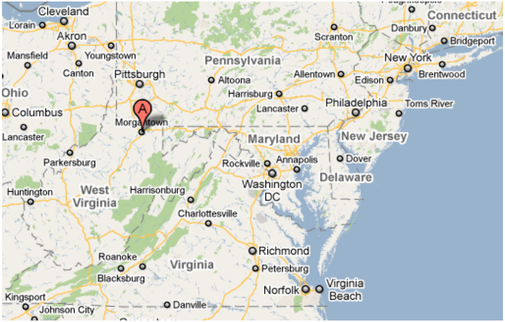
The Transmission Capacity of Frequency-Hopping Ad Hoc Networks

Matthew C. Valenti
Lane Department of Computer Science and Electrical Engineering
West Virginia University

June 13, 2011

About Me

- From the state of Maryland, in the United States.
- Educated at
  - Virginia Tech
  - Johns Hopkins University.
- Worked as an Electronics Engineer at the U.S. Naval Research Laboratory.
- Professor at West Virginia University.



Acknowledgements

Funding Sources:

- National Science Foundation (NSF).
- U.S. Army Research Laboratory (ARL).

People:

- Dr. Don Torrieri, ARL.
- Dr. Shi Cheng.
- Mr. Salvatore Talarico, Ph.D. student.

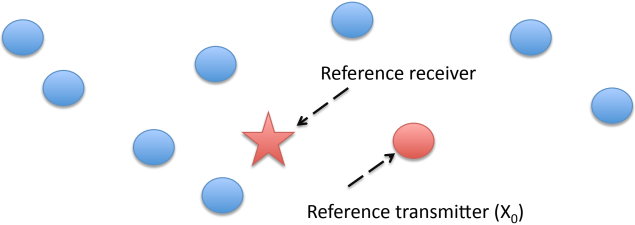
Outline

1. Ad Hoc Networks
2. Spread Spectrum
3. Outage Probability
4. Transmission Capacity
5. CPFSK Modulation
6. Optimization Results
7. Conclusion and Future Directions

Outline

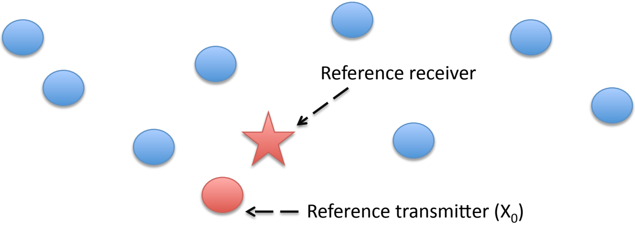
1. Ad Hoc Networks
2. Spread Spectrum
3. Outage Probability
4. Transmission Capacity
5. CPFSK Modulation
6. Optimization Results
7. Conclusion and Future Directions

Ad Hoc Networks



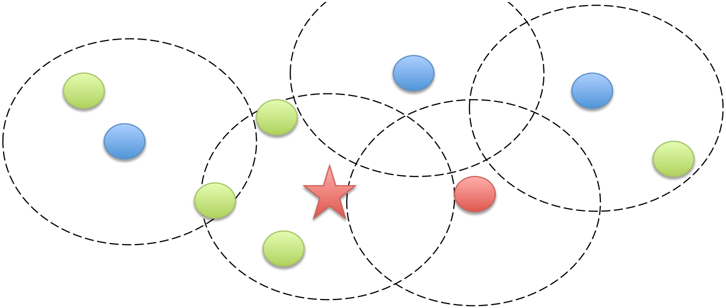
- Transmitters are randomly placed in 2-D space.
  - X_i denotes 2-D location of i^{th} node.
  - Spatial model important (usually Poisson Point Process).
- Each node transmits to a random receiver.
  - Reference receiver located at the origin.
  - $|X_i|$ is distance to i^{th} node.
  - X_0 is location of reference transmitter.
  - M interfering transmitters, $\{X_1, ..., X_M\}$.

