Coherent and Multi-symbol Noncoherent CPFSK: Capacity and Code Design

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Outline

1. Coherent CPFSK
   - System Model and Trellis Decoding
   - Capacity of Coherent CPFSK
   - Capacity Approaching Code Design
2. Multi-symbol Noncoherent CPFSK
   - Capacity of Multi-symbol Noncoherent CPFSK
   - Code Design
3. Conclusion

Discrete Time Model

$$\mathbf{y} = e^{j\phi}\sqrt{\mathcal{E}_s}\mathbf{x} + \mathbf{n}$$

- $\mathbf{y}$ is the output from M complex filters matched to the tones,
- $\mathbf{x}$ is chosen from columns of $\mathbf{K} = [\mathbf{k}_0, \mathbf{k}_1, \cdots, \mathbf{k}_{M-1}]$
- $\mathbf{n}$ is colored noise, with $E(\mathbf{n}\mathbf{n}^H) = N_0\mathbf{K}$.

- Decoding metric

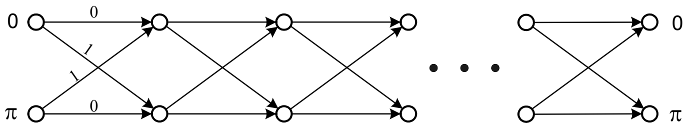
$$\log p(\mathbf{y}|\mathbf{x} = \mathbf{k}_\nu, \phi) = 2\frac{\sqrt{\mathcal{E}_s}}{N_0}\mathrm{Re}(e^{-j\phi}y_\nu) + \mathrm{Constant}.$$

CPFSK Trellis

- $h = \frac{P}{Q}$, which means the trellis has Q states
- Suppose the initial phase is 0,

$$\phi_i \in \Phi = \{\frac{2k\pi}{Q}, k = 0, 1, \cdots, Q-1\}$$

- Example of MSK: $M = 2$, $h = 1/2$



Capacity of Coherent CPFSK Detection

- i.u.d. capacity calculation through trellis

$$
\begin{aligned}
C^{(c)} &= \lim_{N\to\infty} \frac{1}{N} I(\mathbf{x}_0^{N-1}, \mathbf{y}_0^{N-1}) \\
&= \lim_{N\to\infty} \frac{1}{N} \left[\sum_{i=0}^{N-1} H(\mathbf{x}_i) - \sum_{i=0}^{N-1} H(\mathbf{x}_i | \mathbf{x}_0^{i-1}, \mathbf{y}_0^{N-1}) \right]
\end{aligned}
$$

- Treating CPFSK as a finite state markov channel (FSMC)
- Using Monte Carlo simulation to find $E\left[-\log p(\mathbf{x}_i|\mathbf{x}_0^{i-1}, \mathbf{y}_0^{N-1})\right]$

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BCJR Algorithm

- Finding $p(\mathbf{x}_i|\mathbf{y}_0^{N-1})$
- Define α, β and γ as

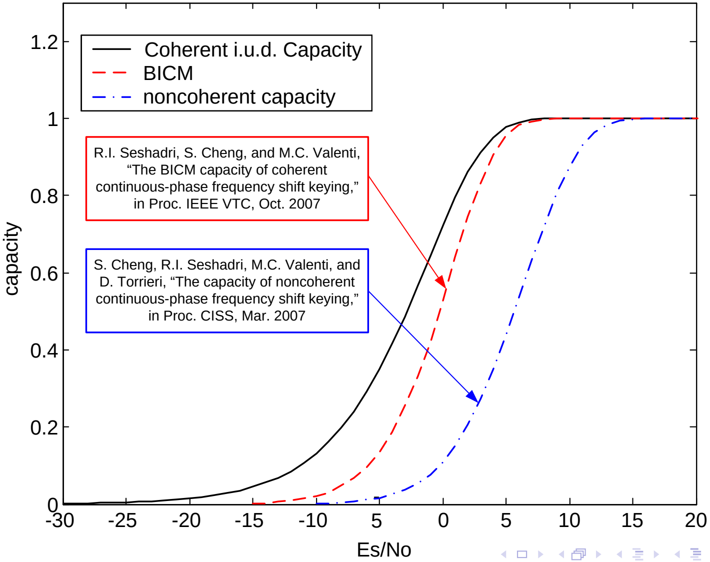
$$\begin{aligned}\alpha_i(\phi_i) &\triangleq p(\phi_i, \mathbf{y}_0^{i-1})\\ \beta_{i+1}(\phi_{i+1}) &\triangleq p(\mathbf{y}_{i+1}^{N-1}|\phi_{i+1})\\ \gamma(\phi_i \rightarrow \phi_{i+1}, \mathbf{y}_i) &\triangleq p(\mathbf{y}_i, \phi_{i+1}|\phi_i).\end{aligned}$$

