Some New Adaptive Protocols for the Wireless Relay Channel

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• System constraints
  – “Cheap” and “cheaper” relay networks = TDD and TDM modes.

• Protocol alternatives
  – Nonadaptive: Always relay.
  – Source adaptive: During second time slot, source or relay transmits.
  – Relay adaptive: Relay switches between decode-forward and amplify-forward.

• Performance analysis
  – Capacity in AWGN.
  – Outage event probability in block fading.

• Conclusions
Cheap and Cheaper Relay Networks

- Unconstrained relay channel (van der Muelen; Cover & El Gamal)
  - Relay may simultaneously receive and transmit.
  - Source and relay transmit coherently (perfect transmit CSI): Beamforming.
  - Relay networks: Generalization to multiple relays (Gastpar & Vetterli).

- Cheap relay networks (Khojastepour & Aazhang; Høst Madsen)
  - Relay cannot receive and transmit simultaneously: TDD mode.
  - Source and relay may still transmit coherently: Beamforming.

- Cheaper relay networks (c.f. Laneman; Kramer, Gastpar & Gupta)
  - TDD relay.
  - Source and relay may not transmit coherently due to phase fading.
  - Orthogonal transmissions, e.g. TDM mode.
  - Besides being more feasible technically, the analysis is greatly simplified.
TDM Relay Networks

- General strategy:
  - Source creates a rate $r$ codeword of length $N$.
  - Source begins by transmitting a fraction $\alpha$ of code symbols during the first time slot $s_1$.
  - During time slot $s_2$ either the source or relay may transmit the remaining fraction $\bar{\alpha} = 1 - \alpha$ of the codeword, depending on the protocol.

- Coding alternatives:
  - Repetition coding:
    * An identical rate $2r$ code is transmitted during each slot.
    * $\alpha = 1/2$.
    * Relay may simply amplify-and-forward (AF) or may decode, re-encode, and forward (DF).
    * Diversity-combining at destination (MRC).
    * SNRs add at the destination: $SNR$ accumulation.
  - Incremental redundancy:
    * Through puncturing, different symbols of a rate $r$ mother code are transmitted during each slot.
    * $0 < \alpha \leq 1$.
    * Relay must decode, re-encode, and transmit the remaining portion of codeword (DF).
    * Code-combining at destination (CC).
    * Capacities add at the destination: $Information$ accumulation.
Outage Event Probability

- Block fading model:
  - SNR fixed for a codeword.
  - SNR varies independently from codeword-to-codeword.

- SNR definitions:
  - Instantaneous source-relay SNR: $\gamma_{s,r}$
  - Source-destination: $\gamma_{s,d}$
  - Relay-destination: $\gamma_{r,d}$
  - Average SNR: $\Gamma = E[\gamma]$

- Outage event probability (OEP)

\[
P_o = \int \int \int_A p(\gamma_{s,r}, \gamma_{s,d}, \gamma_{r,d}) d\gamma_{s,r} d\gamma_{s,d} d\gamma_{r,d}
\]

\[
= \frac{1}{\Gamma_{s,r,\Gamma_{s,d},\Gamma_{r,d}}} \int \int \int_A \exp \left\{ \left( \frac{\gamma_{s,r}}{\Gamma_{s,r}} + \frac{\gamma_{s,d}}{\Gamma_{s,d}} + \frac{\gamma_{r,d}}{\Gamma_{r,d}} \right) d\gamma_{s,r} d\gamma_{s,d} d\gamma_{r,d} \right\}
\]

- $A = \{\gamma_{s,r}, \gamma_{s,d}, \gamma_{r,d} : C < r\}$ is the outage event region.
- $C$ is the AWGN capacity of the protocol.
Decode-Forward Relaying with MRC

- Capacity:
  \[
  C_{MRC}^* = \begin{cases} \min \left\{ \frac{1}{2} C(\gamma_{s,d} + \gamma_{r,d}), \frac{1}{2} C(\gamma_{s,r}) \right\} : \gamma_{s,d} < \gamma_{s,r} \\ \frac{1}{2} C(\gamma_{s,d}) : \text{otherwise} \end{cases}
  \]  

- Protocol (c.f. Laneman):
  - Source TX a rate $2r$ code during the first slot.
  - If relay decodes correctly, it will TX the same codeword during second slot.
  - *SNR accumulation* at the destination.

- Outage event region:
  \[
  \mathcal{A} = \{ C_{MRC}^* \leq r \} = \left\{ \left[ (C(\gamma_{s,r}) \geq 2r) \cap (C(\gamma_{s,d} + \gamma_{r,d}) < 2r) \right] \cup \left[ (C(\gamma_{s,r}) < 2r) \cap (C(\gamma_{s,d}) < 2r) \right] \right\}
  \]  

  Relay not in outage  
  Relay in outage

(3)
Decode-Forward Relaying with CC

- Capacity:
  \[ C_{CC}^* = \begin{cases} 
  \min \{ \alpha C(\gamma_{s,d}) + \bar{\alpha} C(\gamma_{r,d}), \alpha C(\gamma_{s,r}) \} : \gamma_{s,d} < \gamma_{s,r} \\
  \alpha C(\gamma_{s,d}) : \text{otherwise} 
  \end{cases} \]  
  \hfill (4)

- Protocol:
  - Source TX first part of codeword at rate \( r/\alpha \) during first slot.
  - If relay decodes correctly, it will TX the second part of codeword at rate \( r/\bar{\alpha} \) during second slot.
  - Information accumulation at the destination.

