Interference Correlation in Fixed and Finite Networks

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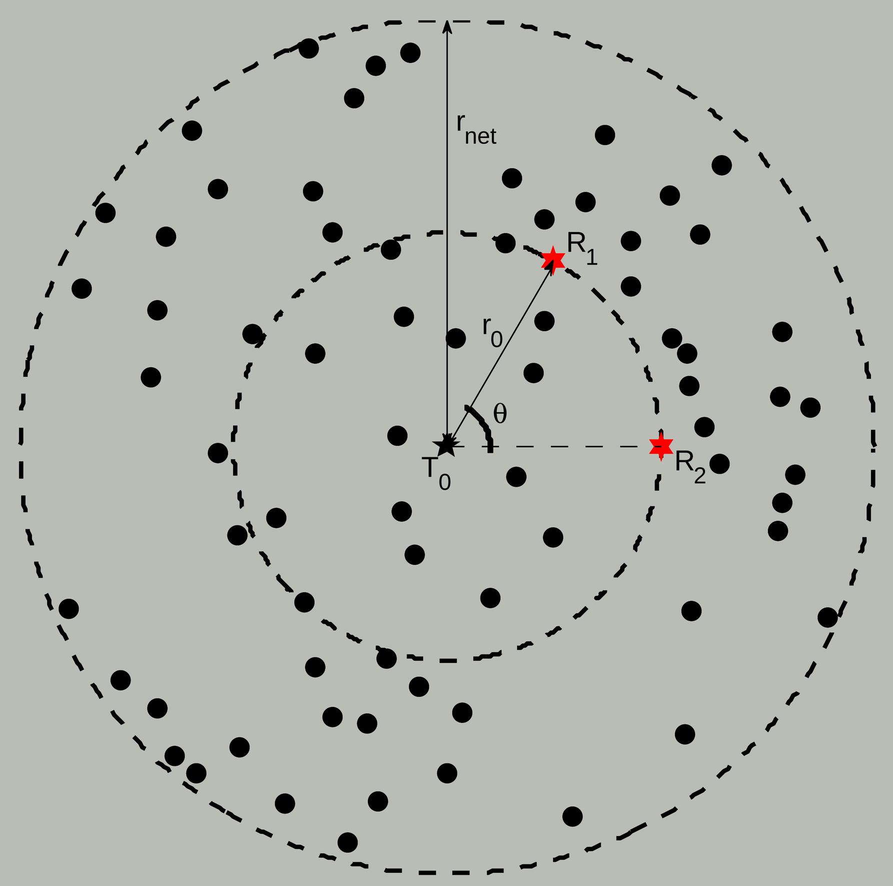
Spatial Correlation of the Interference

- Interference exhibits a spatial correlation [1], that can be captured by evaluating the correlation coefficient of the outage probability at two reference receivers:



- R_1 and R_2 are two reference receivers that are located on a circle of radius r_0 and separated by θ radians.
- T_0 is a reference transmitter.
- The network is circular and finite.
- M interferers are randomly distributed.

- The SINR at receiver R_i is:

$$\gamma_i = \frac{g_{i,0} r_0^{-\alpha}}{\mathsf{SNR}^{-1} + \sum_{j=1}^{M} I_j g_{i,j} r_{i,j}^{-\alpha}}$$

where:
$\{r_{i,j}\}$ are the distances between the i^{th} receiver and the M interferers.
$r_0 = r_{i,0}$ is the distance from T_0 to either R_i.
$\{g_{i,j}\}$ are the fading gains (e.g., i.i.d. exponential).
$\{I_j\}$ are Bernoulli variables indicating interference ($\mathbb{P}[I_j = 1] = p$).
$\alpha \geq 2$ is the path-loss exponent.

