Coverage and Rate in Finite-Sized Device-to-Device Millimeter Wave Networks

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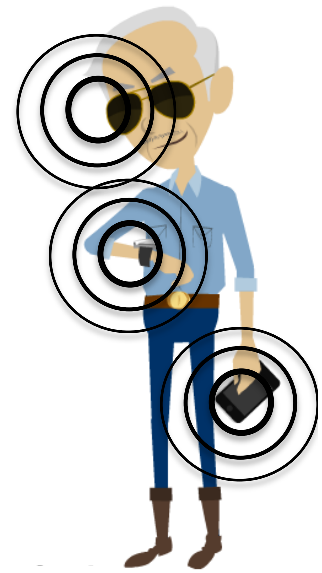
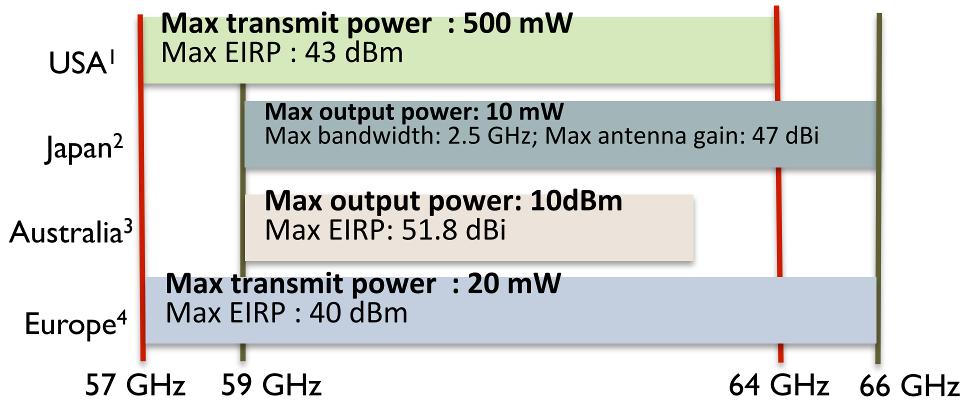
Wearable communication networks



- The next frontier for wireless communications
  - Multiple devices in and around human body
  - Low-rate fitness monitors to high-rate infotainment devices
- Critical challenge
  - Supporting Gbps per user in dense environments
  - Effective operation in finite areas like trains, trolleys, or buses

[1] http://www.bombardier.com/en/transportation/products-services/railvehicles/metros.html
[2] "Smart wearable devices: Fitness, healthcare, entertainment & enterprise 2013-2018," Juniper Research, Oct. 2013.

MmWave as solution for wearable networks



Several GHz of spectrum available for worldwide operation

- High bandwidth and reasonable isolation
- Compact antenna arrays to provide array gains via beamforming
- Commercial products already available: IEEE 802.11ad, WirelessHD

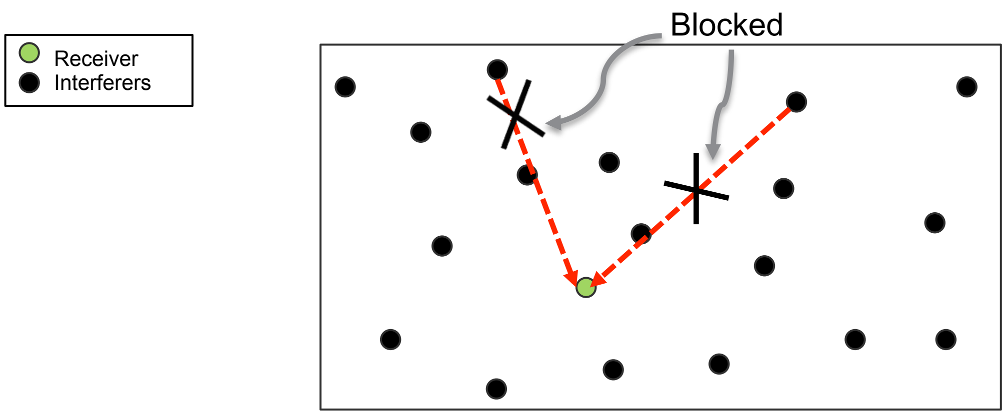
[1] 47 CFR 15.255; [2] ARIB STD-T69, ARIB STD-T74; [3] Radiocommunications Class License 2000; [4] CEPT : Official journal of the EU;

Motivating prior work

- Stochastic geometry models for mmWave cellular networks [1]-[3]
  - Infinite spatial extent and number of nodes
  - Did not consider people as a source of blockage
- Performance analysis for finite ad-hoc networks [4]
  - Does not include directional antennas or blockage
- Self-blockage model for mmWave [5]
  - Considers a 5G cellular system
  - User's own body blocks the signal, not other users

[1] T. Bai and R. W. Heath Jr., “Coverage and rate analysis for millimeter wave cellular networks,” IEEE Trans. Wireless Comm., 2014.
[2] S. Singh, M. N. Kulkarni, A. Ghosh, and J. G. Andrews, “Tractable model for rate in self-backhauled millimeter wave cellular networks,” online
[3] T. Bai, A. Alkhateeb, and R. W. Heath Jr., “Coverage and capacity of millimeter-wave cellular networks,” IEEE Commun. Magazine, 2014.
[4] D. Torrieri and M. C. Valenti, “The outage probability of a finite ad hoc network in Nakagami fading,” IEEE TCOM, 2012.
[5] T. Bai and R. W. Heath Jr., “Analysis of self-body blocking effects in millimeter wave cellular networks,” in Proc. Asilomar 2014.

What is different for mmWave wearable networks?