Ad Hoc vs. Cellular



- In a cellular network, the reference receiver will associate with the closest transmitter (base station).
    - $|X_0| < |X_i|, \forall i \neq 0$.
    - The desired signal is usually stronger than any interferer.
- In an ad hoc network, some interferers may be closer than reference transmitter.
    - $|X_i| < |X_0|$ possible for some transmitters.
    - Near-far effect.

Guard Zones



- To prevent close interferers, interference-avoidance protocols are used.
    - Carrier-sense multiple access with collision avoidance (CSMA-CA).
- If one transmitter is too close to another, it will deactivate.
    - Each transmitter is surrounded by a circular *guard zone* of radius r_{min}.
    - Other nodes in the guard zone are forbidden to transmit.
- Equivalent to *thinning* the spatial model.
    - Thinned PPP.
    - Matern-hard process.

SINR

The performance at the reference receiver is characterized by the signal-to-interference and noise ratio (SINR), given by:

$$\gamma = \frac{g_0 \Omega_0}{\Gamma^{-1} + \sum_{i=1}^{M} I_i g_i \Omega_i} \quad (1)$$

where:

- Γ is the SNR at unit distance.
- g_i is the power gain due to fading (i.e. Rayleigh or Nakagami fading).
- I_i is the fraction of X_i's power in the same band as X_0.
- $\Omega_i = \frac{P_i}{P_0} 10^{\xi_i/10} ||X_i||^{-\alpha}$ is the normalized receiver power.
- P_i is the power of transmitter i.
- ξ_i is the dB shadowing gain (i.e. log-normal shadowing).
- α is the path loss.

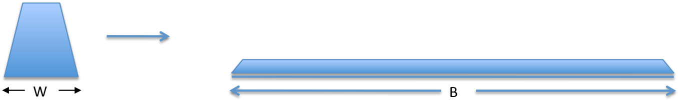
Outline

1. Ad Hoc Networks
2. Spread Spectrum
3. Outage Probability
4. Transmission Capacity
5. CPFSK Modulation
6. Optimization Results
7. Conclusion and Future Directions

Spread Spectrum

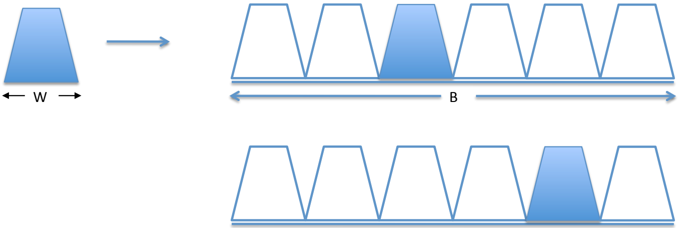
- To control interference, spread spectrum is often used in ad hoc networks.
- There are several types of spread spectrum
    - Direct sequence (DS).
    - Frequency hopping (FH).
    - Hybrid DS/FH.

Direct Sequence



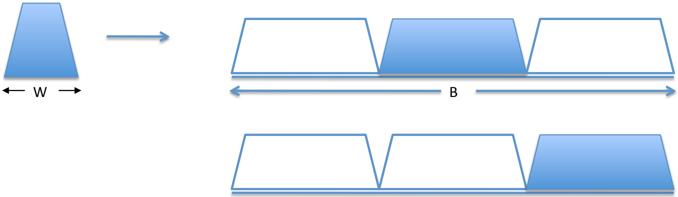
- Spread bandwidth of each signal by a factor G.
- Amount of power in the reference channel is effectively reduced.
- G is called the *processing gain* and is the amount of reduction.
- $I_i = 1/G, \forall i \neq 0$.
- Interference *averaging*.
- Preferred for cellular networks.

Frequency Hopping



- Transmitters randomly pick from among L frequencies.
- I_i is a Bernoulli random variable with probability $p = 1/L$.
- Interference *avoidance*.
- Preferred for ad hoc networks.

Hybrid DS/FH



- Spread bandwidth of each signal by a factor G.
- Sent DS-spread signal over randomly selected frequency.
- $G > 1$ and $p < 1$.