Modification to BCJR Algorithm

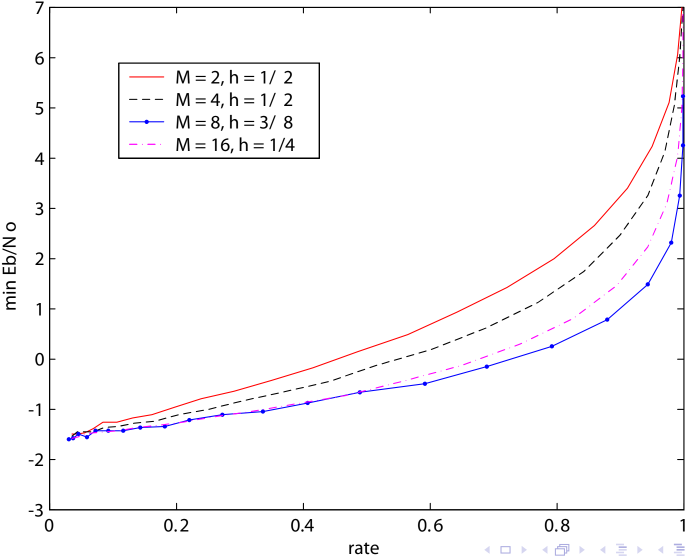
- Finding $p(\mathbf{x}_i|\mathbf{x}_0^{i-1}, \mathbf{y}_0^{N-1})$
- Define α, β and γ as

$$\begin{aligned}
\alpha_i(\phi_i) &\triangleq p(\phi_i, \mathbf{y}_0^{i-1}, \mathbf{x}_0^{i-1}) \\
\beta_{i+1}(\phi_{i+1}) &\triangleq p(\mathbf{y}_{i+1}^{N-1}|\phi_{i+1}) \\
\gamma(\phi_i \rightarrow \phi_{i+1}, \mathbf{y}_i, \mathbf{x}_i) &\triangleq p(\mathbf{y}_i, \phi_{i+1}|\phi_i, \mathbf{x}_i).
\end{aligned}$$

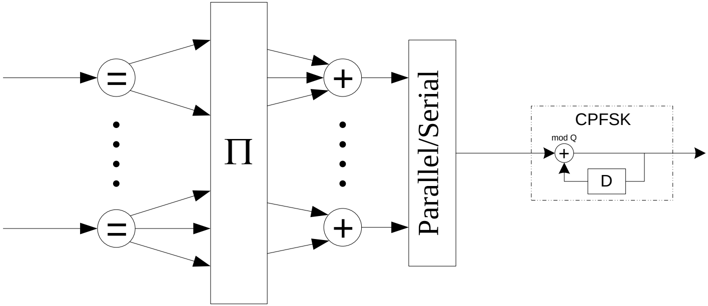
Capacity Curves: MSK



Coherent Capacity Curves

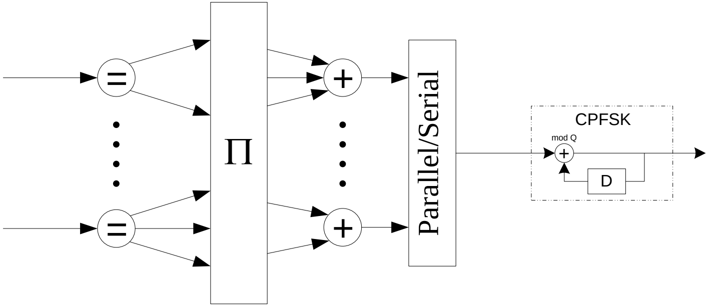


Code Design



- Irregular repeat-accumulate (IRA) code structure
- Using linear programming to design capacity approaching code

