Constellation Shaping for LDPC-Coded APSK

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Outline

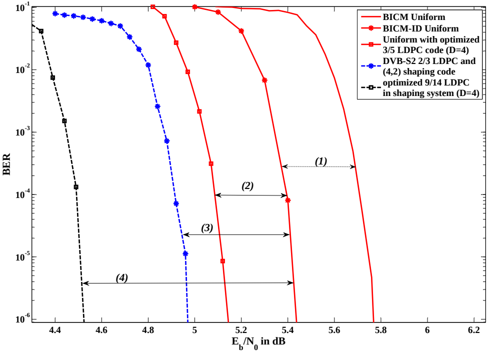
1. Introduction
2. APSK Modulation
3. LDPC Coding
4. Iterative Reception
5. LDPC Degree Optimization
6. Constellation Shaping
7. Conclusion

Outline

Motivation for this Work

- DVB-S2 is a popular system for satellite broadcast and data transmission, and uses a combination of APSK modulation and LDPC coding.
- Goal of this work is to improve performance of LDPC-coded APSK by combining the following ideas:
    - Iterative receiver implementation (a.k.a. BICM-ID).
    - Constellation shaping.
    - LDPC code optimization.

Preview of Our Results

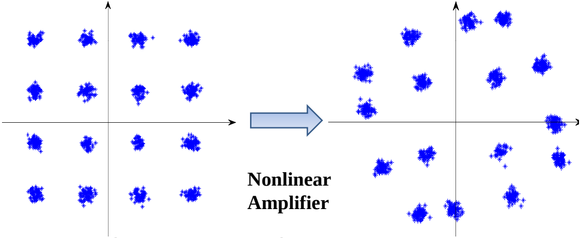


- Baseline system:
  - 32-APSK.
  - $R = 3$ bits/symbol.
  - AWGN channel.
- Performance improvements:
  1. BICM-ID decoder: *0.3 dB gain*.
  2. Optimized LDPC code's degree distribution: *0.3 dB gain*.
  3. Constellation shaping: *0.5 dB gain*.
  4. Both code optimization *and* constellation shaping: *0.9 dB gain*.

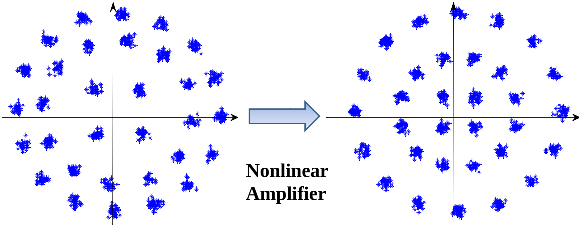
Outline

APSK vs. QAM for Nonlinear Channels

- Due to the use of TWTA, satellite channels are nonlinear.
- QAM constellations become highly distorted.

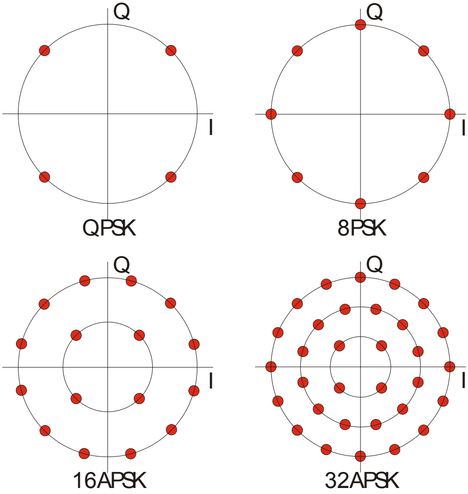


- APSK maintains distinct rings despite nonlinearity.