- Outage event region:
  \[ A = \{ C_{CC}^* \leq r \} \]
  \[ = \left\{ \left[ C(\gamma_{s,r}) \geq \frac{r}{\alpha} \right] \cap \left( \alpha C(\gamma_{s,d}) + \bar{\alpha} C(\gamma_{r,d}) < r \right) \right\} \cup \left\{ \left( C(\gamma_{s,r}) < \frac{r}{\alpha} \right) \cap \left( C(\gamma_{s,d}) < \frac{r}{\alpha} \right) \right\} \]
  \hfill (5)
Amplify-Forward Relaying

- Capacity:

\[ C^*_{AF} = \frac{1}{2} C(\gamma_{s,d} + \gamma_{s,r,d}) \]  

\( \gamma_{s,r,d} \) depends on the amplifier gain at the relay.

- With an optimal gain this is:

\[ \gamma_{s,r,d} = \frac{\gamma_{s,r} \gamma_{r,d}}{\gamma_{s,r} + \gamma_{r,d}} \]  

- Protocol:

  - Source TX a rate 2\( r \) code during the first slot.
  - Relay amplifies its received signal by relay gain \( G \), and forwards during the second slot.
  - SNR accumulation.

- Outage event region:

\[ A = \{ C^*_{AF} \leq r \} \]
Source Adaptive Protocol A with MRC

- Capacity:

\[
C_{SA1}^* = \begin{cases} 
\frac{1}{2} C(2\gamma_{s,d}) & \text{if } \gamma_{s,r} \leq 2^{4r} - 1 \\
\frac{1}{2} C(\gamma_{s,d} + \gamma_{r,d}) & \text{otherwise}
\end{cases}
\]  \(9\)

- Protocol (c.f. Laneman):
  - The source TX a rate 2\(r\) code during the first slot.
  - If \(C(\gamma_{s,r}) \geq 2r\), the relay TX the same codeword during the second slot.
  - Otherwise, the source TX the same codeword during the second slot.

- Outage event region:

\[
\mathcal{A} = \{C_{SA1}^* \leq r\} = \left\{ [(C(\gamma_{s,r}) \geq 2r) \cap (C(\gamma_{s,d} + \gamma_{r,d}) < 2r)] \cup [(C(\gamma_{s,r}) < 2r) \cap (C(2\gamma_{s,d}) < 2r)] \right\}
\]  \(10\)

\text{Relay not in outage} \quad \text{Relay in outage}
Source Adaptive Protocol A with CC

- Capacity:
  \[
  C_{SA2}^* = \begin{cases} 
  C(\gamma_{s,d}) & \text{if } \gamma_{s,r} \leq 2^{2r/\alpha} - 1 \\
  \alpha C(\gamma_{s,d}) + \bar{\alpha} C(\gamma_{r,d}) & \text{otherwise}
  \end{cases}
  \]  
  (11)

- Protocol:
  - Source TX first part of codeword at rate $r/\alpha$ during first slot.
  - If $C(\gamma_{s,r}) \geq 2r$, the relay will TX the second part of codeword at rate $r/\bar{\alpha}$ during second slot.
  - Otherwise, the source will transmit the remainder.

- Outage event region:
  \[
  A = \{ C_{SA2}^* \leq r \}
  = \left\{ \left( C(\gamma_{s,r}) \geq \frac{r}{\alpha} \right) \cap \left( \alpha C(\gamma_{s,d}) + \bar{\alpha} C(\gamma_{r,d}) < r \right) \right\} \cup \left\{ \left( C(\gamma_{s,r}) < \frac{r}{\alpha} \right) \cap \left( C(\gamma_{s,d}) < r \right) \right\}
  \] 
  Relay not in outage
  Relay in outage
  (12)
Figure 1: User Cooperative Coding (c.f Hunter & Nostratinia)

- Interchannel SNR: $\Gamma_{s,r}$
- Percent user cooperation: $\bar{\alpha} = 1 - \alpha$
- Protocol: SA-CC.

