The Impact of Channel Estimation Errors on Space Time Block Codes

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Abstract

In this paper, we demonstrate the performance of space-time block codes when the decoding is performed using imperfect estimates of the channel. A brief explanation of the system model is given and the results of performance simulations discussed. Simulation results are introduced to characterize the performance of a space-time block code using two transmit antennas, one receive antenna, and QPSK modulation. The performance of this system is shown when there is perfect channel state information, and also when there are amplitude and/or phase errors in the channel estimates.

1. Introduction

In order to adequately demonstrate the performance of a system via simulation, it is necessary to develop an accurate model of the system as it would be physically implemented. The assumption that perfect channel state information (CSI) would be available to the receiver is inappropriate when simulating a physical system, because in a real system the effects of the channel can never be known exactly. Rather, some form of estimation is performed to find an approximation to the channel.

Under the assumption that perfect CSI is available to the decoder, the performance of space-time block codes has been shown in [1-4]. In this paper, the performance of space-time block codes is analyzed under the constraint that the receiver must rely on imperfect estimates of the channel conditions. This is done in order to verify the performance that can be expected in practice by an actual block space-time coded system.

The sensitivity and robustness of space-time block codes to varying levels of error in the amplitude and phase of the estimates is illustrated using a series of simulations. This new information could be used to help design and verify the performance of a channel estimation scheme based on the insertion of pilot sequences into the data stream of each antenna.

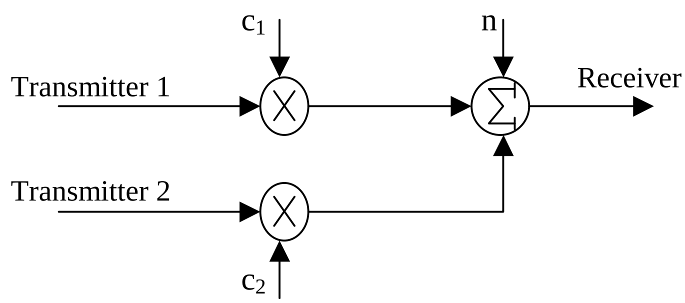

Figure 1: Transmission system consisting of two transmit antennas and one receive antenna.

The imperfect channel estimates are created by taking the actual fading coefficients that characterize the channel and applying some degree of error into the magnitude and/or the phase of those coefficients before decoding. This simulates the inability of an estimation scheme to predict the complex channel gain with perfect accuracy.

The effects of errors in the estimate of the gain and the phase are initially analyzed separately for two reasons. First, it is difficult to represent the resulting changes in BER when errors in both gain and phase occur simultaneously, especially when we have no clear understanding of what specific effects either type of error will have. Second, it is important to understand what effects each of the two components has on performance when its estimate is not exact. This way it is possible to determine if either gain or phase is more important to the decoding and estimating process in terms of Bit Error Rate performance. Also, certain modulation formats may be more or less vulnerable to phase estimation errors than they are to gain estimation errors, and vice versa.

2. System Model

The system model that we use to analyze the performance of space-time block codes with channel estimation errors can be seen in Figure 1. It consists of two transmit antennas and one receive antenna operating in a Rayleigh fading environment. Symbol mapping uses a QPSK or BPSK signal constellation and the generator matrix G_2, developed by Tarokh *et al.* in [1].

The fading coefficient, or path gain, between the i^{th} transmit antenna and the receive antenna is given as

$$c_i = a_i \exp\{j\theta_i\} \quad (1)$$

We ran simulations of the system with errors in the amplitude and phase of the channel estimates. A channel estimate with phase error is of the form

$$\hat{c}_i = a_i \exp\{j\theta_i + \phi_i\} \qquad (2)$$

where ϕ_i is the error introduced into the phase. An estimate with errors in the amplitude is of the form

$$\hat{c}_i = K_i a_i \exp\{j\theta_i\} \qquad (3)$$

where K_i is the error introduced into the amplitude.

The channel is assumed to undergo flat fading and the fading is independent between different transmit antennas. It is also assumed that the fading over a channel is constant over a frame. The assumption of constant fading over a frame is justified if the data rate is high and/or the channel fades relatively slowly. In order to justify the assumption that the fading gains between antennas are uncorrelated requires that the different antennas be physically separated by approximately ten wavelengths.