Code Design



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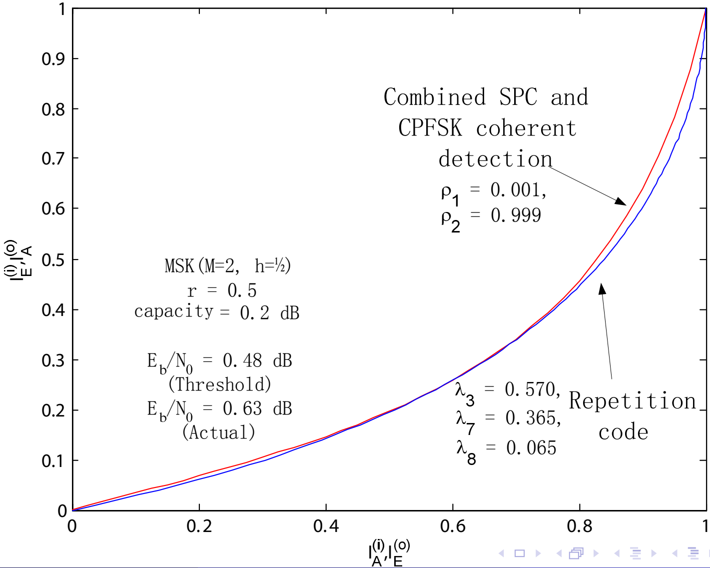
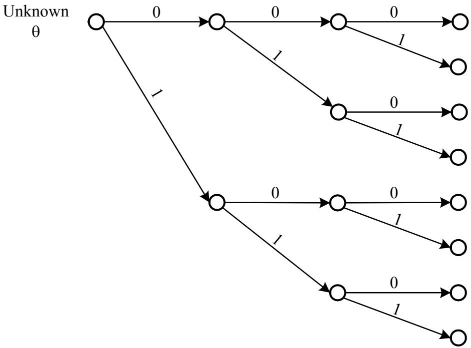
EXIT Curves

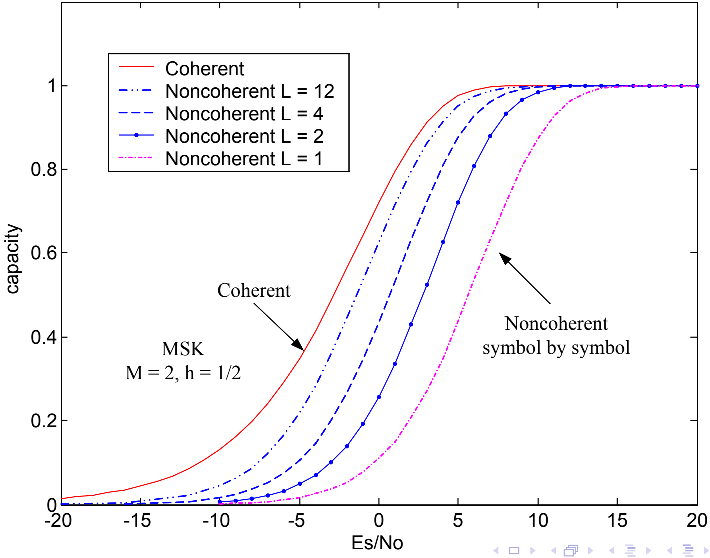

Combined SPC and CPFSK coherent detection
ρ1 = 0.001,
ρ2 = 0.999
MSK(M=2, h=½)
r = 0.5
capacity = 0.2 dB
Eb/N0 = 0.48 dB (Threshold)
Eb/N0 = 0.63 dB (Actual)
λ3 = 0.570,
λ7 = 0.365,
λ8 = 0.065
Repetition code
I(i)E, I(o)A
I(i)A, I(o)E
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Multi-symbol Noncoherent Detector