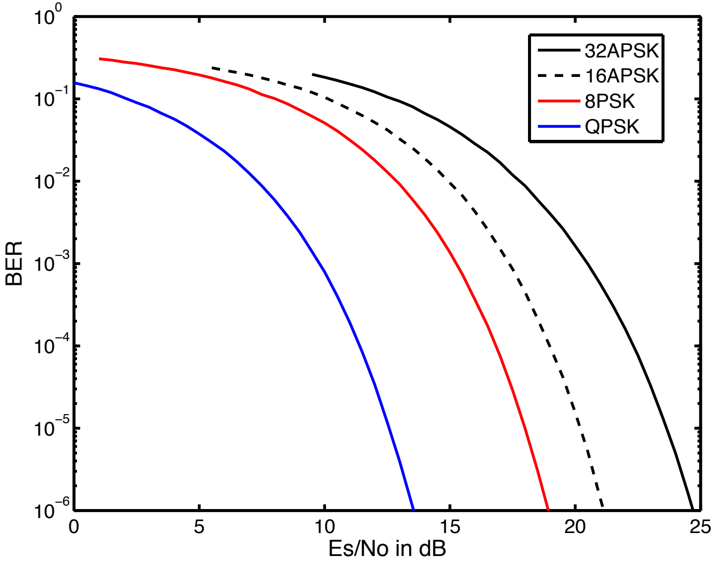


Amplitude Phase Shift Keying

DVB-S2 uses the following APSK constellations:

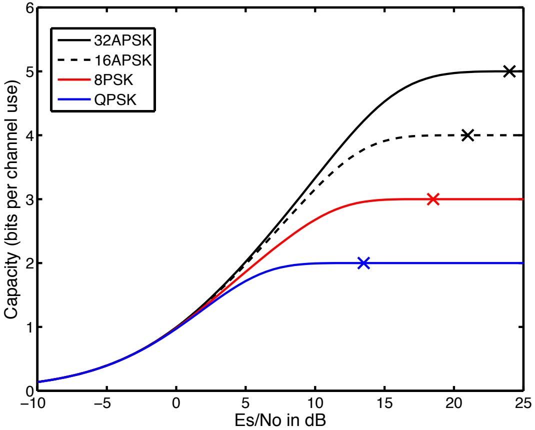


Uncoded BER in AWGN



Symmetric Information Rate of APSK

- Performance can be improved by using error control coding.
- Gains are limited by the modulation-constrained capacity.
- LDPC codes are capable of approaching capacity.



Symmetric information rate (assumes uniform input).

Outline

Single Parity-Check Codes

- Consider the following rate $R = 5/6$ single parity-check code:

$$\mathbf{c} = \left[\underbrace{1 \quad 0 \quad 1 \quad 0 \quad 1}_{\mathbf{u}} \quad \underbrace{\quad 1 \quad}_{\text{parity bit}} \right]$$

- One error in *any* position may be detected:

$$\mathbf{c} = \begin{bmatrix} 1 & 0 & X & 0 & 1 & 1 \end{bmatrix}$$

- Problem with using an SPC is that it can only detect a single error.

Product Codes

- Place data into a k by k rectangular array.
  - Encode each row with a SPC.
  - Encode each column with a SPC.
  - Result is a rate $R = k^2/(k+1)^2$ code.
- Example $k = 2$.

$$\begin{array}{|c|c|c|} \hline c_1 = u_1 & c_2 = u_2 & c_3 = c_1 \oplus c_2 \\ \hline c_4 = u_3 & c_5 = u_4 & c_6 = c_4 \oplus c_5 \\ \hline c_7 = c_1 \oplus c_4 & c_8 = c_2 \oplus c_5 & c_9 = c_3 \oplus c_6 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 1 & 0 & 1 \\ \hline 1 & 1 & 0 \\ \hline 0 & 1 & 1 \\ \hline \end{array}$$

- A single error can be corrected by detecting its row and column location

$$\begin{array}{|c|c|c|} \hline 1 & 0 & 1 \\ \hline 0 & 1 & 0 \\ \hline 0 & 1 & 1 \\ \hline \end{array} \Rightarrow \begin{array}{|c|c|c|} \hline 1 & 0 & 1 \\ \hline 1 & 1 & 0 \\ \hline 0 & 1 & 1 \\ \hline \end{array}$$