The path gains are considered to be independent samples of a complex Gaussian distribution. The variance of the path gains is 0.5 per real dimension. The noise at the receiver is an additive Gaussian noise produced from samples of another Gaussian random variable with a mean of zero and a variance equal to n/(2*SNR). Here, n is the number of transmit antennas and SNR is the signal to noise ratio at the receiver. The average energy is normalized to be one for each symbol leaving each of the n transmitting antennas. This gives the energy of the received signal as n (assuming no path loss) and SNR is measured at the receiver.

The received signal at time t is

$$r_t = \sum_{i=1}^{n} c_i s_i^{(t)} + n_t \, . \qquad (4)$$

Since we are using generator matrix G_2, there will be two sets of transmissions for each set of two input symbols. Therefore in matrix notation we can express the received signal as

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\mathbf{r} = \mathbf{Sc} + \mathbf{n} \qquad (5)$$

The decoding for this system is rather simple and consists of minimizing the metric

$$\sum_{t=1}^{l} \left| r_t - \sum_{i=1}^{n} c_i s_i^{(t)} \right|^2 , \qquad (6)$$

over all possible combinations of transmitted symbols. For the simulations using imperfect channel estimates the metric becomes

$$\sum_{t=1}^{l}\left|r_t - \sum_{i=1}^{n}\hat{c}_i s_i^{(t)}\right|^2 \qquad (7)$$

3. Simulation Results

In this section we provide simulation results for the performance of space-time block codes with channel estimation errors as described in the previous sections. Figure 2 shows the performance of uncoded QPSK and QPSK using G_2, under the assumption that perfect CSI is available at the receiver. All further figures correspond to a QPSK signal constellation, the generator matrix G_2, two transmit antennas, and one receive antenna. Figures 3, 4, and 5 show the bit error rates of our transmission scheme against the phase error in each channel for fixed levels of received SNR of 10, 20, and 25 dB, respectively. The phase errors are measured in radians, with a maximum phase error of $\pi/4$ radians.

These figures show that as the SNR is increased the system can tolerate a larger degree of error and still retain reasonable performance. However, as the degree of error in the phase approaches $\pi/4$, the system performance breaks down regardless of SNR. This is to be expected as the decision regions for QPSK are defined by boundaries that are $\pi/4$ radians between different signals.

Figure 6 shows the bit error rate of our system when the channel estimates contain amplitude, or gain, errors as high as 1.5. If the gain errors are equal in each channel than there is little performance degradation. It is for this reason that further results conserning gain errors will only be concerned with what we call the normalized gain error. This is simply the ratio of the gain error in channel one to the gain error in channel two, or K_1/K_2.

Figures 7 and 8 show the Bit Error Rates for the normalized gain error at fixed SNR of 10 and 20 dB. The performance becomes extremely degraded only when the difference between the gain errors of each channel differ by an order of magnitude. This shows that a large degree of error can be tolerated in the amplitude estimate, especially if the degree of error is relatively equal in each channel.

The results of bit error rate performance with various levels of average phase error per channel are shown in Figure 9. When the average phase error in both channels exceeds 0.6 radians the performance is not acceptable even at large values of SNR.

Figure 10 is a plot of BER versus received SNR for several values of average normalized gain error. Regardless of the degree of error in any individual channel, if the normalized error is close to one, the performance is very close to that of having perfect channel estimates. Even when the error in one channel is nearly double that of the error in the other channel, acceptable performance can still be achieved.

Figure 11 shows the bit error rate performance of the system when there are errors present in both the gain and the phase of the channel estimate. This is a plot of BER versus normalized gain error at several values of average phase error per channel. Once the average phase error exceeds approximately 0.5 radians the degree of gain error is irrelevant, because the performance is already too degraded. When there are relatively small phase errors the degree of gain error can be relatively high without a large performance penalty.

