Outage Correlation in Finite and Clustered Wireless Networks

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Problem Statement:
Quantifying the Outage Correlation

- Suppose you have a finite network:
  - A reference transmitter is at the center.
  - The network has many randomly placed interferers.
  - Signals undergo Rayleigh fading.

- A receiver at location $Y_1$ will have a certain outage probability
  - And so will a receiver at location $Y_2$.

- **Question:** If $Y_1$ is in an outage, is $Y_2$ likely to be in an outage?
  - Can be answered by quantifying the outage correlation.
Contributions

- The outage probability at a given location is a **random variable**.
  - Randomness is due to **fading** and the interferers’ random **location** and **activity**.
- Outage probability at two locations is a **pair** of random variables.
  - Just like any pair of random variables, they can be characterized by a **correlation coefficient**.
  - Main task in this paper is to **compute** the correlation coefficient.
- This problem has been previously solved for **infinite** networks with interferers drawn from a **PPP** [7].
  - Our contribution is to consider more **arbitrary** networks:
    - **Finite** in extent.
    - Interferers drawn from **arbitrary point processes**: BPP, PPP, Clustered PP.
- **Key finding**: **Interference can be correlated even if the fading is independent**.
  - Reason: Interferers’ location and activity are **common sources of randomness**.

The instantaneous signal-to-interference-and-noise ratio (SINR) at the receiver location $Y_j$ is:

\[ \gamma_j = \frac{P_{0}g_{0,j}r_{0,j}^{-\alpha}}{N + \sum_{i=1}^{M} I_{i}P_{i}g_{i,j}r_{i,j}^{-\alpha}} \]

- **Power transmitted by reference transmitter $X_0$**
- **Path-loss exponent ($\alpha \geq 2$)**
- **Number of interferers**
- **Distances between the $i^{th}$ transmitter and the $j^{th}$ receiver**
- **Fading gains (e.g., i.i.d exponential for Rayleigh fading)**
- **Bernoulli random variable indicating interference, where $P[i=1]=p_i$**

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**SINR**

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Talarico, Valenti, Di Renzo
Outage Probability

- The SINR is a function of:
  - **Fading** \((g)\)
  - **Location** of interferers \((r)\)
  - **Number** of interferers \((M)\)
- Like any random variable, the SINR can be characterized by its **CDF**:
  \[
  F_{\gamma_j}(\beta) = P[\gamma_i \leq \beta]
  \]
- By interpreting \(\beta\) to be an **outage threshold**, then the CDF is the **outage probability** of the SINR
  \[
  \epsilon_j = F_{\gamma_j}(\beta)
  \]
- In the above, the dependence on \(\beta\) is suppressed.
From Conditional Outage Probability to Its Spatial Average [5]

Fix $r$ and $M$, and find the conditional OP averaged over fading:

$$\epsilon_j(M, r) = \mathbb{E}_g[1(\gamma_j \leq \beta) \mid M, r]$$

For a given $M$, the spatially averaged OP is:

$$\epsilon_j(M) = \mathbb{E}_r[\epsilon_j(r) \mid M] = \int \epsilon_j(M, r) f_r(r \mid M) \, dr$$

If interferers are uniform on a circle [24]:

$$f_r(r \mid M) = \begin{cases} \frac{2\pi r^2}{|A|} & \text{for } 0 \leq r \leq r_1 \\ \frac{2\pi^2 r_1^2}{|A|} \arccos \left( \frac{r_2^2 + r_0^2 - r_1^2}{2r_0^2r} \right) & \text{for } r_1 \leq r \leq r_2 \end{cases}$$

If $M$ is random, then take the expectation with respect to $M$:

$$\epsilon_j = \mathbb{E}_M[\epsilon_j(M)] = \sum_{m=0}^{\infty} p_M[m] \mathbb{E}_r[\epsilon_i^x(r) \mid m]$$

$p_M[m]$ is the PMF of $M$

For PPP:

$$p_M[m] = \frac{(\lambda |A|)^m}{m!} \exp(-\lambda |A|), \quad \text{for } m \geq 0$$


Averaging Over the Fading

• When **conditioned** on the location and number of interferers, the **outage probability** is:

\[
\epsilon_j(M, r) = \mathbb{E}_g[\mathbf{1}(\gamma_j \leq \beta) \mid M, r] = 1 - \exp\left(-\frac{\beta}{\text{SNR}}\right) \prod_{i=1}^{M} \left(1 - p_i + \frac{p_i d_{i,j}^\alpha}{\beta + d_{i,j}^\alpha}\right)
\]

\[
\text{SNR} = P_0 r_0^{-\alpha} / N \\
d_{i,j} = r_{i,j} / r_0
\]

• The set \(\{\epsilon_1(M, r), \epsilon_2(M, r)\}\) constitutes a **pair of random variables**.
  • Despite the independent fading, the two variables may be **correlated** because they both depend on the same realization of interference locations \(r\)
Outage Probability and Its Dependence Upon Topology

- Fading: Rayleigh
- Network area: \( \pi \cdot r_{\text{out}}^2 \) with \( r_{\text{out}} = 1 \)
- Distance between \( X_0 \) and \( Y_j \): \( r_0 = 0.25 \)
- Path-loss exponent: \( \alpha = 3.5 \)
- SNR: 10 dB
- Number of Interferers: \( M = 2 \)