Linear Codes

$c_1 = u_1$	$c_2 = u_2$	$c_3 = c_1 \oplus c_2$
$c_4 = u_3$	$c_5 = u_4$	$c_6 = c_4 \oplus c_5$
$c_7 = c_1 \oplus c_4$	$c_8 = c_2 \oplus c_5$	$c_9 = c_3 \oplus c_6$

- The example product code is characterized by the set of five linearly-independent equations:

$$\begin{aligned}
c_3 = c_1 \oplus c_2 \quad &\Rightarrow \quad c_1 \oplus c_2 \oplus c_3 = 0 \\
c_6 = c_4 \oplus c_5 \quad &\Rightarrow \quad c_4 \oplus c_5 \oplus c_6 = 0 \\
c_7 = c_1 \oplus c_4 \quad &\Rightarrow \quad c_1 \oplus c_4 \oplus c_7 = 0 \\
c_8 = c_2 \oplus c_5 \quad &\Rightarrow \quad c_2 \oplus c_5 \oplus c_8 = 0 \\
c_9 = c_3 \oplus c_6 \quad &\Rightarrow \quad c_3 \oplus c_6 \oplus c_9 = 0
\end{aligned}$$

- In general, it takes $(n-k)$ linearly-independent equations to specify a *linear* code.

Parity-Check Matrix

- The system of equations may be expressed in matrix form as:

$$\mathbf{c}H^T = \mathbf{0}$$

where H is a *parity-check* matrix.

- Example:

$$\begin{array}{rcl} c_1 \oplus c_2 \oplus c_3 & = & 0 \\ c_4 \oplus c_5 \oplus c_6 & = & 0 \\ c_1 \oplus c_4 \oplus c_7 & = & 0 \\ c_2 \oplus c_5 \oplus c_8 & = & 0 \\ c_3 \oplus c_6 \oplus c_9 & = & 0 \end{array} \quad \Leftrightarrow \quad H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

System of equations — Parity-check matrix

LDPC Codes

- An LDPC code is a code with a large, sparse H matrix.
- A code from MacKay and Neal (1996):

$$
\mathbf{H} = \left[\begin{array}{cccc|cccc|cccc}
1 & & & & & 1 & & 1 & & 1 & & \\
1 & & & 1 & 1 & & & & & & 1 & \\
 & 1 & & & 1 & & 1 & & 1 & & & \\
 & & 1 & & & 1 & & & & & 1 & 1 \\
 & & 1 & & & & 1 & 1 & & & & 1 \\
 & 1 & & & 1 & & & & 1 & & 1 & \\
1 & & & 1 & & & 1 & & & 1 & & \\
 & 1 & & & & 1 & & 1 & & 1 & & \\
 & & 1 & 1 & & & & & 1 & & & 1
\end{array} \right]
$$

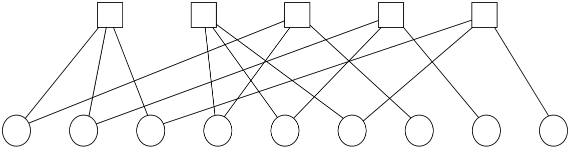
- The code called a $(3,4)$ *regular* code because:
  - Each column has exactly 3 ones.
  - Each row has exactly 4 ones.
- Irregular codes:
  - An *irregular* LDPC code has columns with different *Hamming weights*.
  - An irregular code can outperform a regular code.
  - The DVB-S2 LDPC codes are *irregular*.