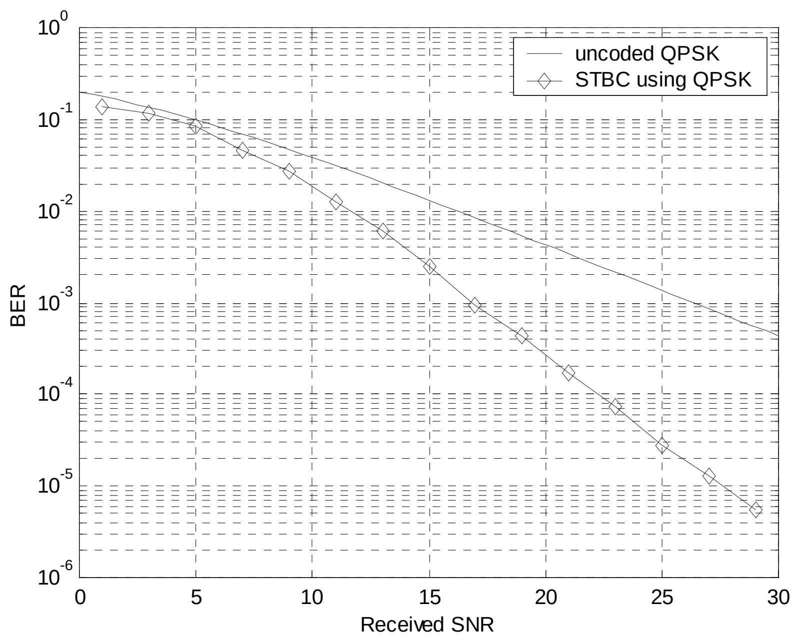

Figure 2. Performance of STBC in Rayleigh flat fading with QPSK modulation, perfect CSI, two transmit antennas, and one receive antenna.

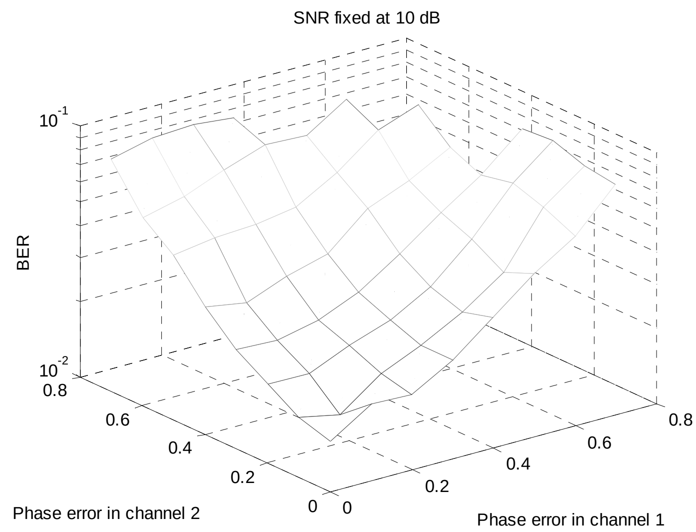

Figure 3. Performance of STBC in Rayleigh flat fading with QPSK modulation, two transmit antennas, one receive antenna, a **fixed SNR of 10 dB**, and a maximum phase error of $\pi/4$ radians.

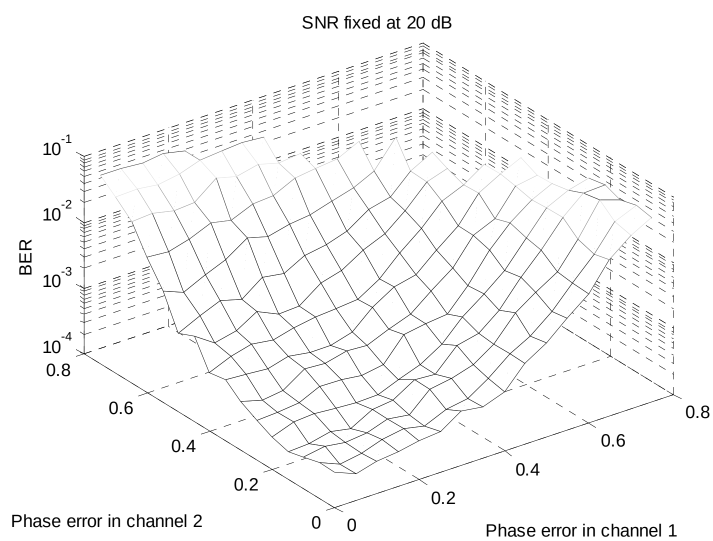

Figure 4. Performance of STBC in Rayleigh flat fading with QPSK modulation, two transmit antennas, one receive antenna, a **fixed SNR of 20 dB**, and a maximum phase error of $\pi/4$ radians.