- Blue dashed lines = conditional outage probability
- Red = spatial average
- The outage probability **varies significantly** with the **location of the interferers**.
Outage Correlation Coefficient

- The correlation coefficient of the outage at the two receivers is:

\[
\zeta[Y_1, Y_2] = \frac{E_{M,r}[\epsilon_1(M, r)\epsilon_2(M, r)] - E_{M,r}[\epsilon_1(M, r)]E_{M,r}[\epsilon_2(M, r)]}{\sqrt{E_{M,r}[\epsilon_1^2(M, r)] - E_{M,r}^2[\epsilon_1(M, r)]\sqrt{E_{M,r}[\epsilon_2^2(M, r)] - E_{M,r}^2[\epsilon_2(M, r)]}}
\]

- The above expectation is with respect to \( r \) and \( M \).
- Requires computation of:
  - First moments (same procedure as for finding spatial average outage probability)
  - Second moments
  - Joint first moment
- If \( M \) is fixed (e.g., a BPP), and if the two SINRs are stochastically equivalent (i.e., \( |Y_1| = |Y_2| = r_0 \)), then the interference correlation coefficient can be written as:

\[
\zeta[Y_1, Y_2] = \frac{E_r[\epsilon_1(r)\epsilon_2(r)] - E_r^2[\epsilon_j(r)]}{E_r[\epsilon_j^2(r)] - E_r^2[\epsilon_j(r)]}
\]
Approach to Evaluate Correlation Coefficient

Fix $r$ and $M$, and find the conditional OP averaged over fading:

$$
\epsilon_j(M,r) = \mathbb{E}_q[1(\gamma_j \leq \beta)|M,r]
$$

For a given $M$, evaluate the spatial average:

- The spatially averaged OP is:
  $$
  \mathbb{E}_r[\epsilon_j(r)|M] = \int \epsilon_j(M,r)f_r(r|M)\,dr
  $$
- The spatially averaged second moment OP is:
  $$
  \mathbb{E}_r[\epsilon_j^2(r)|M] = \int \epsilon_j^2(M,r)f_r(r|M)\,dr
  $$
- The spatially averaged first joint moment is:
  $$
  \mathbb{E}_r[\epsilon_1(r)\epsilon_2(r)|M] = \int \int \epsilon_1(M,l(\rho,\phi,0))\epsilon_2(M,l(\rho,\phi,\theta))f_\rho(\rho)f_\phi(\phi)\,d\rho\,d\phi
  $$

If $M$ is random, then take the expectation with respect to $M$:

$$
\mathbb{E}_{M,r}[\epsilon_i^x(r)\epsilon_j^y(r)] = \sum_{m=0}^{\infty} p_M[m]\mathbb{E}_r[\epsilon_i^x(r)\epsilon_j^y(r)|m]
$$

For BPP:

$$
\begin{align*}
  f_r(r|M) = & \begin{cases} 
    \frac{2\pi r_0^2}{|A|} & \text{for } 0 \leq r \leq r_1 \\
    \frac{2\pi r_0^2}{|A|} \arccos \left( \frac{r_0^2 r^2 + r_0^2 - r_2^2}{2r_0^2 r} \right) & \text{for } r_1 \leq r \leq r_2 
  \end{cases} \\
  l(\rho,\phi,\theta) = & ||r_0 \exp(j\theta) + r_{\text{out}}\sqrt{\rho} \exp(j\phi)|| \\
  \rho & \sim U(0,1) \\
  \phi & \sim U(0,2\pi)
\end{align*}
$$

For PPP:

$$
p_M[m] = \frac{(\lambda|A|)^m}{m!} \exp(-\lambda|A|), \quad \text{for } m \geq 0
$$

$p_M[m]$ is the PMF of $M$.
Numerical Results

- Analytical results are verified via simulation.
- Three point processes are investigated:
  - Binomial Point Process (BPP) w/ M interferers.
  - Poisson Point Process (PPP) w/ intensity $\lambda$
  - Thomas Cluster Process (TCP)

**Fading** | Rayleigh
---|---
Radius of network | $r_{\text{out}}=1$
Distance between $X_0$ and $Y_j$ | $r_0=0.25$
Path-loss exponent | $\alpha=3.5$
SNR | 10 dB
SINR threshold | $\beta=0$ dB
• Sparser networks are more correlated than denser networks (denser network corresponds to higher $\lambda_p = \frac{E[M]p}{|A|}$).
• A PPP experiences a higher correlation than a BPP.
Results for Thomas Cluster Process

- A more compact cluster (a cluster characterized by a lower $\sigma$) experiences higher correlation.
- As the network becomes denser the correlation decreases faster, since there are more dominant interferers.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fading</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>Network area</td>
<td>$\pi \cdot r_{out}^2$ with $r_{out}=1$</td>
</tr>
<tr>
<td>Distance between $X_0$ and $Y_j$</td>
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Conclusions

• Summary:
  • An analytical framework to **evaluate** in closed-form the **correlation coefficient** of the outage probability of a **finite** wireless network is provided.
  • The proposed approach is used to evaluate analytical expressions for **several point processes** (i.e., BPP, PPP, TCP).

• Remarks:
  • **Sparser** networks have a **higher** spatial correlation than denser networks.
  • PPP networks have a **higher** correlation than BPP.
  • The spatial correlation in a TCP is **smaller** when the offspring points are **more highly dispersed**.

• Applications:
  • Cooperative protocols
  • Caching
  • User-centric communications