, it is equal to the sum of the probabilities of transitioning into the constituent CBR states.



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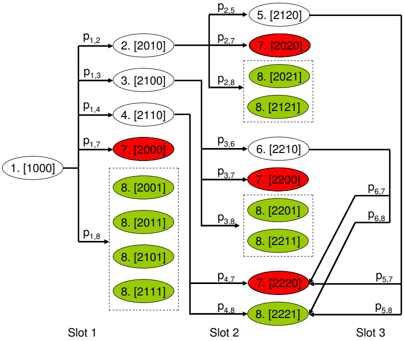
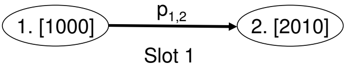


For instance:



$$p_{1,2} = \epsilon_{R_1}^{(1)} \left(1 - \epsilon_{R_2}^{(1)}\right) \epsilon_D^{(1)}$$

where $\epsilon_j^{(t)}$ is found using (3).

CBR Outage and Success Probabilities

- The *state transition matrix* $\boldsymbol{P}$, whose $(i,j)^{th}$ entry is $\{p_{i,j}\}$, is

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{R} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \quad (4)$$

  where

  - $\boldsymbol{Q}$ is the $\tau \times \tau$ *transient matrix*;
  - $\boldsymbol{R}$ is the $\tau \times r$ *absorbing matrix*;
  - $\boldsymbol{I}$ is an $r \times r$ identity matrix.

- The *absorbing probability* $b_{i,j}$ is the probability that the process will be absorbed in the absorbing state s_j if it starts in the transient state s_i.
- The absorbing probability $b_{i,j}$ is the $(i,j)^{th}$ entry of $\boldsymbol{B}$, which is:

$$\boldsymbol{B} = (\boldsymbol{I} - \boldsymbol{Q})^{-1} \boldsymbol{R}. \quad (5)$$

- If the absorbing states are indexed so that the first absorbing state corresponds to a CBR outage, while the second corresponds to a CBR success:
  - The *CBR outage probability* is $\epsilon_{CBR} = b_{1,1}$;
  - The *CBR success probability* is $\hat{\epsilon}_{CBR} = b_{1,2}$.

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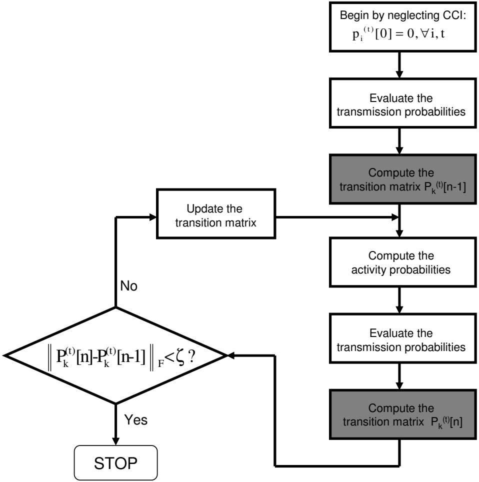
Inter-CBR Interference

- Inter-CBR interference causes a linkage between the transition probabilities in a given CBR and the transmission probabilities of adjacent CBRs.
- In order to compute the transition matrix $\mathbf{P}_k$, an iterative method is used:



- $\mathbf{P}_k[n]$ is the transition matrix at the n^{th} iteration for the k^{th} CBR;
- $\{p_i^{(t)}[n]\}$ is the set of activity probabilities at the n^{th} iteration;
- $|| \cdot ||_F$ is the Frobenius norm;
- ξ is a tolerance.

Example

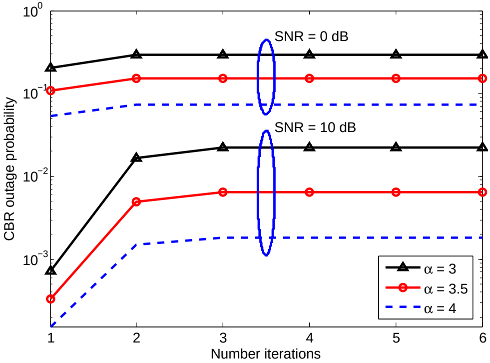


Figure: *CBR outage probability for the k^{th} CBR as function of the number of iterations used. Set of curves at the top: SNR $= 0$ dB. Set of curves at the bottom: SNR $= 10$ dB.*

Settings:

- Line network;
- Two relays $(N = 2)$;
- CCI from two closest active zones;
- Infinite cascade of CBRs;
- All CBRs have identical topology.