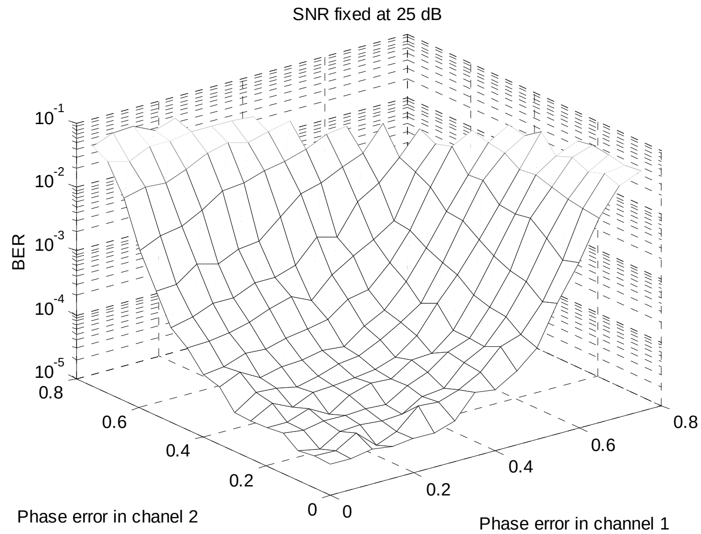

Figure 5. Performance of STBC in Rayleigh flat fading with QPSK modulation, two transmit antennas, one receive antenna, a **fixed SNR of 25 dB**, and a maximum phase error of $\pi/4$ radians.

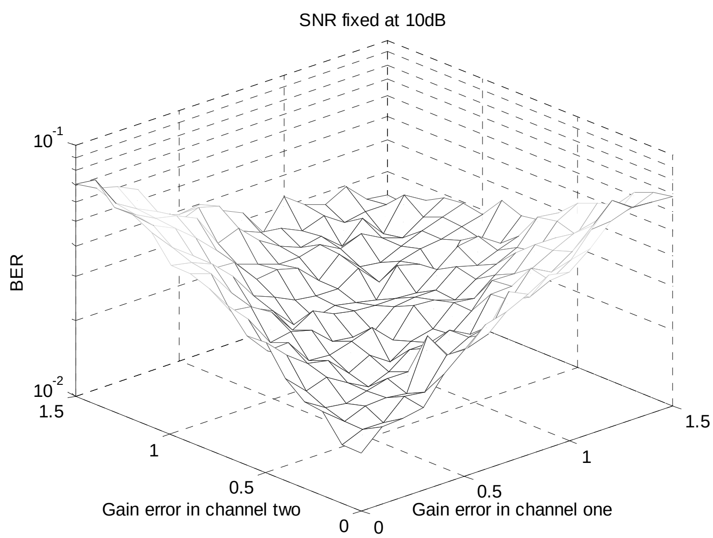

Figure 6. Performance of STBC in Rayleigh flat fading with QPSK modulation, two transmit antennas, one receive antenna, a **fixed SNR of 10 dB**, and **a maximum gain error of 1.5.**