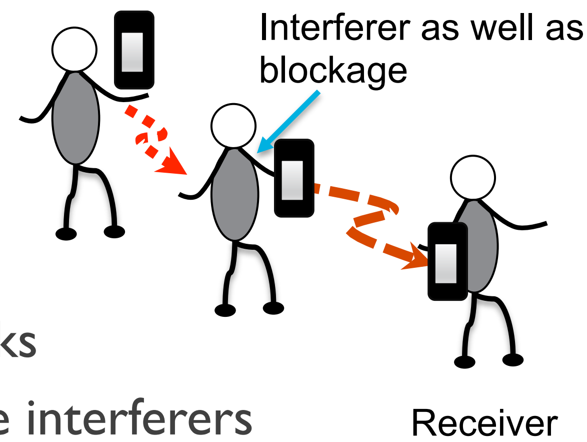


2D geometry

- Finite number of interferers in a finite network region
  - Realistic assumption for the indoor wearable setting w/ mmWave
  - Fixed/random location of interferers (extended in journal version)
- Blockages due to other human bodies
- Both interferer and blockage associated with a user

Contributions

- Model interferers as also potential blockages

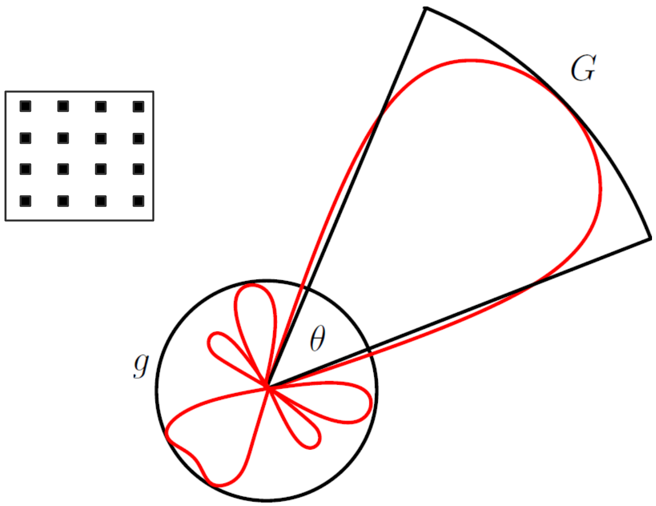


- Analyze SINR distribution and rate
  - Finite-sized mmWave-based wearable networks
  - Initially, conditioned on a fixed location for the interferers
  - Conditioning can be removed by averaging over the spatial distribution

- Assess impact of antenna parameters on performance
  - Factor in array size and gain
  - Incorporate antenna directivity and orientation

SYSTEM MODEL

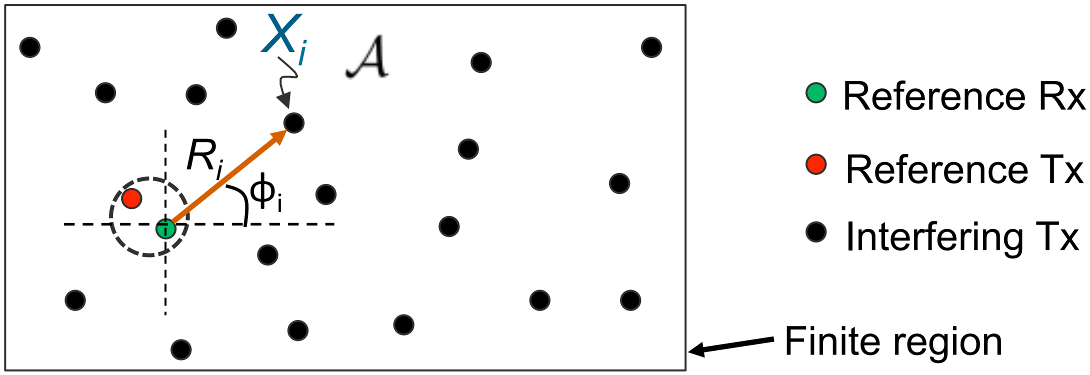
Modeling antenna pattern using a sectored antenna



Number of antenna elements	*N*
Beamwidth θ	$2\pi / \sqrt{N}$
Main-lobe gain *G*	*N*
Side-lobe gain *g*	$1 / \sin^2\left(3\pi / 2\sqrt{N}\right)$

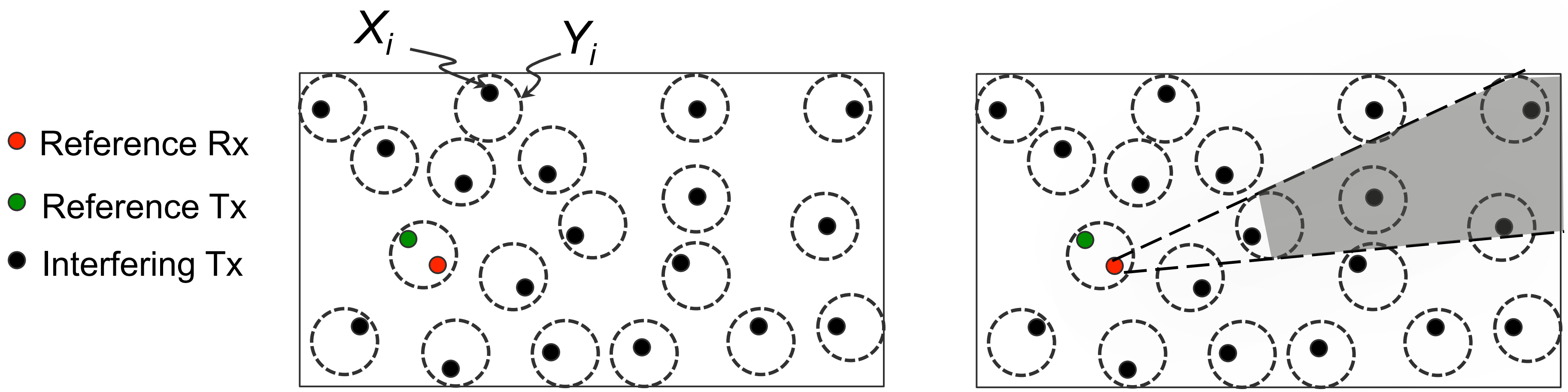
- Use a 2D sectored antenna model to simplify the analysis
  - Parameterize via a uniform planar square array w/ half-wavelength spacing
- Incorporates omni-directional antennas as a special case
  - $N = 1$ → omni-directional antenna, $G = g = 1$
  - Of interest for inexpensive wearable

Network topology



- Finite sized network region $\mathcal{A}$, area = $|\mathcal{A}|$, $K+1$ users
- One interferer per user transmits at a time
  - K interferers + reference transmitter-receiver pair
- $X_i = R_i e^{j\phi_i}$, location of transmitters relative to reference receiver
  - X_0 is location of the reference transmitter
  - $X_1, \ldots, X_K$ are the locations of the interferers.