Outline

Information Outage Probability

- The *frame-error rate* (FER) is a practical performance metric.
- Assuming the use of a capacity-approaching code (turbo, LDPC), the *information outage probability* is a good predictor for the FER.
- The IOP is given by:

$$\epsilon \;=\; P\left[C(\gamma) \leq R\right] = P\Big[\gamma \leq \underbrace{C^{-1}(R)}_{\beta}\Big]. \qquad (2)$$

where

  - $C(\gamma)$ is the *capacity* of an AWGN system with SNR γ.
  - R is the *rate* of the error-correcting code.
  - β is the SINR threshold.

Evaluating IOP

- Substituting (1) into (2) and rearranging yields

$$\epsilon = P\Big[\underbrace{\beta^{-1} g_0 \Omega_0 - \sum_{i=1}^{M} I_i g_i \Omega_i}_{\mathsf{Z}} \leq \Gamma^{-1}\Big].$$

- The outage probability is related to the cumulative distribution function (cdf) of Z,

$$\epsilon = P\left[\mathsf{Z} \leq \Gamma^{-1}\right] = F_{\mathsf{Z}}(\Gamma^{-1}).$$

- To find the IOP, we should find an expression for the cdf of Z.

Rayleigh Fading

- When all links are subject to Rayleigh fading,

$$F_{\mathsf{Z}}(z) \;=\; 1 - e^{-\beta z} \prod_{i=1}^{M} \frac{G + \beta(1-p)\Omega_i}{G + \beta \Omega_i}. \tag{3}$$

where it is assumed that:

  - The reference transmitter is at unit distance, $|X_0| = 1$.
  - There is no shadowing.

Nakagami Fading

If the channel from the i^{th} node to the receiver is Nakagami-m with parameter m_i, then for integer m_0,

$$\begin{aligned}
F_{\mathsf{Z}}(z) &= 1 - \exp\left\{ \beta z \frac{m_0}{\Omega_0} \right\} \sum_{s=0}^{m_0-1} \left(\beta z \frac{m_0}{\Omega_0} \right)^s \sum_{r=0}^{s} \frac{z^{-r} V_r(\mathbf{\Psi})}{(s-r)!} \\
V_r(\mathbf{\Psi}) &= \sum_{\substack{\ell_i \geq 0 \\ \sum_{i=0}^{M} \ell_i = r}} \prod_{i=1}^{M} U_{\ell_i}(\Psi_i) \\
U_\ell(\Psi_i) &= \begin{cases} 1 - p(1 - \Psi_i^{m_i}), & \text{for } \ell = 0 \\ \frac{p\Gamma(\ell+m_i)}{\ell!\Gamma(m_i)} \left(\frac{\Omega_i}{G m_i} \right)^{\ell} \Psi_i^{m_i+\ell}, & \text{for } \ell > 0 \end{cases} \\
\Psi_i &= \left(\beta \left(\frac{m_0}{\Omega_0} \right) \left(\frac{\Omega_i}{G m_i} \right) + 1 \right)^{-1}, \qquad \text{for } i = \{1, ..., M\}.
\end{aligned}$$

An Example

- Reference (source) transmitter placed at distance $|X_0| = 1$.
- $M = 50$ interferers randomly placed in a circle of radius $r_{max} = 4$.
- $L = 200$ hopping frequencies, i.e. $p = 1/200 = 0.005$.
- $\beta = 3.7$ dB SINR threshold.
- Three fading models considered:
  - **Rayleigh fading:** $m_i = 1$ for all i.
  - **Nakagami fading:** $m_i = 4$ for all i.
  - **Mixed fading:** $m_0 = 4$ for source and $m_i = 1$ for interferers.
- Path-loss coefficient $\alpha = 3$.
- No shadowing.