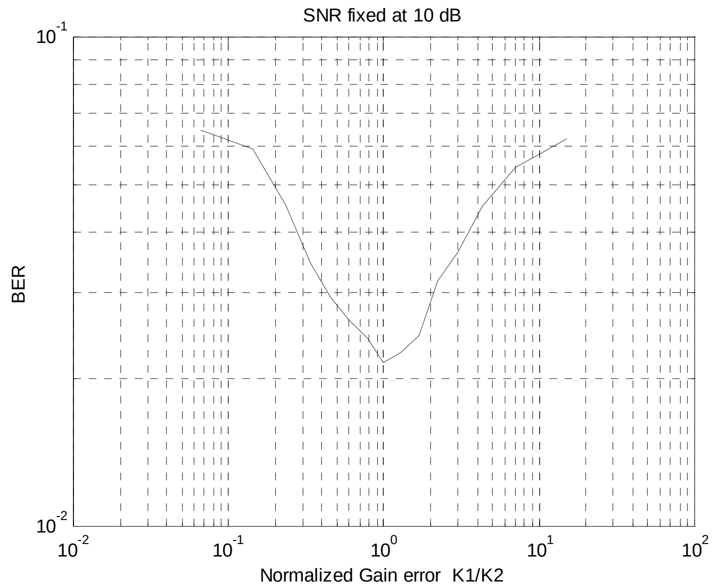

Figure 7. Performance of STBC in Rayleigh flat fading with QPSK modulation, two transmit antennas, one receive antenna, a **fixed SNR of 10 dB**, and **a range of gain error.**

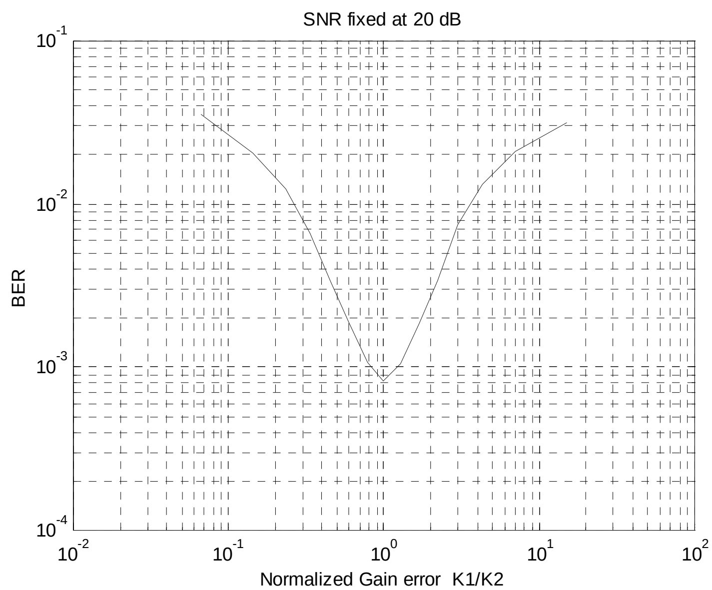

Figure 8. Performance of STBC in Rayleigh flat fading with QPSK modulation, two transmit antennas, one receive antenna, a **fixed SNR of 20 dB**, and **a range of gain error.**

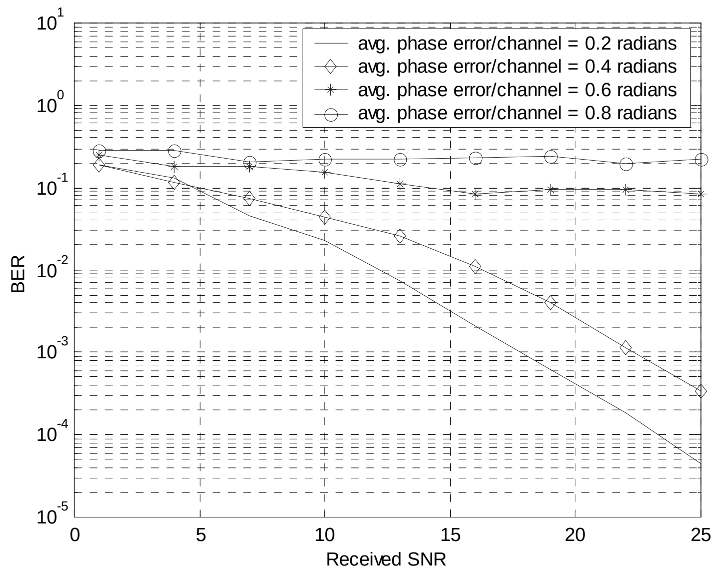

Figure 9. Performance of STBC in Rayleigh flat fading with QPSK modulation, two transmit antennas, one receive antenna, and **various levels of phase errors per channel.**