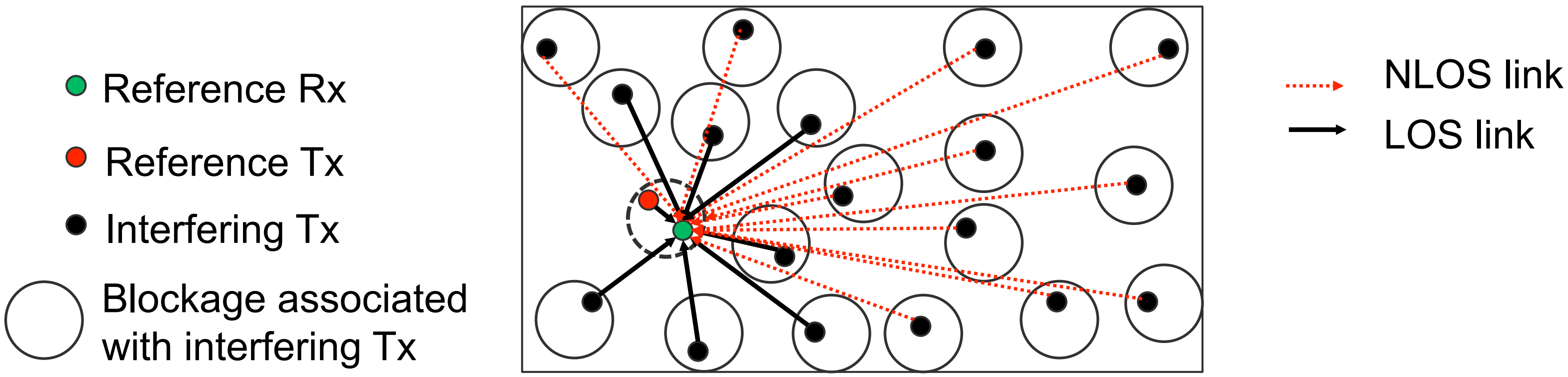
Modeling human body blockages



- Associate diameter W circle with each user – denoted Y_i
- Determine blocking cone for each Y_i
- X_i blocked if it falls in one of the blocking cones
- Assume Y_i does not block X_i, i.e., no self-blocking

SIGNAL MODEL

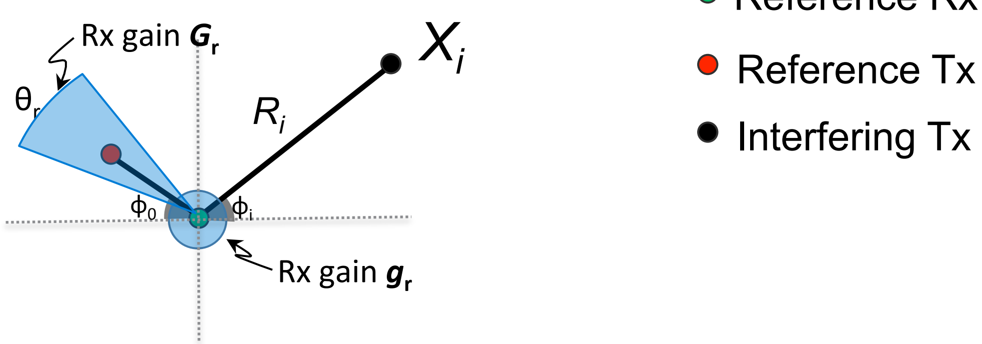
Received signal model



- h_i - Nakagami fading with parameter m_i from X_i
- Link is NLOS if blocked and LOS otherwise

NLOS ⇒ $m_i = m_N$ LOS ⇒ $m_i = m_L$

Path-loss model and power gains



- α_i - path-loss exponent from X_i
- Define

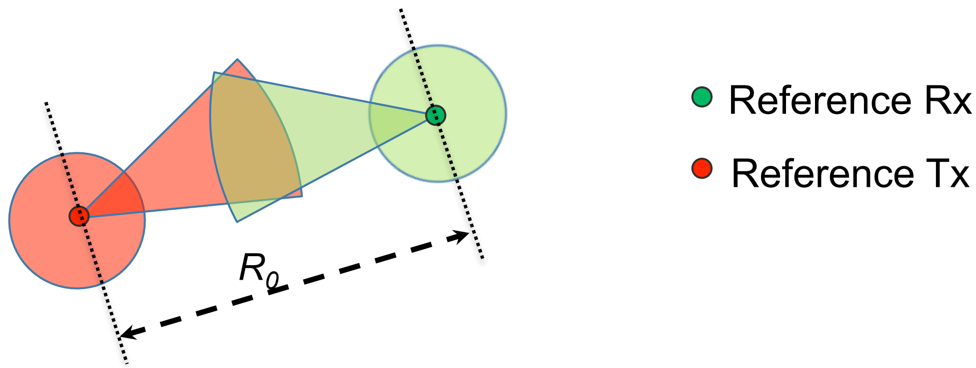
Tx power of X_i

Ref. receiver's main-lobe points towards X_i

$$\Omega_i = \begin{cases} \frac{P_i}{P_0} G_r R_i^{-\alpha_i} & \text{if } -\frac{\theta_r}{2} \leq \phi_i - \phi_0 \leq \frac{\theta_r}{2} \\ \frac{P_i}{P_0} g_r R_i^{-\alpha_i} & \text{otherwise} \end{cases}$$