Tanner Graphs

- The parity-check matrix may be represented by a *Tanner* graph.
- Bipartite graph:
  - Check nodes: Represent the $n-k$ parity-check equations.
  - Variable nodes: Represent the n code bits.
- If $H_{i,j}=1$, then i^{th} check node is connected to j^{th} variable node.
- Example: For the parity-check matrix:

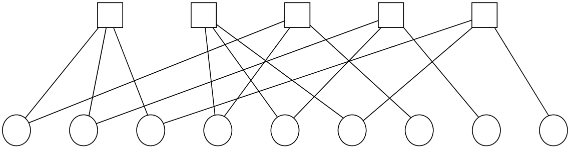
$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The Tanner Graph is:



Degree Distribution

- Edge-perspective degree distributions:
  - ρ_i is the fraction of *edges* touching degree i check nodes.
  - λ_i is the fraction of *edges* touching degree i variable nodes.
- For example, consider the Tanner graph:



  - 15 edges.
  - All are connected to degree-3 check nodes, so $\rho_3 = 15/15 = 1$.
  - Three are connected to degree-1 variable nodes, so $\lambda_1 = 3/15 = 1/5$.
  - Twelve are connected to degree-2 variable nodes, so $\lambda_2 = 12/15 = 4/5$.

DVB-S2 standardized LDPC code

Key features of the DVB-S2 LDPC code:

- Variable rate: $R_c = \frac{k_c}{n_c} = \{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{8}{9}, \frac{9}{10}\}$.
- Two lengths: $n_c = 16,200$ (short) and $n_c = 64,800$ (long).
- Systematic encoding.
  - Last $m_c = n_c - k_c$ columns of $\mathbf{H}$ are a *dual diagonal* submatrix, making it an *extended irregular repeat accumulate* (eIRA) code[1].

- Constant row weight; i.e., *check regular*.
- Variable column weight, with $D = 3$ different values[2].

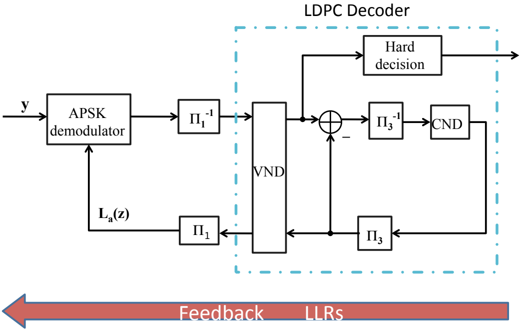
[1] M. Yang, W. E. Ryan, and Y. Li, "Design of efficiently encodable moderate-length high-rate irregular LDPC codes," *IEEE Trans. Commun.*, vol. 52, pp. 564–571, Apr. 2004.

[2] Not including the last column, which has a weight of 1.

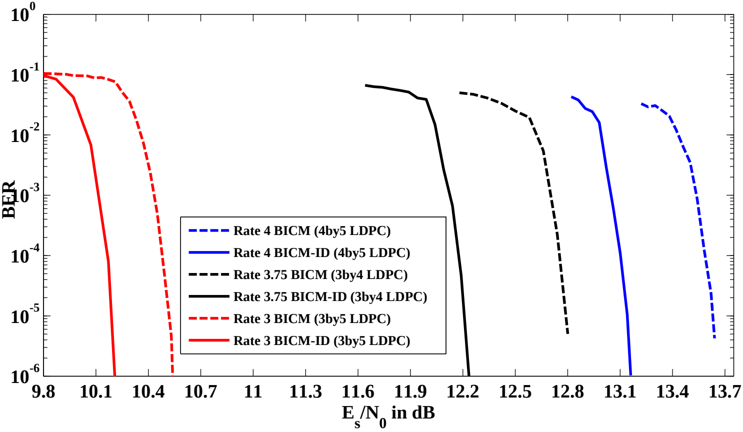
Outline

Iterative Demodulation and Decoding

- Conventional receivers first demodulation, then decode.
- Performance is improved by iterating between the demodulator and decoder.
- BICM-ID: bit-interleaved modulation with iterative decoding.



BICM vs. BICM-ID



Curves show performance of 32APSK in AWGN.

Outline

EXIT charts

The *convergence threshold* is the SNR value in which the bit error rate of an LDPC-coded system starts dropping sharply.

- The value of the threshold depends on the *degree distribution*.

EXIT charts[3]

- Predict the convergence threshold.
- Can be used to identify good candidate degree distributions.
- However, because it is just a prediction, the candidate codes still need to be simulated to determine which is best.