- For a SINR threshold of β, the outage probability (OP) is

$$\epsilon_i(M, \boldsymbol{r}) = \mathbb{E}_{\boldsymbol{g}}\left[\mathbf{1}(\gamma_i \leq \beta) \,|\, M, \boldsymbol{r}\right]$$

where
$\mathbf{1}(\cdot)$ is an indicator function.
$\mathbb{E}_{\boldsymbol{g}}[\cdot]$ is the expectation operator, evaluated over the fading.

- The spatially averaged correlation coefficient of the OP at the two receivers is

$$\zeta(R_1, R_2) = \frac{\mathbb{E}_{M,\boldsymbol{r}}[\epsilon_1\epsilon_2] - \mathbb{E}^2_{M,\boldsymbol{r}}[\epsilon_i]}{\mathbb{E}_{M,\boldsymbol{r}}[\epsilon_i^2] - \mathbb{E}^2_{M,\boldsymbol{r}}[\epsilon_i]} \qquad (1)$$

where $\mathbb{E}_{M,\boldsymbol{r}}[\cdot]$ is the expectation operator, evaluated over the number of interferers and their distances to the reference receivers.

Approach to Spatial Averaging [2]

Fix $\boldsymbol{r}$ and M and find the conditional OP averaged over just the fading:

$$\epsilon_i(M, \boldsymbol{r}) = \mathbb{E}_{\boldsymbol{g}}\left[\mathbf{1}(\gamma_i \leq \beta) \,|\, M, \boldsymbol{r}\right]$$

For a given M, evaluate the spatial average using $f_{\boldsymbol{r}}(\boldsymbol{r}|M)$ from [3] :

- The *spatially averaged OP* is

$$\mathbb{E}_{\boldsymbol{r}}[\epsilon_i|M] = \int \epsilon_i(M, \boldsymbol{r}) f_{\boldsymbol{r}}(\boldsymbol{r}|M)\, \mathrm{d}\boldsymbol{r}$$

- The *spatially averaged second moment OP* is

$$\mathbb{E}_{\boldsymbol{r}}[\epsilon_i^2|M] = \int \epsilon_i^2(M, \boldsymbol{r}) f_{\boldsymbol{r}}(\boldsymbol{r}|M)\, \mathrm{d}\boldsymbol{r}$$

- The *spatially averaged first joint moment OP* is

$$\mathbb{E}_{\boldsymbol{r}}[\epsilon_1\epsilon_2|M] = \iint \epsilon_i(M, l(\boldsymbol{\rho}, \boldsymbol{\phi}, 0))\, \epsilon_i(M, l(\boldsymbol{\rho}, \boldsymbol{\phi}, \theta))\, f_{\boldsymbol{\rho}}(\boldsymbol{\rho})\, f_{\boldsymbol{\phi}}(\boldsymbol{\phi})\, \mathrm{d}\boldsymbol{\rho}\, \mathrm{d}\boldsymbol{\phi}$$

where
$\rho \sim \mathcal{U}(0, 1)$.
$\phi \sim \mathcal{U}(0, 2\pi)$.
$l(\boldsymbol{\rho}, \boldsymbol{\phi}, \theta) = \| r_0 \exp(j\theta) + r_{\text{net}}\sqrt{\boldsymbol{\rho}} \exp(j\boldsymbol{\phi}) \|$.

If M is random (i.e PPP), then take the expectation with respect to M:

$$\mathbb{E}_{M,\boldsymbol{r}}\left[\epsilon_i^x \epsilon_j^y\right] = \sum_{m=0}^{\infty} p_M[m] E_{\boldsymbol{r}}\left[\epsilon_i^x \epsilon_j^y | m\right]$$

where $p_M[m]$ is the PMF of M.