Example #1

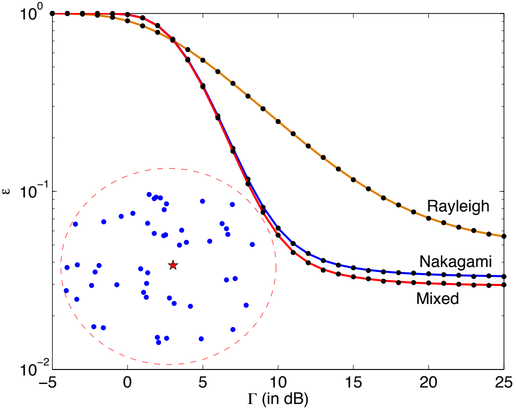
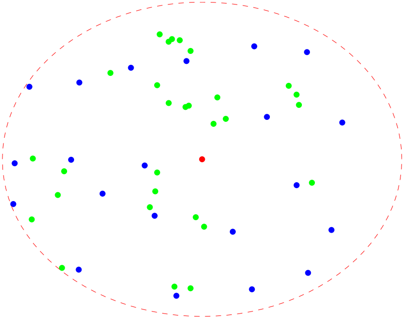


Figure: Outage probability ϵ as a function of SNR Γ. Analytical curves are solid, while • represents simulated values. The network geometry is shown in the inset, with the reference receiver represented by $\star$ and interferers by •.

Reducing Outage



- The outage probability can be reduced several ways:
  1. Impose a **guard zone** of radius r_{min}.
  2. Increase number of **hopping frequencies** L, which reduces $p = 1/L$.
  3. Decrease the **threshold** β, which can be done by using a lower rate channel code.
- For example, by using a guard zone with $r_{min} = 1$, the number of interferers decreases to 21.

Performance with a Guard Zone

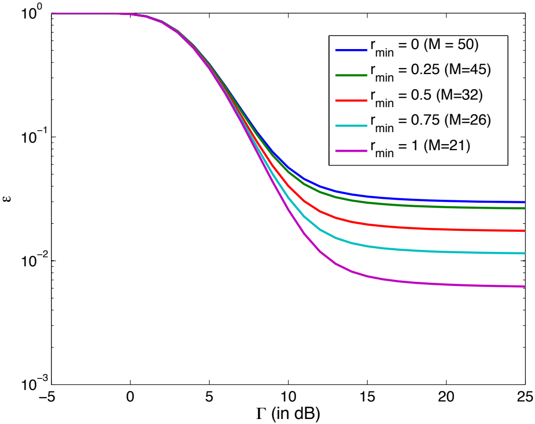


Figure: Outage probability ϵ over the mixed fading channel when a guard zone of radius r_{min} is imposed. Note that although ϵ is reduced, the network now supports far fewer transmissions.

Outline

1. Ad Hoc Networks
2. Spread Spectrum
3. Outage Probability
4. Transmission Capacity
5. CPFSK Modulation
6. Optimization Results
7. Conclusion and Future Directions

Transmission Capacity

- Reducing M reduces ϵ, which improves the per-link throughput.
  - However, fewer links are supported, so less total data might be transmitted within the network.
- *Transmission capacity* is a metric that quantifies this tradeoff.
- The transmission capacity is defined as

$$\tau = \zeta(1-\epsilon)\lambda$$

  where
  - ζ is the *throughput efficiency* of the link (bps/Hz).
  - $\lambda = M/A$ is the *density* of the network.
  - A is the *area* of the network.
- It is interpreted as the *area spectral efficiency* of the network.
  - Units of bps/Hz/m^2.
  - The rate that bits are successfully transmitted over 1 Hz BW and 1 square meter of area.

Example #2

- $M = 100$ interferers placed randomly on circle of radius $r_{max} = 4$.
- Guard zone r_{min} gradually increased, thinning the network.
- Channel and network parameters:
  - Path-loss exponent $\alpha = 3.5$.
  - Mixed fading, i.e. $m_0 = 4$ and $m_i = 1$ for all interferers.
  - SINR threshold $\beta = 0$ dB.
  - Collision probability $p = 0.5$.
  - SNR $\Gamma = 25$ dB (high SNR regime).
  - Bandwidth efficiency $\zeta = 1$.

Example #2

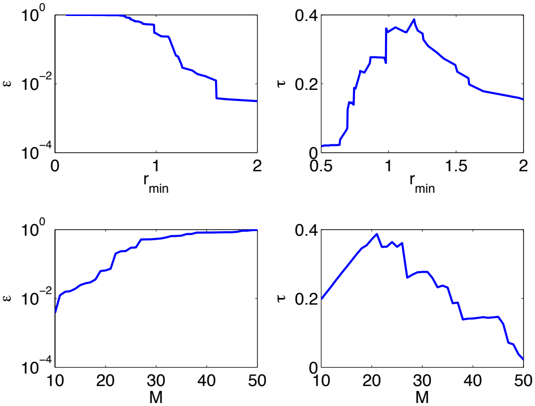


Figure: Performance of Example #2.