Captures path loss and Rx orientation

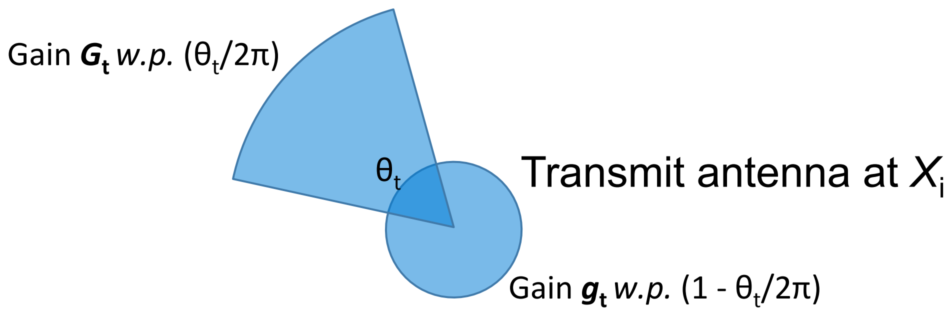
Signal from reference transmitter



- h_0 – Nakagami fade gain from reference with parameter m_0
- Assume that there is always LOS communication
- Reference Tx is within the main beam of the reference Rx

$$\Omega_0 = G_r R_0^{-\alpha_0}$$

Relative transmit power



- X_i transmits with probability p_t (Aloha-like medium access)
- X_i points its main-lobe in a (uniform) random direction
- Define

$$I_i = \begin{cases} 0 & w.p.\ (1 - p_t) \\ G_t & w.p.\ p_t \left(\frac{\theta_t}{2\pi}\right) \\ g_t & w.p.\ p_t \left(1 - \frac{\theta_t}{2\pi}\right) \end{cases}$$

Captures p_t and random Tx orientation

Probability that ref. receiver is within main-lobe of X_i

SINR and ergodic spectral efficiency

Evaluate CCDF of SINR → Derive ergodic spectral efficiency

- SINR is

$$\gamma = \frac{h_0 \Omega_0}{\sigma^2 + \sum_{i=1}^{K} I_i h_i \Omega_i}$$

Noise power normalized by P_0

CCDF of SINR

- SINR coverage probability for a given $\boldsymbol{\Omega} = [\Omega_0, ..., \Omega_K]$

$$P_{\mathsf{c}}(\beta) = \mathbb{P}\left[\gamma > \beta | \boldsymbol{\Omega}\right]$$

threshold

$$P_{\mathsf{c}}(\beta) = \mathbb{P}\left[\mathsf{S} > \sigma^2 + \sum_{i=1}^{K} \mathsf{Y}_i \,\middle|\, \boldsymbol{\Omega}\right],$$

where

$$\mathsf{S} = \beta^{-1} h_0 \Omega_0, \quad \mathsf{Y}_i = I_i h_i \Omega_i.$$

$$P_{\mathsf{c}}(\beta) = \int \cdots \int_{\mathbb{R}^K} \left(\int_{\sigma^2 + \sum_{i=1}^{K} y_i}^{\infty} f_{\mathsf{S}}(s) \mathrm{d}s \right) f_{\mathbf{Y}}(\mathbf{y}) \mathrm{d}\mathbf{y}$$

CCDF of SINR

- SINR coverage probability for a given $\mathbf{\Omega} = [\Omega_0, ..., \Omega_K]$

$$P_{\mathsf{c}}(\beta) = \mathbb{P}\left[\gamma > \beta | \mathbf{\Omega}\right]$$

threshold

$$P_{\mathsf{c}}(\beta) = e^{-\beta_0 \sigma^2} \sum_{\ell=0}^{m_0-1} \frac{(\beta_0 \sigma^2)^\ell}{\ell!} \sum_{t=0}^{\ell} \binom{\ell}{t} \frac{t!}{\sigma^{2t}} \sum_{\substack{t_i \geq 0 \\ \sum_{i=1}^{K} t_i = t}} \left(\prod_{i=1}^{K} \mathcal{G}_{t_i}(\Omega_i) \right),$$

where

$$\mathcal{G}_{t_i}(\Omega_i) = p_{\mathsf{t}} \left(\frac{\Omega_i}{m_i}\right)^{t_i} \frac{\Gamma(m_i + t_i)}{t_i! \Gamma(m_i)} \left[\frac{\theta_{\mathsf{t}}}{2\pi} \psi_{t_i}(G_{\mathsf{t}}) + \left(1 - \frac{\theta_{\mathsf{t}}}{2\pi}\right) \psi_{t_i}(g_{\mathsf{t}})\right] + (1 - p_{\mathsf{t}})\, \delta[t_i],$$

$$\delta[t_i] = \begin{cases} 1 & \text{if } t_i = 0 \\ 0 & \text{if } t_i \neq 0 \end{cases},$$

$$\psi_{t_i}(x) = x^{t_i} \left(1 + \frac{\beta_0 x \Omega_i}{m_i}\right)^{-(m_i + t_i)}$$

Rate (Spectral Efficiency)

- For a threshold β, the spectral efficiency is

$$\eta = \log_2(1+\beta)$$

- The ccdf of the spectral efficiency is found by defining equivalent rates

$$\{\gamma > \beta\} \Leftrightarrow \underbrace{\{\log_2(1+\gamma) > \eta\}}_{\{\gamma > 2^\eta - 1\}}$$

- Since they are equivalent

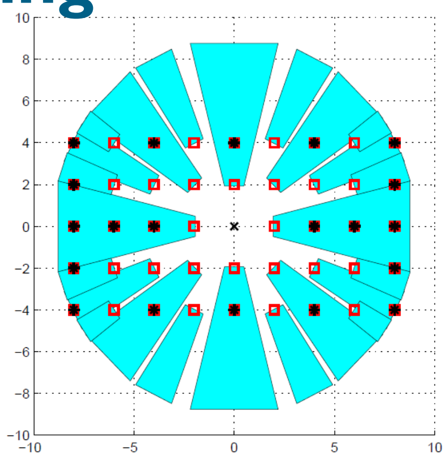
$$P_\eta(r) = P_{\mathrm{c}}(2^r - 1)$$

- And the ergodic spectral efficiency is found from:

