Cooperative Diversity using Distributed Turbo Codes

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Abstract— **A novel coding technique, termed distributed turbo coding, is proposed for the quasi-static relay channel. The source broadcasts a recursive code to both relay and destination. After detecting the data broadcasted by the source, the relay interleaves and re-encodes the message prior to forwarding it to the destination. Because the destination receives both codes in parallel, a distributed turbo code is embedded in the relay channel. Simulation results show that the proposed code performs close to the information-theoretic bound on outage event probability.**

I. Introduction

A classic *relay channel* is a three terminal network consisting of a source, a relay, and a destination [1]. The source broadcasts a message to both relay and destination, while the relay forwards the message to the destination. Relaying has received renewed interest due to its ability to achieve *distributed* spatial diversity in wireless networks [2] to overcome the effects of fading and interference.

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In point-to-point links, spatial diversity is typically implemented with antenna arrays and space time coding at the transmitter and/or receiver. For certain types of ad hoc wireless networks, such as embedded networks and wireless sensor networks, antenna arrays are too cumbersome to be employed at each device. However, the low device costs associated with these networks allows the coverage area to be blanketed with a dense deployment of devices. The performance of these densely deployed ad hoc wireless networks can be improved by leveraging the intrinsic spatial diversity due to the presence of multiple devices. This spatial diversity could be readily exploited to achieve significant macrodiversity gain by cross layer protocol design and conventional diversity techniques [3].

One strategy for achieving distributed spatial diversity is termed *cooperative diversity* [4]. With cooperative diversity, a message is broadcast by the source and received simultaneously by the destination as well as one or more relays. Once the relays have received the message, they may then forward the information to the destination. The destination can combine the information received from the source and all the relays. Such a technique is related to the classic relay channel although it is more general because multiple relays may be used. The relays themselves may be regenerative (decode-and-forward) or nonregenerative (amplify-and-forward). Regenerative relays may use the same code as the source, in which case the destination receives the same code over all channels (i.e. a type of repetition code). Alternatively, the relay may employ a different code. The cooperation between source and relay may be reciprocal, since the two may periodically exchange roles (i.e. the source could also forward information that originated at the relay) or each node in the network could simultaneously act as source and relay by using separate channels.

A space-time coded approach for cooperative diversity was considered in [2]. Although [2] showed the benefits of such a technique from an information theoretic standpoint, it did not fully address the practical aspects of how to construct and detect the code. In particular, synchronization seems to be an issue because space-time coding requires the relays and source to transmit at the same time. While it is relatively easy to ensure that the bit epochs are aligned when a space-time code originates from a conventional antenna array, it is much more difficult to do so with a distributed array, due to the differential propagation delays and distributed clocks. One solution to this problem could be to use macrodiversity space time codes [5].

Another technique for achieving cooperative diversity was proposed in [6] and is termed user cooperative coding. Each user encodes blocks of K source bits into N bit codewords using a prescribed Rate Compatible Punctured Convolutional (RCPC) code [7]. Each N bit codeword is partitioned into two sets, with the first set being a N_1 bit punctured convolutional code, and the second set being the $N_2 = N - N_1$ remaining parity bits for the same codeword. Initially, user 1 transmits its own first set of N_1 bits. If user 2 successfully decodes the data from user 1, it will calculate and transmit user 1's N_2 remaining parity bits in the second frame. Otherwise, user 1's own N_2 parity bits are transmitted. Essentially, it is an adaptive user cooperation protocol that tends to choose the channel with the best instantaneous SNR to transmit the data. To apply this mechanism in the wireless relay channel, we could simply assign the source as user 1 and the relay as user 2.

In this paper, we focus on coding for the relay channel. First, the information theoretic limit on outage probability is analyzed under new channel constraints. Then a simple coding strategy inspired by the turbo principle is proposed and shown to approach the capacity of the constrained relay channel. In particular, a recursive systematic convolutional (RSC) code is transmitted from the source and decoded by both the relay and destination. Rather than re-encoding the data directly, the relay first interleaves the data prior to forwarding it. It will be shown that by interleaving at the relay, the relay channel itself has been transformed into a distributed turbo code. The destination can detect the source data by introducing both the direct and relayed messages into an iterative decoder. Simulation results indicate that the proposed code comes within 4.5 dB of the corresponding information theoretic bound.