Spatial Averaging

- Until now, we have only considered *specific* network topologies.
  - The network is drawn once from a random process.
- However, we may be interested in performance of a system across a wide range of realizations.
- Can draw multiple realizations of the network and average the outage probabilities.
  - Draw N networks, each of size M.
  - Let $\mathbf{\Omega}_j$ be the set of Ω_i's for the j^{th} network.
  - Let $F_{\mathsf{Z}}(z|\Omega_j)$ be the the cdf of Z for the j^{th} network.
  - Take the average of the N cdfs

$$F_{\mathsf{Z}}(z) = \frac{1}{N} \sum_{j=1}^{N} F_{\mathsf{Z}}(z|\mathbf{\Omega}_j).$$

  - As before, the outage probability is $\epsilon = F_{\mathsf{Z}}(\Gamma^{-1})$.
- **Shadowing** can be modeled by including the factor $10^{\xi_i/10}$ in each Ω_i. For log-normal shadowing ξ_i is zero mean Gaussian with variance σ_s^2.

Example #3

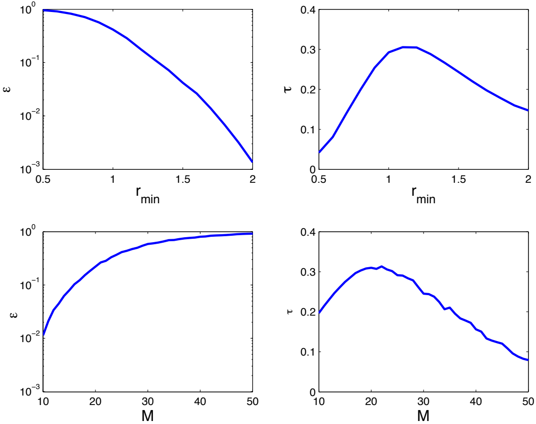


Figure: Performance averaged over $N = 1000$ network realizations. Parameters are the same as used in Example #2.

Outline

1. Ad Hoc Networks
2. Spread Spectrum
3. Outage Probability
4. Transmission Capacity
5. CPFSK Modulation
6. Optimization Results
7. Conclusion and Future Directions

SINR Threshold

- Until now, we have picked the SINR threshold β arbitrarily.
- β depends on the choice of modulation.
    - For *ideal* signaling

      $$C(\gamma) \quad = \quad \log_2(1+\gamma)$$

      β is the value of γ for which $C(\gamma) = R$ (the code rate),

      $$\beta \quad = \quad 2^R - 1$$

    - For other modulations, the *modulation-constrained* capacity must be used.
- The code rate and modulation influence ζ, the throughput efficiency.

Modulation Choices for Frequency Hopping

$$s_d(t) = \frac{1}{\sqrt{T_s}} e^{j2\pi dt/T_s}, \; d = 0, 1, \cdots, q-1$$

- Philosophy #1: Orthogonal FSK
  - Suitable for noncoherent reception.
  - Reasonable energy efficiency.
  - Poor bandwidth efficiency because adjacent tones are $1/T_s$ apart.
- Philosophy #2: Nonorthogonal CPFSK
  - Reduce bandwidth by using modulation index $h < 1$.
  - Adjacent frequency tones are h/T_s apart.
  - Continuous-phase constraint controls the spectrum.
  - Transmitted $x(t) = e^{j\phi} s_d(t)$ where phase ϕ is accumulated