![Graph showing outage event probability vs average received SNR for different cooperation percentages and average interuser channel SNR values](image)
Source Adaptive Protocol B with MRC

- Capacity:

\[
C_{SB1}^* = \max\{C_{MRC}^*, C_{DTR}^*\} \\
= \frac{1}{2} \max \{\min \{C(\gamma_s,d + \gamma_r,d), C(\gamma_s,r)\}, C(2\gamma_s,d)\}
\]

\(C_{DTR}^* \triangleq \frac{1}{2} C(2\gamma_s,d)\) is the capacity of direct transmission with MRC

- Protocol:
  - The source TX a 2r code during the first slot.
  - If \(C(\gamma_s,r) > 2r\) and \(\gamma_r,d > \gamma_s,d\), the relay TX the same codeword during the second block.
  - Otherwise, the source TX the same codeword during the second block

- Outage event region:

\[
A = \{C_{SB1}^* \leq r\} \\
= \left\{ [(C(\gamma_s,r) < 2r) \cap (C(2\gamma_s,d) < 2r)] \cup [(C(\gamma_s,r) > 2r) \cap (C(\gamma_s,d + \gamma_r,d) < 2r) \cap (C(2\gamma_s,d) < 2r)] \right\}
\]  

(13)
Source Adaptive Protocol B with CC

• Capacity:

\[ C_{SB2}^* = \max \{ C_{CC}^*, C_{DT}^* \} \]
\[ = \max \{ \min \{ \alpha C(\gamma_s,d) + \bar{a}C(\gamma_r,d), \alpha C(\gamma_s,r) \}, C(\gamma_s,d) \} \]  

(14)

• Protocol:

− The source TX the first part of the codeword at rate \( r/\alpha \) during the first slot.
− If \( C(\gamma_s,r) \geq 2r \) and \( \alpha C(\gamma_s,d) + \bar{a}C(\gamma_r,d) > C(\gamma_s,d) \), the relay TX the remainder of the codeword at rate \( r/\bar{a} \) during the second slot.
− Otherwise, the source TX the remainder of the codeword during the second slot.

• Outage event region:

\[ \mathcal{A} = \{ C_{SB2}^* \leq r \} \]
\[ = \{ \left[ \left( C(\gamma_s,r) < \frac{r}{\alpha} \right) \cap (C(\gamma_s,d) < r) \right] \]
\[ \cup \left[ \left( C(\gamma_s,r) > \frac{r}{\alpha} \right) \cap (\alpha C(\gamma_s,d) + \bar{a}C(\gamma_r,d) < r) \cap (C(\gamma_s,d) < r) \right] \}\]

(15)
Relay Adaptive Protocol

• Capacity:

\[
C_{RA}^* = \max\{C_{CC}^*, C_{AF}^*\} \\
= \max \left\{ \min \left\{ \frac{1}{2} C(\gamma_{s,d}) + \frac{1}{2} C(\gamma_{r,d}), \frac{1}{2} C(\gamma_{s,r}) \right\}, \frac{1}{2} C \left( \gamma_{s,d} + \frac{\gamma_{r,d} \gamma_{s,r}}{\gamma_{r,d} + \gamma_{s,r}} \right) \right\}. \quad (16)
\]

• Protocol:
  
  – The source TX during the first slot, while the relay TX during the second slot.
  
  – If \( \gamma_{s,r} < 2^{4r} - 1 \), the relay will use amplify-forward relaying.
  
  – If \( \gamma_{s,r} \geq 2^{4r} - 1 \), the relay will decode and forward the source message using incremental redundancy.

• Outage event region:

\[
\mathcal{A} = \{ C_{RA}^* \leq r \} \\
= \left\{ \left[ (C(\gamma_{s,r}) < 2r) \cap \left( C \left( \gamma_{s,d} + \frac{\gamma_{r,d} \gamma_{s,r}}{\gamma_{r,d} + \gamma_{s,r}} \right) < 2r \right) \right] \cup \left[ (C(\gamma_{s,r}) > 2r) \cap (C(\gamma_{s,d}) + C(\gamma_{r,d}) < 2r) \right] \right\} \quad (17)
\]
Comparison of Different Protocols

- Wireless propagation model
  - The received power $P_r$ at distance $d_m$ is related to transmitted power $P_t$ by
    \[ P_r = \left( \frac{c}{4\pi d_o f_c} \right)^2 \frac{P_t}{d_m^n} = 10^{-4} \frac{P_t}{d_m^3} \]  
    where $f_c = 2.4$ GHz, $d_o = 1$ m, and path loss coefficient $n = 3$.
  - Define “transmit” SNR as $\frac{P_t}{N}$

- independent quasi-static Rayleigh fading.
- Assume source and destination are separated by 10 m.
- Assume a single relay lies halfway between source and destination.
- Find two dimensional contours of source/relay transmit SNRs required to achieve desired OEP.
Figure 2: Transmit SNR of Source Adaptive Protocols Required to Achieve $OEP = 10^{-2}$
Figure 3: Transmit SNR of Relay Adaptive Protocol Required to Achieve $\text{OEP} = 10^{-2}$
Conclusions

- Message # 1: Protocols with incremental redundancy and code combining outperform their counterparts with repetition coding and diversity combining.
  - Difference is on the order of 1 dB.
  - With incremental redundancy, can adjust $\alpha$.

- Message # 2: Adaptive protocols are more energy efficient.
  - SA uses $\gamma_{s,r}$ and is more efficient than nonadaptive strategies.
    * Only beneficial at low $\gamma_{s,r}$.
  - SB uses all three SNRs and is more efficient than SA.

- Extensions:
  - Adaptively adjust $\alpha$.
  - Combination of source and relay adaptive.
  - Networks with multiple relays.
  - Allowing ARQ to guide the adaptation.