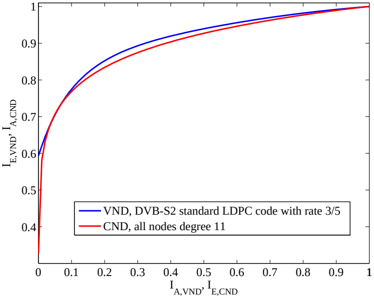


Figure : EXIT chart for the uniform system at $\mathcal{E}_b/N_0 = 4.93$ dB.

[3] S. ten Brink, G. Kramer, and A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," *IEEE Trans. Commun.*, vol. 52, pp. 670–678, Apr. 2004.

Optimal Degree Distributions

- Degree distributions for uniform 32-APSK.
- The DVB-S2 standard rate $R_c = 3/5$ LDPC code has degree distributions:

$$\begin{aligned} \lambda_2 &= 0.182 \\ \lambda_3 &= 0.273 \\ \lambda_{12} &= 0.545 \end{aligned}$$

- The optimized degree distributions with $D = 3$ are:

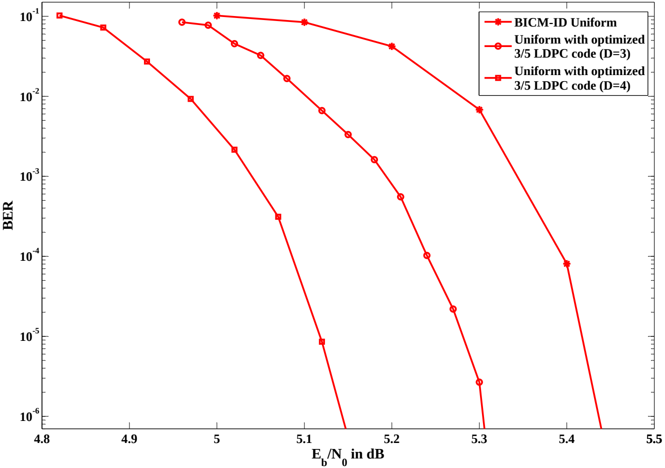
$$\begin{aligned} \lambda_2 &= 0.182 \\ \lambda_4 &= 0.473 \\ \lambda_{19} &= 0.345 \end{aligned}$$

- The optimized degree distributions with $D = 4$ are:

$$\begin{aligned} \lambda_2 &= 0.182 \\ \lambda_3 &= 0.066 \\ \lambda_4 &= 0.402 \\ \lambda_{25} &= 0.351 \end{aligned}$$

- All codes are check regular with $\rho_{11} = 1$.

BER with Optimized Degree Distributions



- BER of 32-APSK in AWGN at rate R=3 bits/symbol.
- Comparison of standard vs. optimized LDPC codes.

Outline

Constellation Shaping

The energy efficiency can be improved by transmitting lower-energy signals more frequently than higher-energy signals.

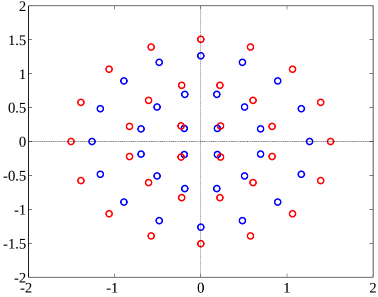


Figure : Uniform 32APSK vs. shaped 32APSK. Both constellations have the same energy.

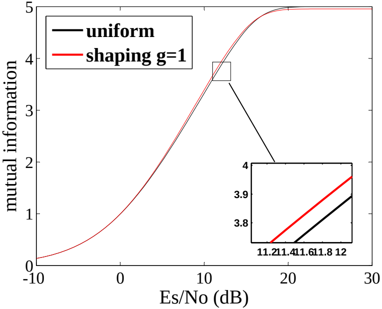
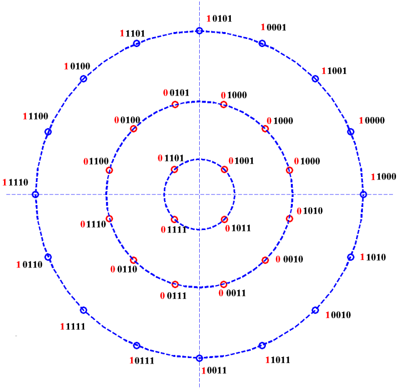


Figure : The capacity of shaped 32APSK is about 0.3 dB better than uniform 32APSK.