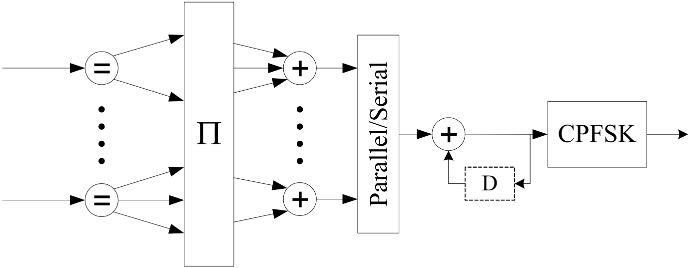
- Noncoherent detector over L symbols
- Approach capacity of coherent detector when L is large enough
- Detector structure



Multi-symbol Noncoherent Detector vs Coherent (MSK)

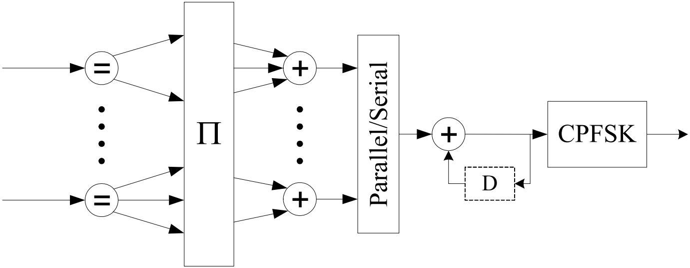


IRA Coded System



- Irregular repeat-accumulate (IRA) code structure
- Using differential encoder

IRA Coded System



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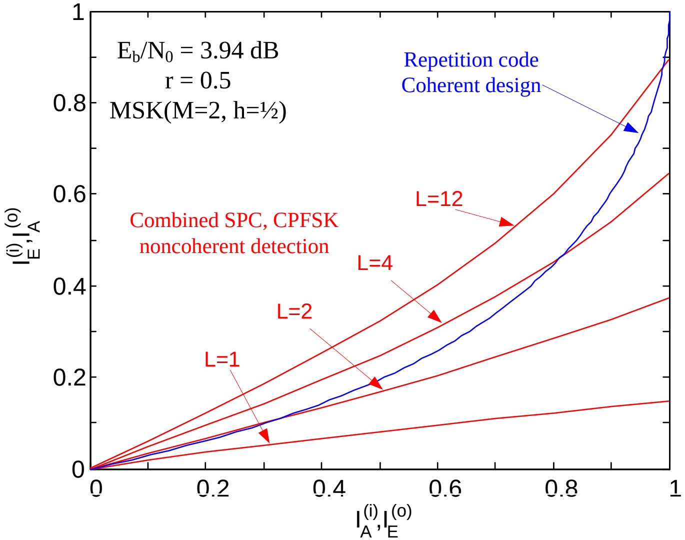
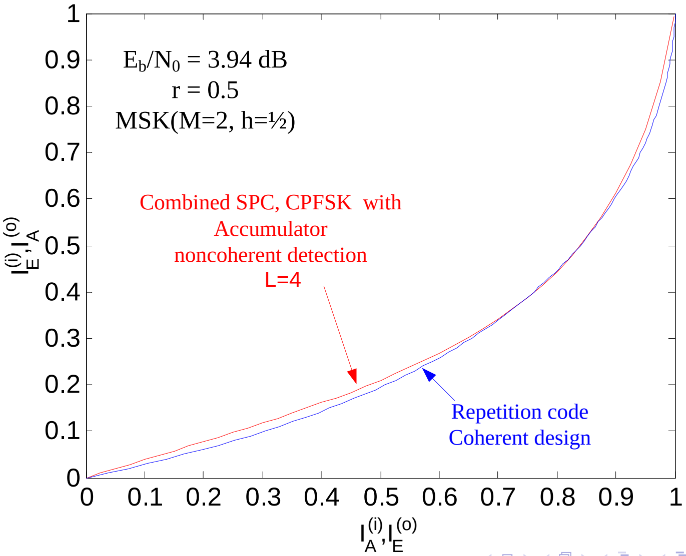
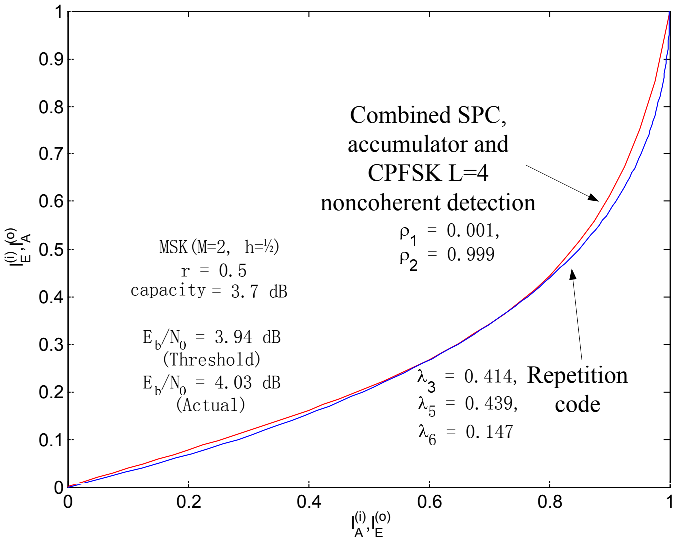
Without Accumulator