Specific Cases

- In Rayleigh fading [2, 4],

$$\epsilon_i = 1 - e^{-\beta r_0^\alpha/(\mathsf{SNR})} \prod_{j=1}^{M} \left(1 - p + \frac{p r_{i,j}^\alpha}{\beta r_0^\alpha + r_{i,j}^\alpha}\right).$$

- When the M interferers are i.u.d. (i.e., a binomial point process), then:

$$\mathbb{E}_{\boldsymbol{r}}[\epsilon_i|M] = 1 - \exp\left(-\frac{\beta r_0^\alpha}{\mathsf{SNR}}\right)\left[1 - p + \frac{2p}{\pi r_{\text{net}}^2}(\mathcal{T} + \Phi_1)\right]^M$$

$$\mathbb{E}_{\boldsymbol{r}}[\epsilon_i^2|M] = 2E_{\boldsymbol{r}}[\epsilon_i|M] - 1 + \exp\left(-\frac{2\beta r_0^\alpha}{\mathsf{SNR}}\right)\mathcal{S}^M$$

$$\mathbb{E}_{\boldsymbol{r}}[\epsilon_1\epsilon_2|M] = 2E_{\boldsymbol{r}}[\epsilon_i|M] - 1 + \exp\left(-\frac{2\beta r_0^\alpha}{\mathsf{SNR}}\right)\mathcal{W}^M$$

- When M is Poisson (i.e., Poisson point process) with intensity λ , then:

$$\mathbb{E}_{M,\boldsymbol{r}}[\epsilon_i] = 1 - \exp\left[-\frac{\beta r_0^\alpha}{\mathsf{SNR}} - 2\lambda p\left(\frac{\pi r_{\text{net}}^2}{2} - \mathcal{T} - \Phi_1\right)\right]$$

$$\mathbb{E}_{M,\boldsymbol{r}}[\epsilon_i^2] = 2\mathbb{E}_{M,\boldsymbol{r}}[\epsilon_i] - 1 + \exp\left(-\frac{2\beta r_0^\alpha}{\mathsf{SNR}} - \pi r_{\text{net}}^2 \lambda [1 + \mathcal{S}]\right)$$

$$\mathbb{E}_{M,\boldsymbol{r}}[\epsilon_1\epsilon_2] = 2E_{M,\boldsymbol{r}}[\epsilon_i] - 1 + \exp\left(-\frac{2\beta r_0^\alpha}{\mathsf{SNR}} - \pi r_{\text{net}}^2 \lambda [1 + \mathcal{W}]\right)$$

- The spatially averaged correlation coefficient can be evaluated by substituting the expressions provided above in (1), where

$$\mathcal{T} = \frac{1}{r_{\text{net}}^2}\frac{(r_{\text{net}} - r_0)^{\alpha+2}}{2+\alpha}\Psi_1(r_{\text{net}} - r_0)$$

$$\Phi_i = \int_{r_{\text{net}}-r_0}^{r_{\text{net}}+r_0} \frac{r}{\beta r_0^\alpha r^{-\alpha}[\beta r_0^\alpha + 2(i-1) r^\alpha] + 1} \arccos\left(\frac{r^2 + r_0^2 - r_{\text{net}}^2}{2 r_0 r}\right) \mathrm{d}r$$

$$\Psi_k(y) = {}_2F_1\left(\left[1, \frac{2}{\alpha} + k\right]; \frac{2}{\alpha} + 1 + k; -\frac{y^\alpha}{\beta r_0^\alpha}\right)$$

$$\mathcal{S} = (1-p)^2 + \frac{4}{\pi r_{\text{net}}^2}(1-p)\,p\,(\mathcal{T} + \Phi_1) + \frac{2p^2}{\pi r_{\text{net}}^2}(\mathcal{Z}(r_{\text{net}} - r_0) + \Phi_2)$$

