An Accurate and Efficient Analysis of a MBSFN Network

Salvatore Talarico       Matthew C. Valenti

West Virginia University
Morgantown, WV

May 9, 2014
1. Introduction

2. Spatial Model

3. Network Model

4. Performance Analysis

5. Conclusion
Outline

1 Introduction

2 Spatial Model

3 Network Model

4 Performance Analysis

5 Conclusion
Multicast-Broadcast Single-Frequency Network (MBSFN)

- MBSFN is a transmission mode in the LTE standard.
- MBSFN allows multimedia content to be broadcast over a cellular network (no additional license spectrum, no new infrastructure and end-user devices).
- Different MBSFN Areas can broadcast different contents.
- A cell can be part of multiple (up to eight) MBSFN Areas.

Diagram:
- MBSFN Service Area
- A Cell Can Belong to Multiple MBSFN Areas
- MBSFN Area (A)
- MBSFN Area (B)
- Same data All Synchronized
In an MBSFN area, it is also required the use of the same radio resources. The coordination is provided by a logical node called *Multi-cell/multicast Coordination Entity* (MCE). Inside a radio frame, certain sub-frames are reserved as MBSFN subframes. The MBSFN subframes use the *extended cyclic prefix* (16.7 µs).
Outline

1. Introduction
2. Spatial Model
3. Network Model
4. Performance Analysis
5. Conclusion
MBSFN Areas: Deployment

- In absence of real data, a MBSFN network can be created as follows:
  - Deploy $M$ base stations according to a *uniform clustering model* characterized by an exclusion zone of radius $r_{bs}$ [15];

In absence of real data, a MBSFN network can be created as follows:

1. Deploy $M$ base stations according to a *uniform clustering model* characterized by an exclusion zone of radius $r_{bs}$ (●) [15];

2. Pick $Z$ points $\{Z_1, \ldots, Z_S\}$ according to a regular hexagonal grid, which are equally separated by $d_{sfn}$ (★);

---

MBSFN Areas: Deployment

- In absence of real data, a MBSFN network can be created as follows:
  1. Deploy $M$ base stations according to a uniform clustering model characterized by an exclusion zone of radius $r_{bs}$ ($\bullet$) [15];
  2. Pick $Z$ points $\{Z_1, ..., Z_S\}$ according to a regular hexagonal grid, which are equally separated by $d_{sfn}$ ($\star$);
  3. Form MBSFN areas by grouping the radio cells of all base stations that are closer to each of the points $\{Z_1, ..., Z_S\}$.

---

Network Model

- The Network comprises:
  - \(S\) MFSFN areas \(\{Z_1, \ldots, Z_S\}\) which are equally separated by \(d_{sfn}\);
  - \(M\) cellular base stations \(\{X_1, \ldots, X_M\}\).
- Finite network area discretized into \(N\) points, \(\{Y_1, \ldots, Y_N\}\).
- The instantaneous power of \(X_i\) received at position \(Y_j\) is
  \[
  \rho_{i,j} = P_0 g_{i,j} 10^{\xi_{i,j}/10} f (||X_i - Y_j||)
  \]  
  (1)

where

- \(P_0\) is the transmit power;
- \(g_{i,j}\) is the power gain due to Nakagami fading;
- \(f(\cdot)\) is a path-loss function:
  \[
  f (d) = \left( \frac{d}{d_0} \right)^{-\alpha}
  \]
  - \(\alpha\) is the path loss exponent;
  - \(d \geq d_0\);
- \(\xi_{i,j}\) is a shadowing factor and \(\xi_{i,j} \sim N\left(0, \sigma_s^2\right)\) with Gudmundson’s autocorrelation function
  \[
  R(\Delta x) = \exp \left\{ - \frac{||\Delta x||}{d_{corr}} \ln 2 \right\}
  \]
  (2)

with the decorrelation length \(d_{corr} = 20\text{ m}\) as suggested by the 30.03 UMTS standard.
Inter-Symbol Interference (ISI)

- In a MBSFN OFDMA network, given a MBSFN area $Z_k$, there are two sources of ISI:
  - **Inter-MBSFN area interference**: all the base stations outside $Z_k$;
  - **Intra-MBSFN area interference**: a transmission results in ISI if
    \[ ||X_i - Y_j|| > \frac{c}{T_{ECP}} \approx 5 \text{ km} \]
    where
    - $c = 3 \times 10^8 \text{ m/s}$, which is the speed of light;
    - $T_{ECP} = 16.7 \mu\text{s}$, which is the *extended cyclic prefix*.
Signal-To-Interference-And-Noise Ratio (SINR)

- Let $G_{j,z}$ denote the set of the indexes of the base stations that belong to the $z^{th}$ MBSFN area and serving location $Y_j$, and let $N_j = |G_{j,z}|$ denote the cardinality of $G_{j,z}$.