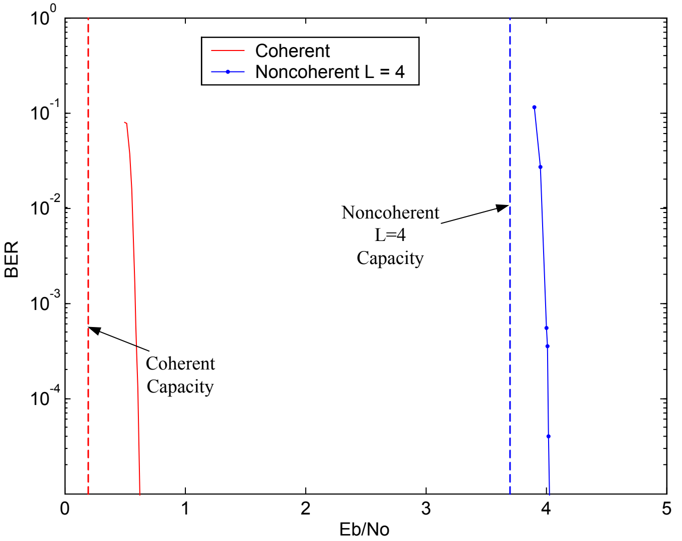
With Accumulator



A New Design


Combined SPC, accumulator and CPFSK L=4 noncoherent detection
ρ1 = 0.001,
ρ2 = 0.999
MSK(M=2, h=½)
r = 0.5
capacity = 3.7 dB
Eb/N0 = 3.94 dB (Threshold)
Eb/N0 = 4.03 dB (Actual)
λ3 = 0.414,
λ5 = 0.439,
λ6 = 0.147
Repetition code
I(i)E, I(o)A
I(i)A, I(o)E
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

BER Curves


Coherent
Noncoherent L = 4
Noncoherent
L=4
Capacity
Coherent
Capacity
BER
Eb/No
10^0
10^-1
10^-2
10^-3
10^-4
0
1
2
3
4
5

Conclusion

- The i.u.d. capacity of coherent detected CPFSK in AWGN channel is found by treating CPFSK as a finite-state Markov channel.
- The capacity of multi-symbol noncoherent detected CPFSK is studied. When the block size L increases, this capacity gets closer to the coherent capacity. For example, at coding rate $r = 0.5$, the $L = 4$ noncoherent MSK detector is $3.5dB$ worse than the coherent one, but $5dB$ better than the symbol-by-symbol noncoherent detector.
- Capacity approaching codes are designed for both detectors. IRA code structure is utilized, and the degree distribution is optimized through the linear programming EXIT curve fitting approach.

Thank you