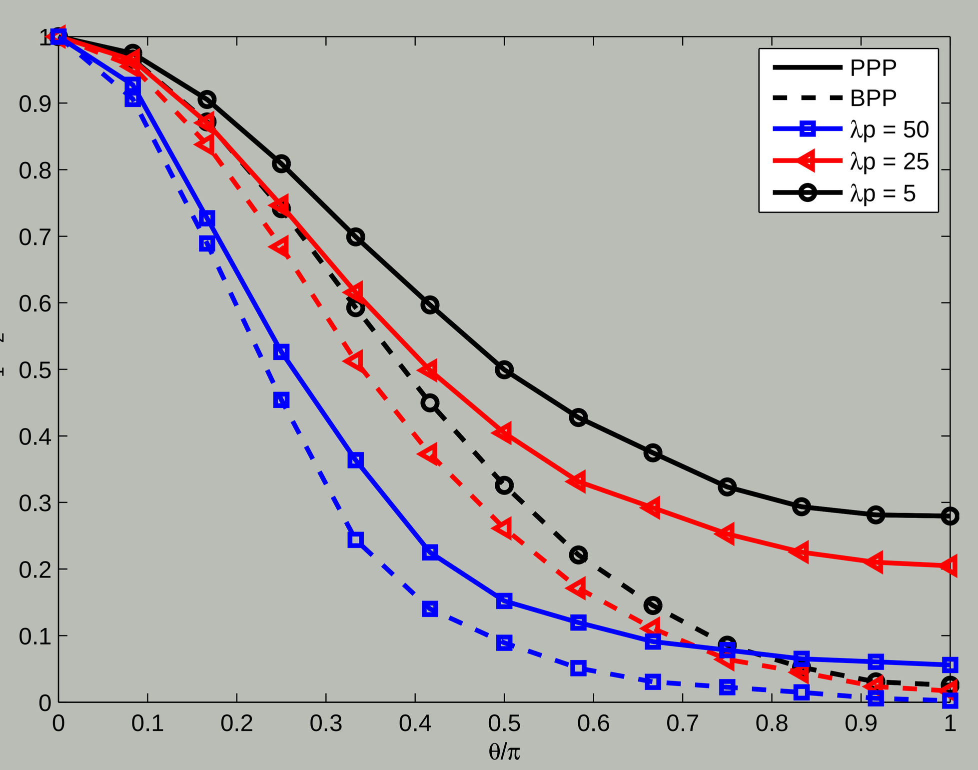
$$\mathcal{Z}(y) = \frac{y^2}{2\alpha[y^\alpha + \beta r_0^\alpha]}[\alpha + 2\beta r_0^\alpha + \alpha y^\alpha - (\alpha+2)\Psi_0(y)]$$

$$\mathcal{W} = (1-p)^2 + \frac{4}{\pi r_{\text{net}}^2}(1-p)\,p\,(\mathcal{T} + \Phi_1) + p^2 \mathcal{X}$$

$$\mathcal{X} = \frac{1}{2\pi}\int_0^1 \int_0^{2\pi} \frac{l(\rho,\phi,0)\, l(\rho,\phi,\theta)}{[\beta r_0^\alpha + l(\rho,\phi,0)][\beta + l(\rho,\phi,\theta)]} \mathrm{d}\rho\, \mathrm{d}\phi.$$

Numerical Results

Fading	Rayleigh
Network area	$\pi r_{\text{net}}^2 = 1$
Distance between T_0 and R_i	$r_0 = \frac{r_{\text{net}}}{2}$
Path loss exponent	$\alpha = 3.5$
SNR	SNR = 10 dB
SINR threshold	β = 0 dB



Benefits and Conclusions

- The spatial correlation of the interference is quantified for a finite wireless network.
- The approach used is particularly effective when the interferers are drawn from a BPP or PPP, for which semi-closed form expressions are derived.

References

[1] R. K. Ganti and M. Haenggi, "Spatial and temporal correlation of the interference in ALOHA ad hoc networks," *IEEE Communications Letters*, vol. 13, pp. 631-633, Sept. 2009.

[2] M.C. Valenti, D. Torrieri, and S. Talarico, "A direct approach to computing spatially averaged outage probability," *IEEE Communications Letters*, vol. 18, pp. 1103-1106, Jul. 2014.

[3] Z. Khalid and S. Durrani, "Distance distributions in regular polygons," *IEEE Transactions on Vehicular Technology*, vol. 62, pp. 2363-2368, Jun. 2013.

[4] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge University Press, 2012.

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