$$\phi = \phi' + 2\pi dh$$

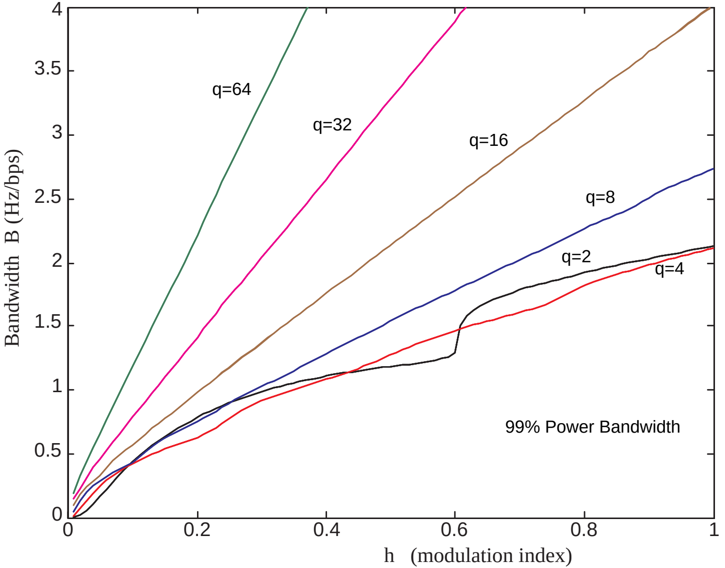
Modulation Choices for Frequency Hopping

$$s_d(t) = \frac{1}{\sqrt{T_s}} e^{j2\pi dht/T_s}, \ d = 0, 1, \cdots, q-1$$

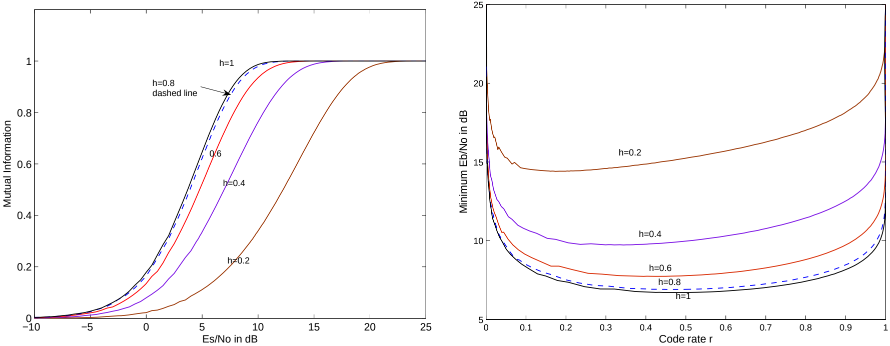
- Philosophy #1: Orthogonal FSK
  - Suitable for noncoherent reception.
  - Reasonable energy efficiency.
  - Poor bandwidth efficiency because adjacent tones are $1/T_s$ apart.
- Philosophy #2: Nonorthogonal CPFSK
  - Reduce bandwidth by using modulation index $h < 1$.
  - Adjacent frequency tones are h/T_s apart.
  - Continuous-phase constraint controls the spectrum.
  - Transmitted $x(t) = e^{j\phi} s_d(t)$ where phase ϕ is accumulated

$$\phi = \phi' + 2\pi dh$$

Bandwidth of CPFSK



Capacity of Noncoherent Binary CPFSK



(a) channel capacity versus $\mathcal{E}_S/N_0$ (b) minimum $\mathcal{E}_b/N_0$ versus coding rate

Reference: S. Cheng, R. Iyer Sehshadri, M.C. Valenti, and D. Torrieri, "The capacity of noncoherent continuous-phase frequency shift keying, in *Proc. Conf. on Info. Sci. and Sys. (CISS)*, (Baltimore, MD), Mar. 2007.

Throughput Efficiency

- The throughput over the frequency subchannel is:

$$T = \eta R W$$

  where
    - R is the rate of the channel code.
    - η the (uncoded) modulation's spectral efficiency (bps/Hz).
    - $W = B/L$ is the bandwidth of the subchannel.
- Throughput efficiency is throughput divided by the overall bandwidth B,

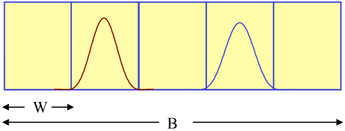
$$\zeta = \frac{T}{B} = \frac{\eta R}{L}$$

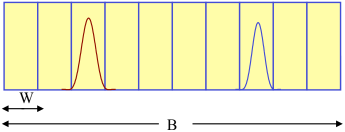
  and has units of bps/Hz.
- For a given L, there is a tradeoff between R, η, and β.