Shaping Through Signal Set Partitioning

- Partition the constellation into **two** equal-sized sub-constellations.
- Use a **shaping bit** to select between the two sub-constellations.
  - The lower-energy sub-constellation is selected more frequently than the higher-energy sub-constellation.
  - Requires the shaping bit to be encoded so that it is not uniform.
- The remaining bits select from among the $M/2$ symbols in the selected sub-constellation with equal probabability.



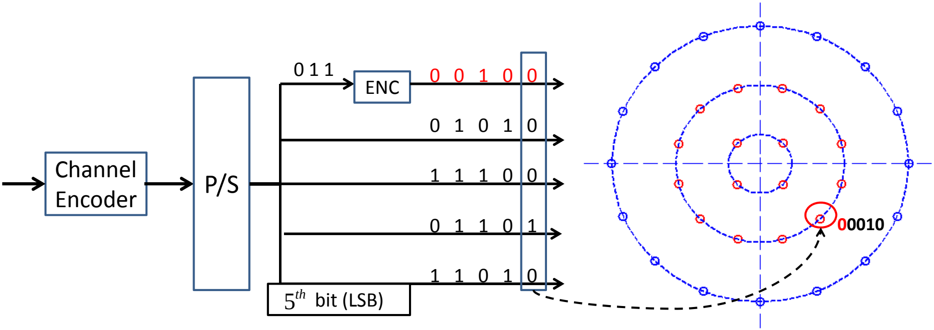
Shaping Encoder

- The shaping encoder maps k_s bits to a n_s bit shaping codeword.
- Code is designed with the goal of having more zeros than ones.
- Example $(k_s = 3, n_s = 5)$ code:

3 input data bits	5 output codeword bits
0 0 0	0 0 0 0 0
0 0 1	0 0 0 0 1
0 1 0	0 0 0 1 0
0 1 1	0 0 1 0 0
1 0 0	0 1 0 0 0
1 0 1	1 0 0 0 0
1 1 0	0 0 0 1 1
1 1 1	1 0 1 0 0

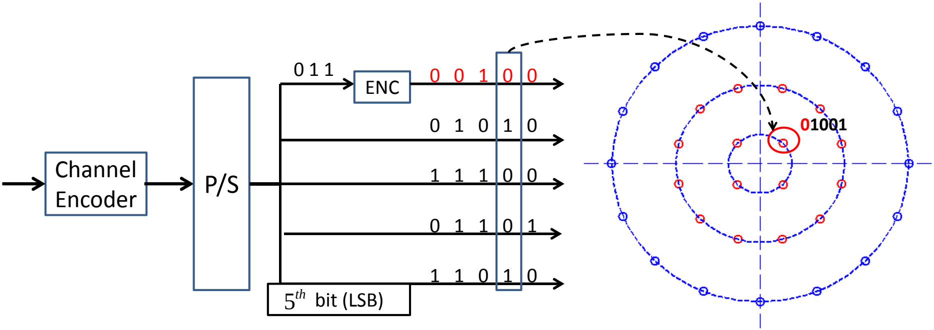
- $p_0 = 31/40$ is the probability of 0.
- $p_1 = 9/40$ is the probability of 1.

Shaping Operation



- Here, the $(5,3)$ shaping code is used as an example.
- The $\boxed{\text{P/S}}$ block segments groups of 23 bits.
- Three bits delivered to the shaping encoder.