- The signal from base station $X_i, i \in G_{j,z}$ to the UE at location $Y_j$ is included in the maximal-ratio combining (MRC) combined signal passed to the demodulator and the instantaneous SINR at location $Y_j$ by using (1) and (2) can be expressed as

$$\gamma_j = \sum_{i \in G_{j,z}} g_{i,j} \Omega_{i,j} \frac{1}{\Gamma^{-1} + \sum_{i \notin G_{j,z}} g_{i,j} \Omega_{i,j}} \tag{3}$$

where

- $\Gamma = d_0^\alpha N_j P_0 / N$ is the signal-to-noise ratio (SNR) at a mobile located at unit distance when fading and shadowing are absent, where $N$ is the noise power;

- $\Omega_{i,j} = \frac{10^{\xi_{i,j}/10} ||X_i - Y_j||^{-\alpha}}{N_j}$ is the normalized power of $X_i$ at receiver $Y_j$. 
Conditional Outage Probability

- An outage occurs when the SINR is below a threshold $\beta$.
  - $\beta$ depends on the choice of modulation and coding.
- The outage probability for the mobile $Y_j$ conditioned over the network is
  \[ \epsilon_j = P[\gamma_j \leq \beta_j | \Omega_j]. \] (4)
- The conditional outage probability is found in closed form [5] for non-identical Nakagami-m parameters $\{m_{i,j}\}$:
  - characterize the fading from the base station $X_k$ to the mobile $Y_j$;
  - selected based on a distance-depending fading model:
    \[ m_{i,j} = \begin{cases} 
    3 & \text{if } \|X_i - Y_j\| \leq r_f/2 \\
    2 & \text{if } r_f/2 < \|X_i - Y_j\| \leq r_f \\
    1 & \text{if } \|X_i - Y_j\| > r_f 
    \end{cases} \] (5)

where $r_f$ is the line-of-sight radius.

Outline

1. Introduction
2. Spatial Model
3. Network Model
4. Performance Analysis
5. Conclusion
Area Below An Outage Threshold (ABOT)

- The *area below an outage threshold* (ABOT) is defined as the fraction of the network realization $t$ that provides an outage probability (averaged over the fading) that meets a threshold $\hat{\epsilon}$ following

$$A_{\text{bot}}^{(t)} = P[\epsilon_j < \hat{\epsilon}] \quad (6)$$

*Figure:* Close-up of an example network topology. The white areas are the portion of the network for which the outage probability is above a typical value of $\hat{\epsilon} = 0.1$. 
Performance Analysis

Area Below An Outage Threshold (ABOT)

- The *area below an outage threshold* (ABOT) is defined as the fraction of the network realization $t$ that provides an outage probability (averaged over the fading) that meets a threshold $\hat{\epsilon}$ following

$$A_{\text{bot}}^{(t)} = P[\epsilon_j < \hat{\epsilon}].$$  \hspace{1cm} (6)

- After computing $A_{\text{bot}}^{(t)}$ for $\Upsilon$ network topologies, its *spatial average* can be computed as

$$\bar{A}_{\text{bot}} = \frac{1}{\Upsilon} \sum_{t=1}^{\Upsilon} A_{\text{bot}}^{(t)}. \hspace{1cm} (7)$$

- Let $R = C(\beta_j)$ represent the relationship between the code rate $R$ (in bit per channel used [bpcu]) and SINR threshold $\beta_j$. For modern cellular systems, it is reasonable to use:

$$C(\beta_j) = \log_2(1 + \beta_j)$$
ABOT vs Rate

**Settings:**
- Square arena of side $d_{\text{net}} = 20$ km;
- SNR: $\Gamma = 10$ dB;
- Path loss exponent: $\alpha = 3.5$;
- Distance among MBSFN areas: $d_{\text{sfn}} = 6$ km;
- Line-of-sight radius: $r_f = 0.5$ km;
- Exclusion zone: $r_{bs} = 0.5$ km;
- Outage constraint: $\hat{\epsilon} = 0.1$.

**Figure:** ABOT as function of the rate for both a shadowed ($\sigma_s = 8$ dB) and unshadowed environment.
**Performance Analysis**

**ABOT vs Minimum Separation Among Base Stations**

**Figure:** ABOT as a function of the minimum separation among base stations, for both Rayleigh fading and a distance-depending fading when $r_{bs} = r_f$.

**Settings:**
- Square arena of side $d_{net} = 20$ km;
- SNR: $\Gamma = 10$ dB;
- Path loss exponent: $\alpha = 3.5$;
- Distance among MBSFN areas: $d_{sfn} = 6$ km;
- Code Rate: $R = 0.1$;
- Density of base station: $\lambda = 0.1$ #bs/km$^2$
- Outage constraint: $\hat{\epsilon} = 0.1$. 

**Area below an outage threshold (ABOT)**

Unshadowed, distance-dependent fading

$\sigma = 8$ dB, distance-dependent fading

Unshadowed, Rayleigh fading

$\sigma = 8$ dB, Rayleigh fading
ABOT vs Outage Constraint

Figure: ABOT as a function of the outage threshold \( \hat{\epsilon} \) for both a shadowed \((\sigma_s = 8 \text{ dB})\) and unshadowed environment.

Settings:
- Square arena of side \( d_{net} = 20 \text{ km} \);
- SNR: \( \Gamma = 10 \text{ dB} \);
- Path loss exponent: \( \alpha = 3.5 \);
- Code Rate: \( R = 0.5 \);
- Density of base station: \( \lambda = 0.5 \text{ #bs/km}^2 \);
- Line-of-sight radius: \( r_f = 0.5 \text{ km} \);
- Exclusion zone: \( r_{bs} = 0.5 \text{ km} \);
Outline

1. Introduction
2. Spatial Model
3. Network Model
4. Performance Analysis
5. Conclusion
Conclusions

- A new approach for modeling and analyzing the performance of multicast-broadcast single-frequency network (MBSFN) has been presented.
- The analysis is driven by a new outage probability closed form expression, which is exact for a given network realization and accounts for path loss, correlated shadowing, and Nakagami-m fading with non-identical parameters.
- Despite other works that characterize the performance of a MBSFN network, the topology of the network is determined by a constrained random spatial model.
- The results show:
  - An increase in the size of an MBSFN areas leads to an improvement in performance until the inter-MBSFN area ISI begins to degrade performance;
  - As expected, densification or an increase in the minimum separation among base stations improve performance.
Thank You