Outline

1. Ad Hoc Networks
2. Spread Spectrum
3. Outage Probability
4. Transmission Capacity
5. CPFSK Modulation
6. Optimization Results
7. Conclusion and Future Directions

Design Considerations





Design # 1

- Wideband hopping channels.
- Large W.
- Fewer hopping channels $L = B/W$.
- More collisions: Higher $p = 1/L$.
- Lower β due to lower R and higher h.
- Better AWGN performance.

Design # 2

- Narrowband hopping channels.
- Small W.
- More hopping channels $L = B/W$.
- Fewer collisions: Lower $p = 1/L$.
- Higher β due to higher R and lower h.
- Worse AWGN performance.

Optimization Objectives

- The parameters L, R, and h are related in a complicated manner.
- Our goal is to find the set of these parameters which provides the best performance.
- We use *transmission capacity* as the objective function for an optimization.
- For each value of M, determine the parameters that maximize τ.
- Due to the large search space, the optimization is computationally demanding.
- We use a 208-core cluster computer to perform the optimization.

Optimization Algorithm

1. Draw N networks, each of size M, according to the spatial distribution.
2. Determine the set $\mathbf{\Omega}_j$ for each network and store it.
3. Pick a value of L.
4. Pick a value of β.
5. Compute the outage probability averaged over the $\mathbf{\Omega}_j$.
6. For each h, determine the rate R corresponding to the current β. This is found by setting $R = C(\beta)$, where $C(\gamma)$ is the modulation-constrained capacity for this h.
7. For the set of (h, R) found in the last step, determine the normalized transmission capacity τ.
8. Return to step 3 until all β are considered.
9. Return to step 4 until all L are considered.

Optimization Results

r_{max}	σ_s^2	m_0	m_i	L	R	h	τ_{opt}	τ_{sub}
2	0	1	1	31	0.61	0.59	15.92	3.31
		4	4	42	0.66	0.59	17.09	4.05
		4	1	36	0.65	0.59	19.82	4.13
	8	1	1	31	0.63	0.59	15.98	3.31
		4	4	41	0.66	0.59	17.43	4.04
		4	1	36	0.66	0.59	20.11	4.12
4	0	1	1	12	0.54	0.59	9.73	0.89
		4	4	15	0.50	0.59	10.65	1.12
		4	1	14	0.51	0.59	11.85	1.12
	8	1	1	12	0.53	0.59	9.41	0.89
		4	4	16	0.51	0.59	10.26	1.12
		4	1	14	0.52	0.59	11.46	1.12

Table: Results of the Optimization for $M = 50$ interferers. The transmission capacity τ is in units of bps/kHz-m^2. τ_{opt} is TC with the optimizer parameters, while τ_{sub} is TC with $(L, R, h) = (200, 1/2, 1)$.

Outline

1. Ad Hoc Networks
2. Spread Spectrum
3. Outage Probability
4. Transmission Capacity
5. CPFSK Modulation
6. Optimization Results
7. **Conclusion and Future Directions**

Conclusions

- The performance of frequency-hopping ad hoc networks is a function of many parameters.
  - Number hopping channels L.
  - Code rate R.
  - Modulation index h (if CPFSK modulation).
  - Guard-zone radius r_{min}.
- These parameters should be jointly optimized.
  - Transmission capacity is the objective function of choice.
  - TC quantifies the tradeoffs involved.
- The approach is general enough to handle a wide variety of conditions.
  - Frequency-hopping and direct-sequence spread spectrum.
  - Rayleigh and Nakagami fading (or mixtures).
  - Shadowing.
  - Any spatial model.

Future Work

- Effect of adjacent-channel interference.
- Cellular networks.
- Influence of location of reference receiver.
- Nonbinary modulation; multisymbol reception.
- Hybrid FH/DS systems.
- Diversity and multiple antennas.
- Cooperative communications.
- Adaptive hopping and cognitive radio systems.

どうもありがとうございます