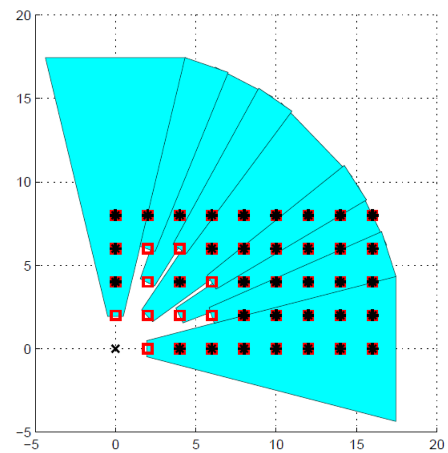
$$\mathbb{E}[\eta] = \int_0^\infty P_{\mathrm{c}}(2^r - 1)\,\mathrm{d}r = \frac{1}{\log(2)} \int_0^\infty \frac{P_{\mathrm{c}}(\beta)}{1+\beta}\mathrm{d}\beta$$

NUMERICAL RESULTS: (FIXED NETWORKS)

Setting



Receiver at center

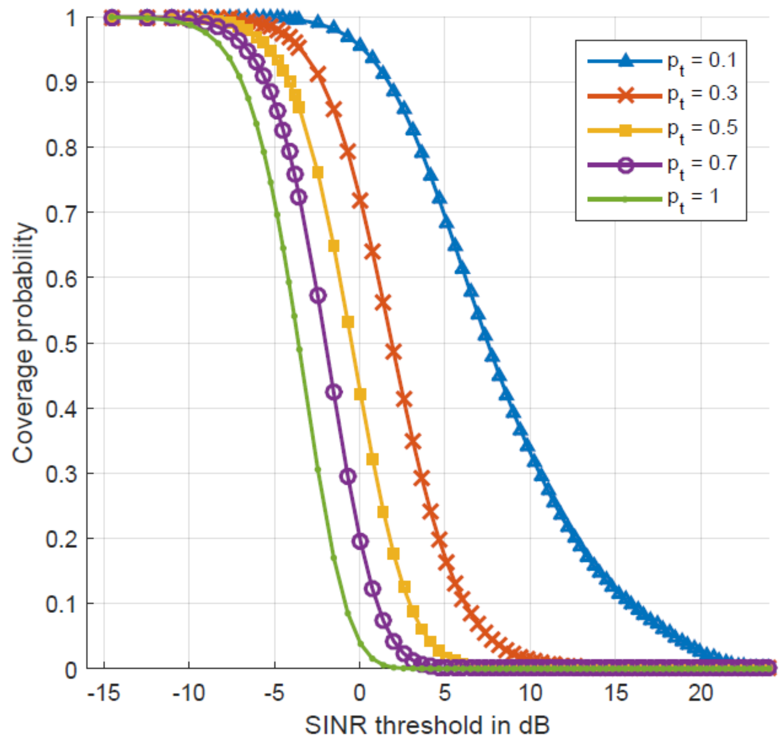


Receiver at a corner

Parameters	Value
R_0	1
m_L	4
m_N	2
α_L	2
α_N	4
W	1
σ^2	-20 dB
K	44

- 5 X 9 rectangular grid
- Separation between nodes = $2R_0$
- No reflection from boundaries
- All nodes transmit with same P_i

CCDF of SINR: Dependence on p_t

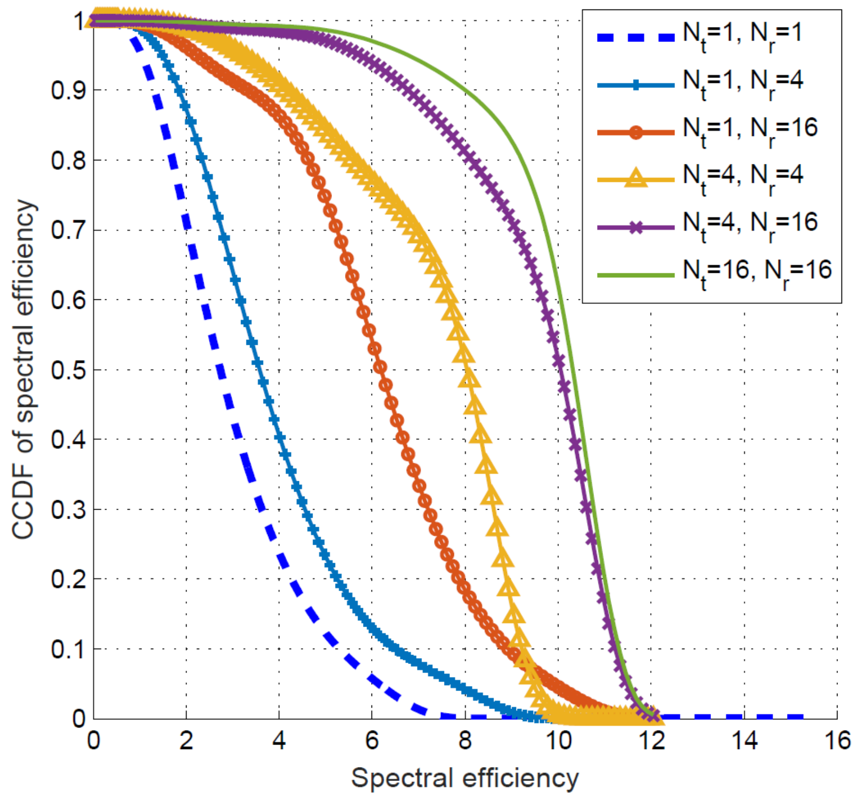


Omni Tx and Rx

Receiver at the center

- Higher transmission probability p_t results in smaller SINR
- Similar trend with other antenna configurations