Shaping Operation



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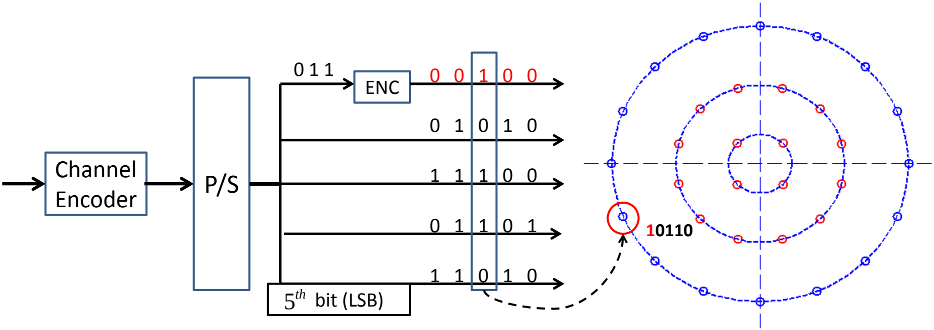
Shaping Operation



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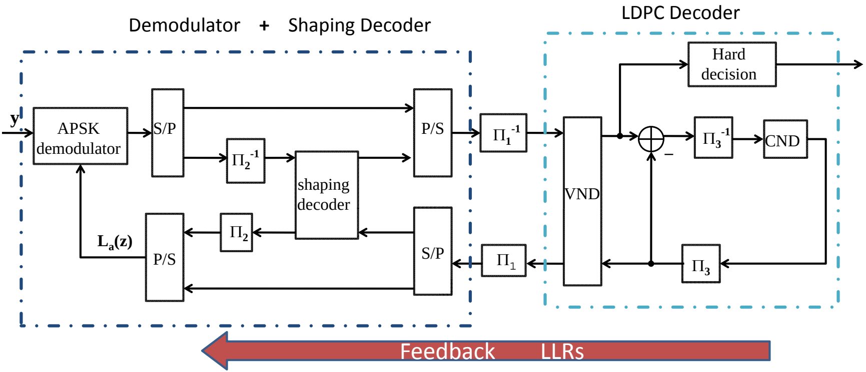
- The $\boxed{\text{P/S}}$ block segments groups of 23 bits.
- Three bits delivered to the shaping encoder.

Receiver Implementation



- Additional complexity relative to BICM-ID due to shaping decoder.
- MAP shaping decoder compares against all 2^{k_s} shaping codewords.

EXIT Charts with Constellation Shaping

When shaping is used, the variable-node decoder (VND) accounts for the effects of shaping.

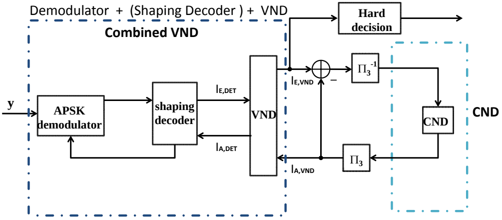


Figure : Model of decoder used for constructing EXIT charts.

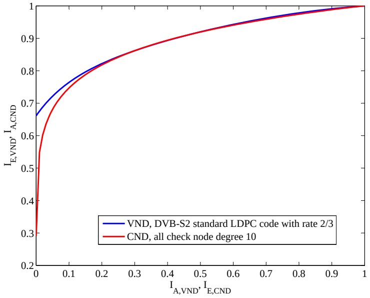


Figure : EXIT chart for the shaped system at $\mathcal{E}_b/N_0 = 4.53$ dB.