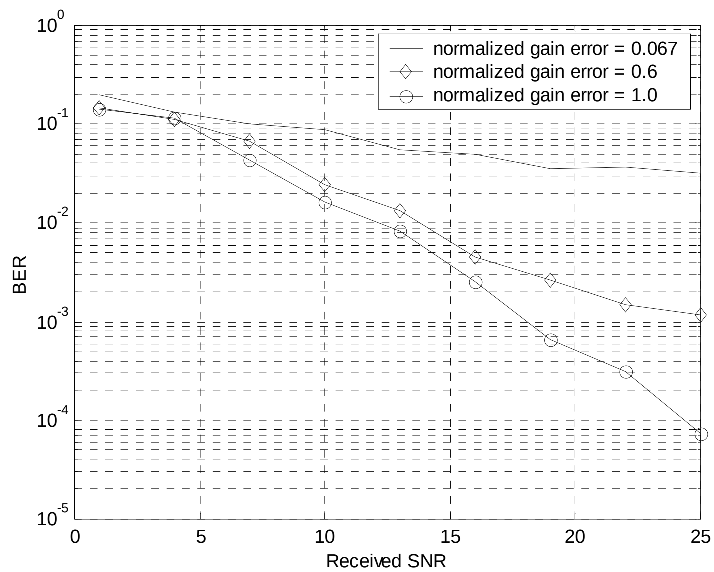

Figure 10. Performance of STBC in Rayleigh flat fading with QPSK modulation, two transmit antennas, one receive antenna, and **various levels of gain errors per channel.**

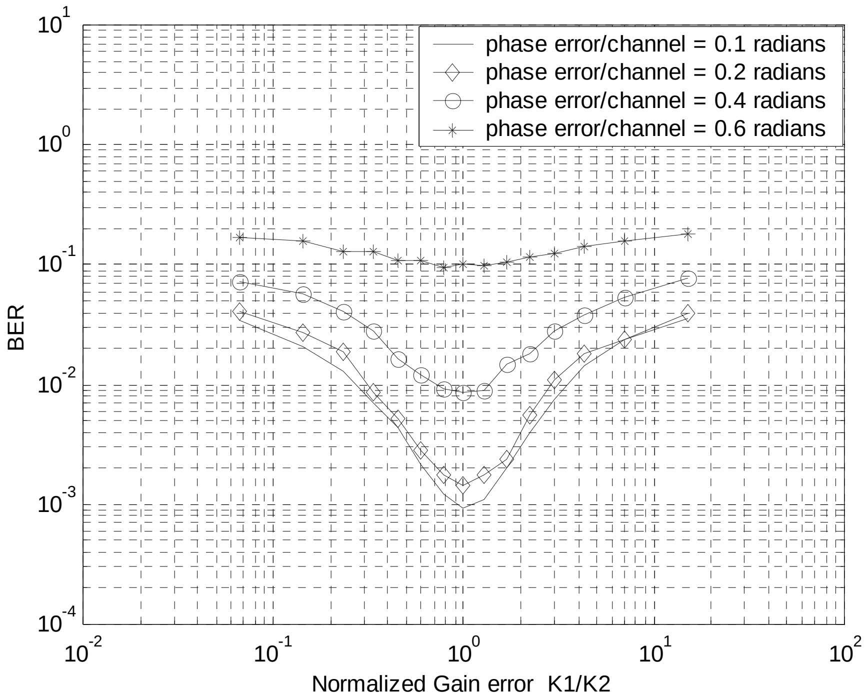

Figure 11. BER versus normalized gain error for STBC in Rayleigh flat fading with QPSK modulation, two transmit antennas, one receive antenna, a **fixed SNR of 20 dB** and **various levels of phase errors per channel.**

4. Conclusions

In this paper we have shown the performance of space-time block codes when decoded using imperfect estimates of the channel. For the case of a two transmit antenna system employing a QPSK constellation we have shown that errors in the amplitude of the channel estimate have a relatively minor effect on the bit error rate performance. If the amount of gain error in each channel is approximately the same there is almost no performance degradation. However, errors in the phase have the predominant effect, as would be expected when using Phase Shift Keying. The amount of error that can be tolerated in the phase of the channel estimate before the performance completely breaks down, is approximately 0.5 radians. Even when the level of error in the phase is 0.4 radians the performance has been greatly degraded.

References

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