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Network Optimization

- The *transport capacity* (TC) of a *typical CBR* is

$$\Upsilon = d \frac{\hat{\epsilon}_{CBR}}{2\left(N+1\right)} R. \tag{6}$$

  where
  - d is the distance between the CBR's source and destination;
  - $\hat{\epsilon}_{CBR}$ is the CBR success probability;
  - N is the number of relays within the CBR;
  - $R = \log_2\left(1+\beta\right)$ is the code rate (in bit per channel used [bpcu]).
- Let $\boldsymbol{X}_k$ be a vector containing the position of the N relays in the k^{th} CBR.
- The goal of the network optimization is to determine the set $\boldsymbol{\Theta} = \left(\boldsymbol{X}_k, R, N, d\right)$ that maximizes the TC.
- The optimization can be solved through a convex optimization over (R, N, d) combined by a stochastic optimization over $\boldsymbol{X}_k$.

Convex Optimization Surface

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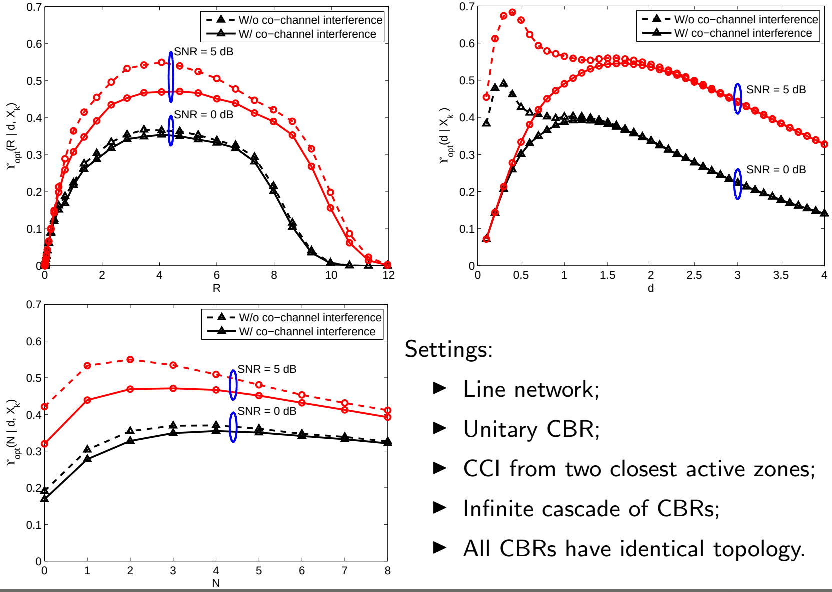


Settings:

- Line network;
- Unitary CBR;
- CCI from two closest active zones;
- Infinite cascade of CBRs;
- All CBRs have identical topology.

Optimization Results

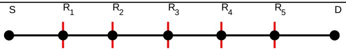
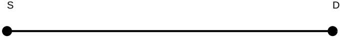
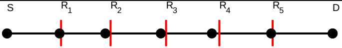
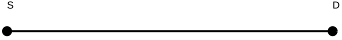
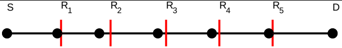
SNR	CCI	R	N	d	Optimal nodes' location	Υ_{opt}
0	✗	4.452	0	0.3		0.490
	✓	4.421	5	1.2		0.402
5	✗	4.611	0	0.4		0.683
	✓	4.452	5	1.7		0.559
10	✗	5.028	0	0.5		0.951
	✓	4.547	5	2.3		0.777

Table: *Results of the optimization for a line network.*

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Conclusions

- This paper presents a new analysis and optimization for unicast in a BRN.
- A BRN is analyzed by describing the behavior of each constituent CBR as a Markov process: path loss, Rayleigh fading, and inter-CBR interference are taken into account.
- The analysis is used to optimize a BRN by finding
  - the optimal number of relays that need to be used in the CBR,
  - the optimal placement of the relays,
  - the optimal size of the CBR,
  - the optimal code rate at which the nodes transmit,

  which maximize TC.
- The analysis and the model are general enough that can be extended to different types of cooperative ad hoc networks, for which multiple source transmissions are diversity combined at the receiver.

Thank You