Optimal Degree Distributions with Shaping

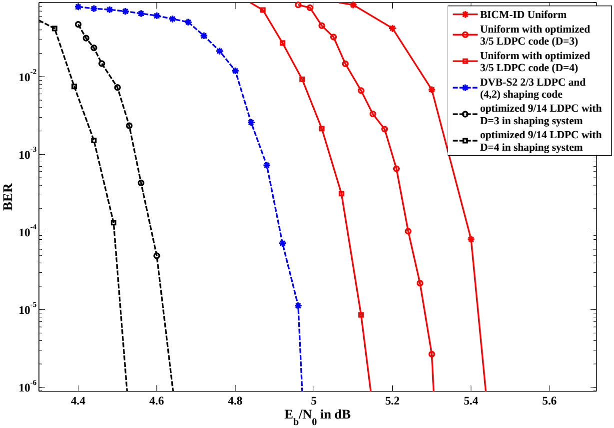
- Spectral efficiency of 3 bits per channel use.
- $(3, 2)$ shaping code.
- rate $r_c = 9/14$ LDPC code.
- Check regular with $\rho_{10} = 1$.
- The optimized degree distributions with $D = 3$ are:

$$\begin{aligned} \lambda_2 &= 0.200 \\ \lambda_3 &= 0.469 \\ \lambda_{14} &= 0.331 \end{aligned}$$

- The optimized degree distributions with $D = 4$ are:

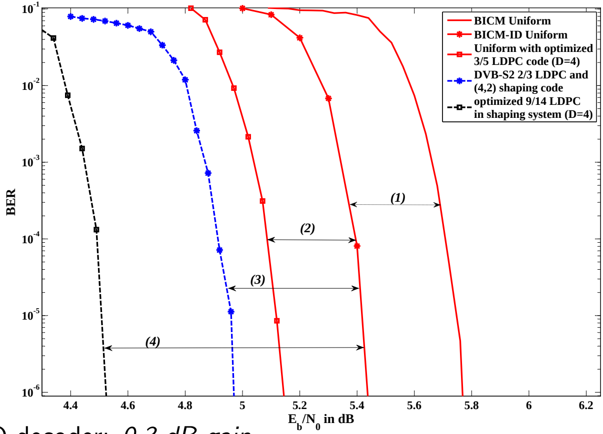
$$\begin{aligned} \lambda_2 &= 0.200 \\ \lambda_3 &= 0.461 \\ \lambda_5 &= 0.002 \\ \lambda_{13} &= 0.337 \end{aligned}$$

BER with Shaping



- BER of 32-APSK in AWGN at rate R=3 bits/symbol.

Summary of Performance Gains



- BICM-ID decoder: *0.3 dB gain*.
- Optimized LDPC degree distribution: *0.3 dB gain*.
- Constellation shaping: *0.5 dB gain*.
- Both code optimization *and* constellation shaping: *0.9 dB gain*.

Outline

Conclusion

- DVB-S2 is already a highly efficient system, thanks to
    - APSK modulation.
    - Capacity-approaching irregular LDPC codes.
- The performance of DVB-S2 can be improved by
    - BICM-ID.
    - Constellation shaping.
    - Optimization of LDPC degree-distribution.
- The cumulative gain is 1 dB with all of these.
- Future work:
    - Application to 64APSK, 128APSK, and beyond.
    - Improved symbol labeling map.

References

1 M.C. Valenti and X. Xiang, "Constellation shaping for bit-interleaved LDPC coded APSK," *IEEE Trans. Commun.*, vol. 60, no. 10, pp. 2960-2970.

2 C. Nannapaneni, M.C. Valenti, and X. Xiang, "Constellation shaping for communication channels with quantized outputs," in *Proc. Conf. on Info. Sci. and Sys. (CISS)*, (Baltimore, MD), Mar. 2011.

3 M.C. Valenti and X. Xiang, "Constellation shaping for bit-interleaved coded APSK," in *Proc. IEEE Int. Conf. on Commun. (ICC)*, (Kyoto, Japan), June 2011.

4 X. Xiang and M.C. Valenti, "Improving DVB-S2 performance through constellation shaping and iterative demapping," in *Proc. IEEE Military Commun. Conf. (MILCOM)*, (Baltimore, MD), Nov. 2011.

5 X. Xiang and M.C. Valenti, "Closing the gap to the capacity of APSK: Constellation shaping and degree distributions," in *P*roc. Int. Conf. on Computing, Networking, and Commun. (ICNC), (San Diego, CA), Jan. 2013

Thank You.