Spectral efficiency for different antenna configurations

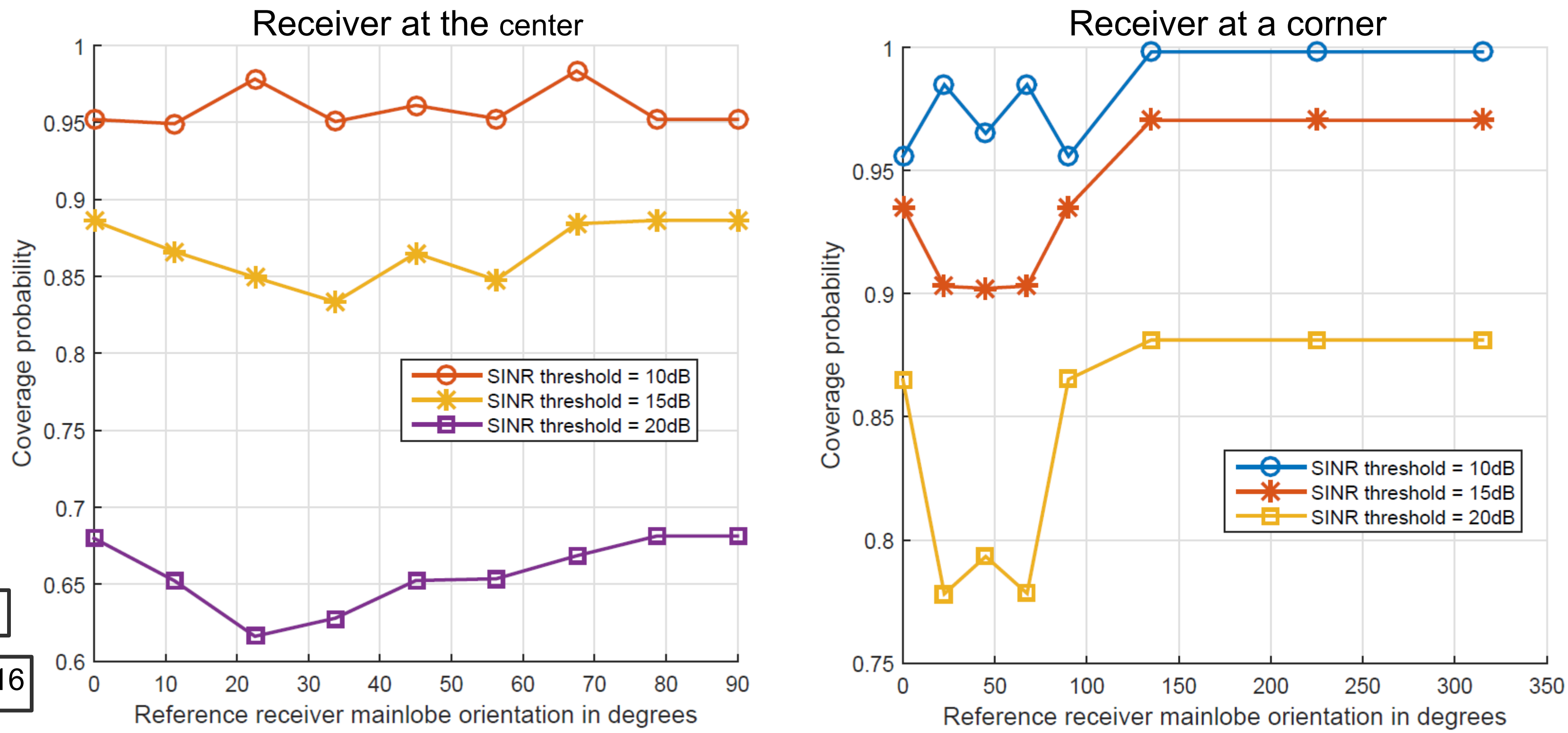


$p_t = 0.1$

Receiver at the center

Larger antenna arrays perform better

Effect of receive antenna orientation



$p_t = 0.7$

$N_t = N_r = 16$

Orientation of receiver more important at corner

Rate trends with N_t and N_r

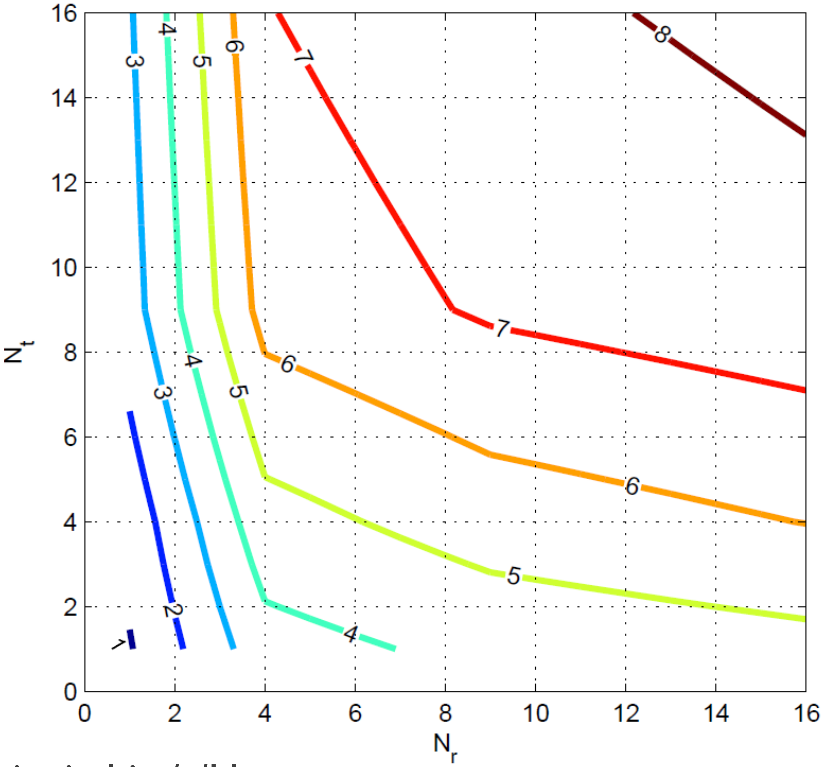
Assume 2.16 GHz BW of IEEE 802.11ad

N_t	N_r	Ergodic spectral efficiency (bits/s/Hz)		Rate (Gb/s)	
		Receiver at center	Receiver at a corner	Receiver at center	Receiver at a corner
1	1	0.499	1.063	1.08	2.30
1	4	0.797	1.405	1.72	3.03
1	16	1.757	2.087	3.80	4.51
4	1	2.449	4.046	5.29	8.74
4	4	3.210	5.072	6.93	10.96
4	16	5.437	7.078	11.74	15.29
16	1	3.618	5.027	7.81	10.86
16	4	4.635	6.396	10.01	13.82
16	16	6.952	8.434	15.02	18.22