II. Performance Limits

The capacity of the relay channel in AWGN was derived in [1] under the assumption that the relay could simultaneously receive and transmit. However, in practical systems, it is more realistic to assume that (a) the relay receives and transmits in different time slots; (b) the relay and the source transmit in orthogonal channels. Orthogonality could be achieved by using time division duplexing (TDD) or frequency division duplexing (FDD). For instance, in many modern systems, such as 3-G cellular in TDD mode, the relay might not be able to simultaneously receive and transmit. The capacity under constraint (a) was analyzed in [8]. As in [1], the source and relay were allowed to transmit coherently, thereby achieving a beamforming effect. However because the source and relay usually have separate oscillators, it is difficult to make their phases add coherently at the destination. Thus we impose the additional constraint (b). Furthermore, we require that the relay decode its received message and re-encode prior to forwarding (*decode-and-forward*). We assume that the relay can perform perfect error detection and will not forward if it cannot decode. While the orthogonal decode-and-forward relaying restriction may reduce the achievable performance, it is a better model of practical systems and allows for a more straightforward capacity analysis.

We assume a quasi-static Rayleigh fading channel. Thus, the channel between two terminals is AWGN for the duration of a particular packet (code word), but the SNR changes from packet-to-packet. In particular, the sequence $\{\gamma\}$ of packet SNRs is i.i.d. exponential, with average received SNR Γ.

First consider a point-to-point link. For a particular packet transmission, the channel is AWGN with SNR γ and capacity $C(\gamma) = \frac{1}{2}\log_2(1+\gamma)$. If a rate r code is used, then the channel will be in an *outage* whenever $C(\gamma) < r$, where $\{C(\gamma) < r\}$ is called the *outage event*. The *outage event probability* (OEP) is found by integrating the pdf of γ over the outage event region,

$$\begin{aligned} P_o &= \int_0^{C^{-1}(r)} p(\gamma)d\gamma \\ &= \int_0^{2^{2r}-1} \frac{1}{\Gamma}\exp\left\{\frac{-\gamma}{\Gamma}\right\} d\gamma \\ &= 1-\exp\left\{\frac{-(2^{2r}-1)}{\Gamma}\right\} \end{aligned} \quad (1)$$

Now consider the relay channel. Computation of the OEP is complicated by the fact that now there are three SNRs, $\gamma_{s,r}, \gamma_{s,d}$, and $\gamma_{r,d}$, corresponding to the source-relay, source-destination, and relay-destination links, respectively. We assume that relaying occurs in two phases. In the first phase, only the source transmits, while during the second phase only the relay transmits. Let α denote the fraction of time that the network is in the first phase. The relay channel conveys a code with overall rate r, with the source using a rate r/α code and the relay a rate $r/(1-\alpha)$ code.

The relay is in an outage if $C(\gamma_{s,r}) < r/\alpha$. When the relay is in an outage, an end-to-end outage occurs if the source-destination link is also in an outage, $C(\gamma_{s,d}) < r/\alpha$. On the other hand, if the relay is not in an outage, then the destination will receive a transmission from both source and relay. The source and relay are then transmitting over orthogonal parallel Gaussian channels. Since capacity adds for parallel channels, an outage will occur if $\alpha C(\gamma_{s,d}) + (1-\alpha)C(\gamma_{r,d}) < r$. Thus, the outage event for the relay channel with orthogonal decode-and-forward relaying is

$$\begin{aligned} E_o &= \left\{ \left[\left(C(\gamma_{s,r}) < \frac{r}{\alpha} \right) \cap \left(C(\gamma_{s,d}) < \frac{r}{\alpha} \right) \right] \right. \\ &\cup \left. \left[\left(C(\gamma_{s,r}) > \frac{r}{\alpha} \right) \cap \left(\alpha C(\gamma_{s,d}) + (1-\alpha) C(\gamma_{r,d}) < r \right) \right] \right\}. \end{aligned} \quad (2)$$

The end-to-end OEP is found by integrating $p(\gamma_{s,d}, \gamma_{s,r}, \gamma_{r,d})$ over the area defined by E_o, yielding for independent channels

$$\begin{aligned} P_o &= \left(1 - \exp \frac{1-2^{r/\alpha}}{\Gamma_{s,r}} \right) \left(1 - \exp \frac{1-2^{r/\alpha}}{\Gamma_{s,d}} \right) \\ &+ \int\int_{\mathcal{A}} \frac{\exp \frac{1-2^{r/\alpha}}{\Gamma_{s,r}}}{\Gamma_{s,d}\Gamma_{r,d}} \exp\left\{ -\frac{\gamma_{s,d}}{\Gamma_{s,d}} - \frac{\gamma_{r,d}}{\Gamma_{r,d}} \right\} d\gamma_{s,d} d\gamma_{r,d} \end{aligned} \quad (3)$$

where $\mathcal{A} = \{(1+\gamma_{s,d})^{\alpha}(1+\gamma_{r,d})^{1-\alpha} < 2^{2r}\}$.