$p_t = 1$

Gigabit throughputs are achieved even with a single transmit and receive antenna

Contour plot of ergodic spectral efficiency



*Units in bits/s/Hz

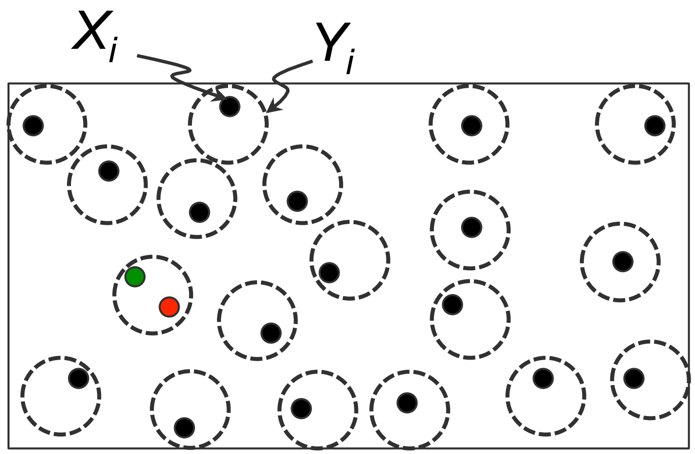
$p_t = 0.5$

Receiver at the center

RANDOM NETWORKS

Stochastic Geometry of the Network

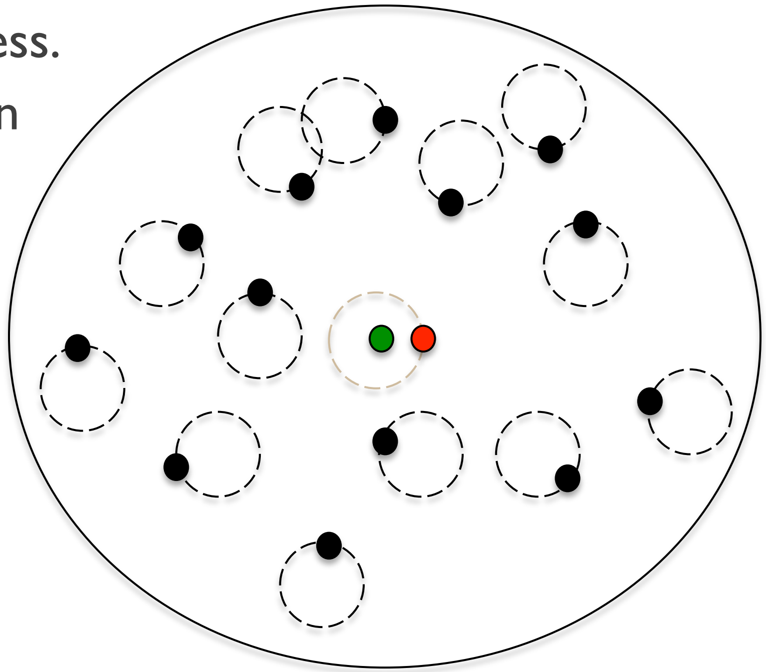
- Can model user location as being drawn from a point process.
  - Poisson Point Process (PPP) or Binomial Point Process (BPP).



- Actually two processes:
  - One process for interferers $\{X_i\}$
  - Another for the blockages $\{Y_i\}$
  - The processes are correlated.
- Analytical approach:
  - Simulation-based: Simulate the location, but use the analytical expressions for coverage and rate for each location.
  - Or, make some approximations for analytical tractability.

Model 1: Orbital Model

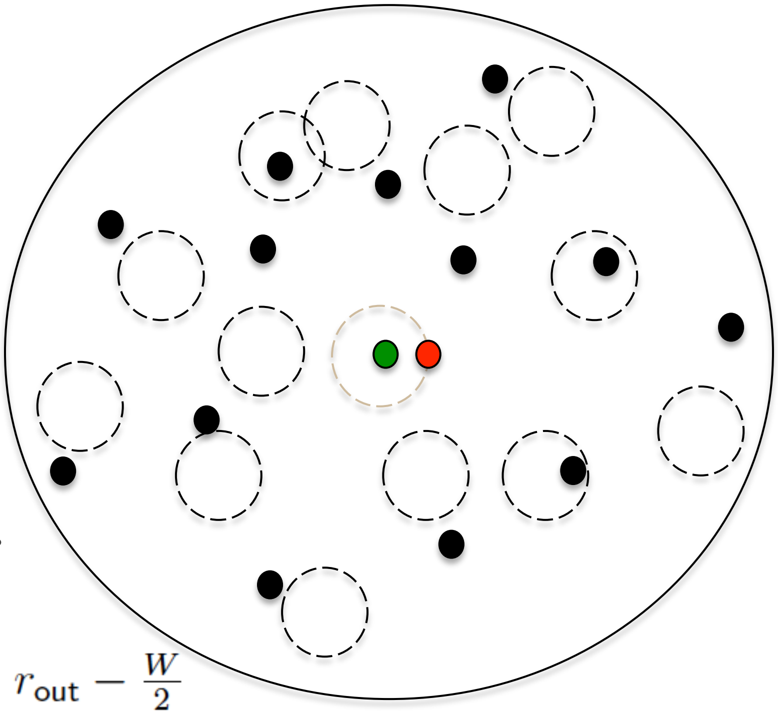
- Orbital model for human body blockage.
  - Blockage Y_i is drawn from a point process.
  - Its transmitter X_i is located randomly on the perimeter of a radius-d circle.
  - Probability of self-blocking easily found.



- Simulation based analysis:
  - Place each blockage
  - Randomly locate each interferer
  - Compute outage probability for each network realization
  - Repeatedly draw many such networks

Model 2/3: Independent Processes

- Draw the interferers and blockages from independent point processes.
  - Assume interferers must be at least distance r_{in} from the reference receiver.