When $\alpha = 1/2$ and $r = 1/4$, (3) can be further reduced to

$$\begin{aligned} P_o &= 1 - \exp\left\{ \frac{-1}{\Gamma_{sd}} \right\} \\ &- \exp\left\{ \frac{-1}{\Gamma_{sr}} \right\} \int_0^1 \frac{1}{\Gamma_{sd}} \exp\left\{ \frac{-\gamma}{\Gamma_{sd}} \right\} \exp\left\{ \frac{\gamma - 1}{(\gamma+1)\Gamma_{rd}} \right\} d\gamma \end{aligned}$$

Note that when orthogonal transmissions are used, the relay channel behaves in a manner similar to a hybrid FEC/ARQ system with code combining. In a hybrid ARQ system, different parts of a single codeword are transmitted through multiple time slots (an orthogonal transmission scheme), while the receiver uses a Maximum Likelihood (ML) decoder to detect the source information. By analogy, in the TDD relay channel, the relay and the source also transmit different parts of the codeword, and the destination uses a ML decoder to detect the information. If the source and the relay transmit the same convolutional codeword, the ML decoder is reduced to a maximal ratio combiner (MRC) followed by a Viterbi decoder. Although it is simpler, MRC will lose about 1.5 dB of energy efficiency compared to code combining at an OEP of 10^{-2}.

III. Distributed Turbo Coding

First consider the classic single-relay channel (the extension of distributed turbo coding to the multiple relay channel is discussed later). The source and relay each employ a very simple code, in this case a two-state rate 1/2 recursive systematic convolutional (RSC) code with octal feedback and feedforward generators (3,2), respectively. After encoding,

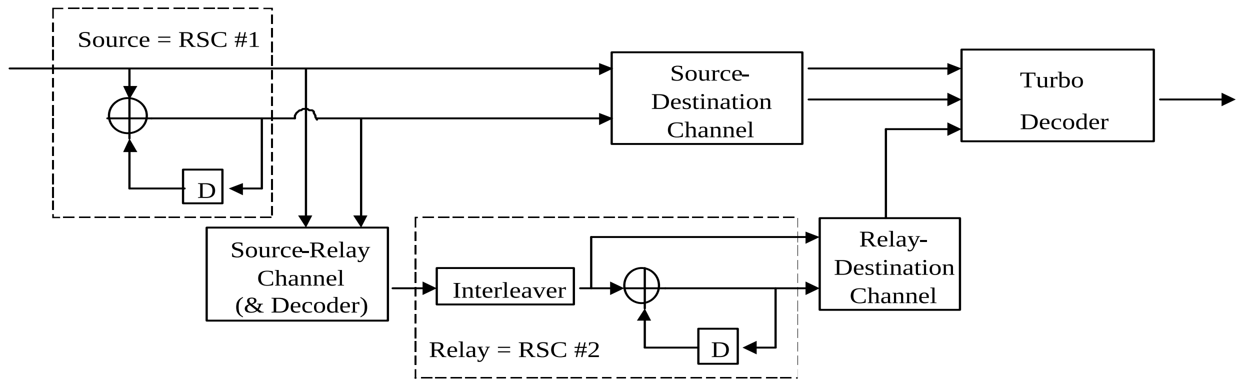

Fig. 1. When an interleaver separates source from relay, the relay channel contains a turbo code.

the signal is BPSK modulated. Note that the parity output of this code is generated by a simple differential encoder (accumulator), so an alternative interpretation is that each terminal transmits BPSK and DPSK in parallel (see Fig. 1).

A conventional decode-and-forward relay will detect the RSC encoded signal and re-encode it with an identical RSC encoder. The destination will receive two versions of the same code word, one directly from the source and the other from the relay. The two signals may be MRC combined and the information bits detected with a Viterbi decoder.

The new twist in our proposed scheme is to add an *interleaver* to the relay, as shown in Fig. 1. If the relay interleaves its estimate of the source's data prior to RSC encoding, then the source and relay have cooperatively constructed a *distributed* turbo code. Recall that with a turbo code, or parallel concatenated convolutional code (PCCC), the data is recursively encoded twice, first in its natural order and again after being interleaved [9]. Thus, the uninterleaved encoding is present in the source-destination path, while the interleaved encoding is present in the relay-destination path. The destination can detect the code iteratively by using a standard turbo decoder [9]. Although the turbo decoder adds some complexity at the destination, the complexity is still reasonable since the constituent encoders only have two states.