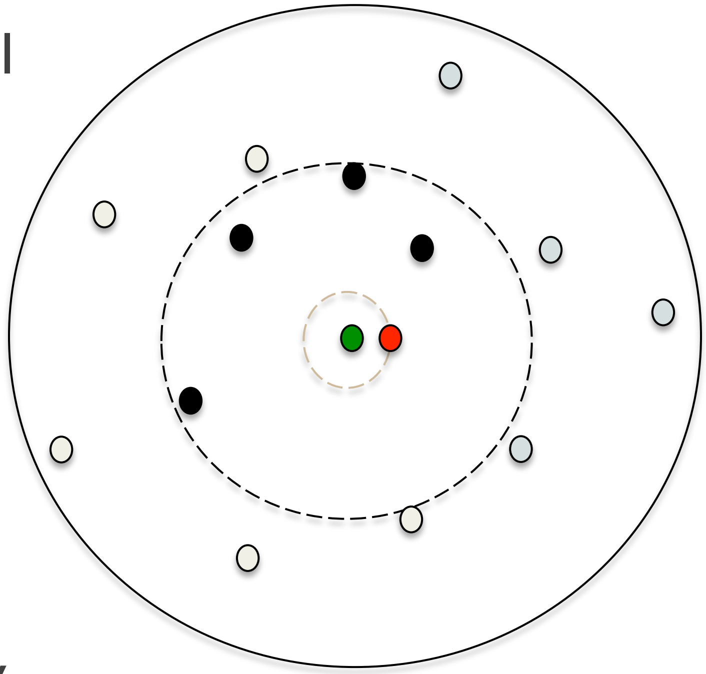


- Under this assumption, we can determine the probability of blocking at distance r when there are K interferers.

$$p_{\mathsf{b}}(r) = \begin{cases} 1 - \left(1 - \frac{rW + \frac{\pi W^2}{8} - \mu}{|\mathcal{A}|}\right)^K & \text{if } r_{\mathsf{in}} \leq r \leq r_{\mathsf{out}} - \frac{W}{2} \\ 1 - \left(1 - \frac{rW - \mu + \nu}{|\mathcal{A}|}\right)^K & \text{if } r_{\mathsf{out}} - \frac{W}{2} \leq r \leq r_{\mathsf{out}} \end{cases}$$

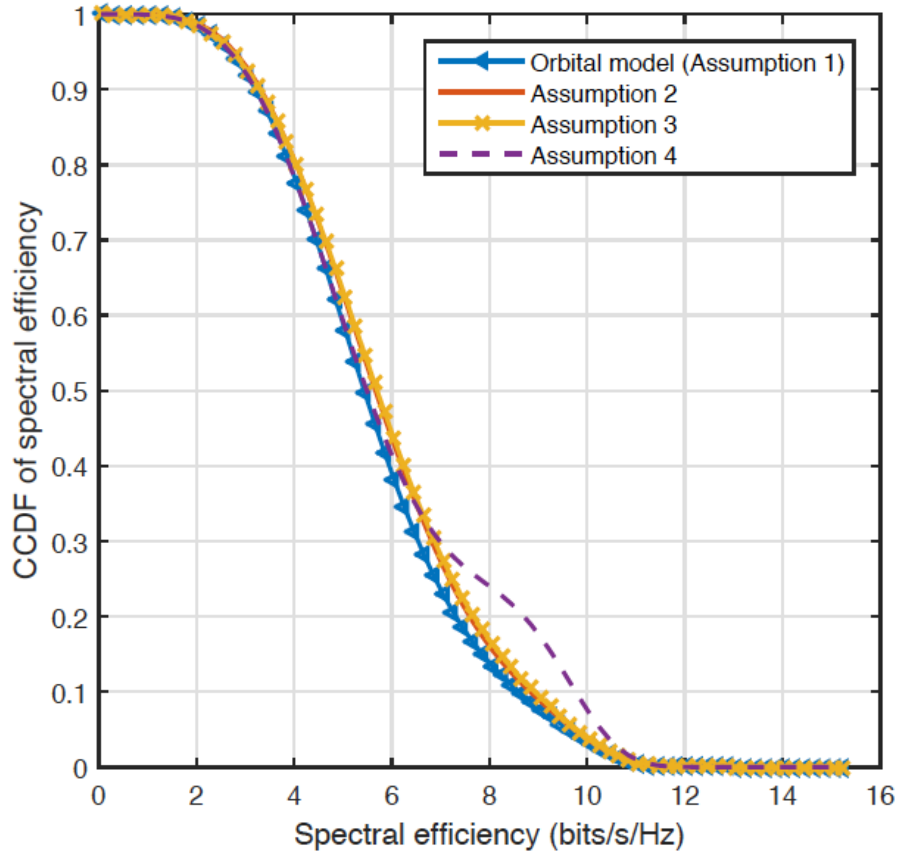
Model 4: All LOS Interferers are Inside a Ball

- Since $p_b(r)$ curve is sharp, can assume all interferers within some critical distance R_B are LOS, and outside are NLOS.
- R_B found as the average blocking distance.
- Under this model, the analysis is tractable by way of stochastic geometry



Comparison of Models

- Parameters:
  - Binomial Point Process
  - $K = 36$
  - $\sigma^2 = -20$ dB
  - $N_t = N_r = 4$
  - $p_t = 1$
- Models are reasonable
  - Overestimates rate.
  - LOS ball even more so.



Concluding remarks

- Human-body blockages should be taken into account at mmWave
  - Proper stochastic models of blockages and interferers is important
- Receive antenna configuration and orientation is critical
  - Users located at a corner can point the antenna away from the "crowd"
- Future work
  - Further analysis of random networks and refinement of their models
- For more information:
  - K. Venugopal, M. C. Valenti, and R. W. Heath, Jr., "Interference in finite-sized highly dense millimeter wave networks," *in Proc. Information Theory and Applications (ITA) Workshop*, (San Diego, CA), Feb. 2015

QUESTIONS?