While this construction maintains the diversity benefit of relaying, the coding gain is far superior than that of a single RSC observed over two independent channels. This extra coding gain is due to the *interleaving* gain of the turbo code construction and the *turbo processing* gain of the iterative decoder. Notice that distributed turbo coding is a rather broad concept. It also includes distributed *serial* concatenated convolutional codes (SCCCs). For instance, if the relay re-encodes the data with a (3,2) RSC encoder prior to interleaving and DPSK encoding, then the relay channel contains a distributed SCCC.

Note that a critical assumption made in cooperative coding is that the link between source and relay is reliable. It is reasonable because the relay is usually much closer to the source than the destination is to the source. Even though the code used on the source-relay link is weak, it is sufficient to overcome the errors encountered during over the short link. Although a complex turbo decoder is needed at the destination, the relay can decode the source transmission by simply using a much less complex Viterbi decoder.

Performance can be further improved by utilizing multiple relays to provide an additional diversity benefit and more interleaving gain. If each relay were to interleave the received data prior to encoding and according to a unique interleaving pattern, then the result would be a distributed *multiple* turbo code [10].

IV. Simulation Results

In quasi-static fading, outage probability is the most important benchmark for system performance. Therefore, frame error rate (FER) curves are more frequently used to characterize simulation results. A simulation campaign was carried out to investigate the performance characteristics of distributed turbo coding. The simulations were used to determine contours illustrating the source/relay transmit power required to achieve an end-to-end FER of 10^{-2} over a single relay channel, as well as the FER of distributed *multiple* turbo codes over a multiple relay channel. In all simulations, a simple rate 1/2 RSC (3,2) code was used at both source and relay, and data was grouped into frames of length $N = 512$ bits. The modulation is assumed to be BPSK at the transmitter and the receiver.

A. Transmit SNR Contour

As an illustration, consider a system with a relay located between a source and destination separated by 10 m. Assuming a transmit frequency of $f_c = 2.4$ GHz, a path loss coefficient $n = 3$, and a free-space reference distance $d_o = 1$ m, the average received power at receiver $j \in \{r, d\}$ is $\Gamma_j^{(r)} = K_o(d_{ij})^{-n}\Gamma_i^{(t)}$, where $K_o = (c/4\pi d_o f_c)^2 \approx 10^{-4}$, d_{ij} is the transmitter-receiver separation, and $\Gamma_i^{(t)}$ is the transmitted power of node $i \in \{s, r\}$.

In the simulations, the transmit power of the source ($\Gamma_s^{(t)}$) and relay ($\Gamma_r^{(t)}$) were varied independently, and it was noted which ($\Gamma_s^{(t)}, \Gamma_r^{(t)}$) pairs achieved a target source-destination frame error rate (FER) of 10^{-2}. Concerning the effect of network topology on the performance of the relay channel, we investigate two different layouts of the relay channel: (a) the relay located halfway between the source and destination, and (b) the relay is 1 m away from the source and 9 m away from the destination. The corresponding simulation results are shown in Fig. 2 and Fig. 3.

In case (a), the information theoretic bound based on (3) with $r = 1/4$ and $\alpha = 1/2$ is computed and shown in the lower-left corner of Fig. 2 to benchmark the performance of different coding strategies. A total of five relaying strategies were studied. The

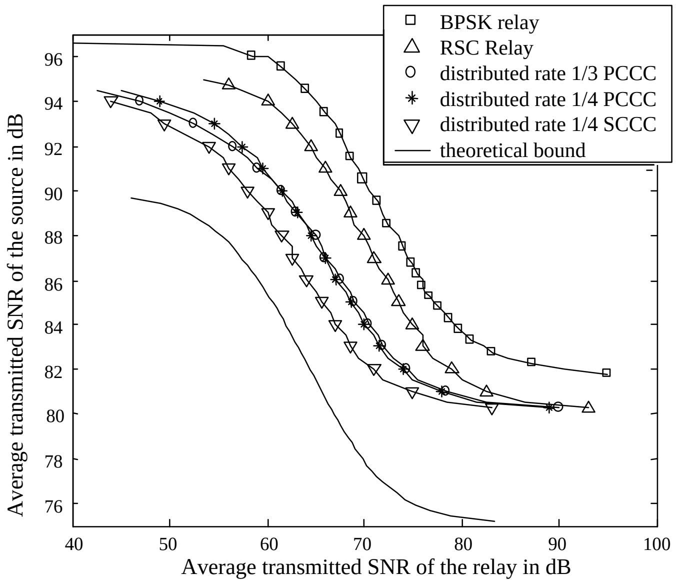

Fig. 2. Minimum transmit SNR at source and relay required to acheive an end-to-end FER of 10^{-2} when the relay is halfway between the source and destination.

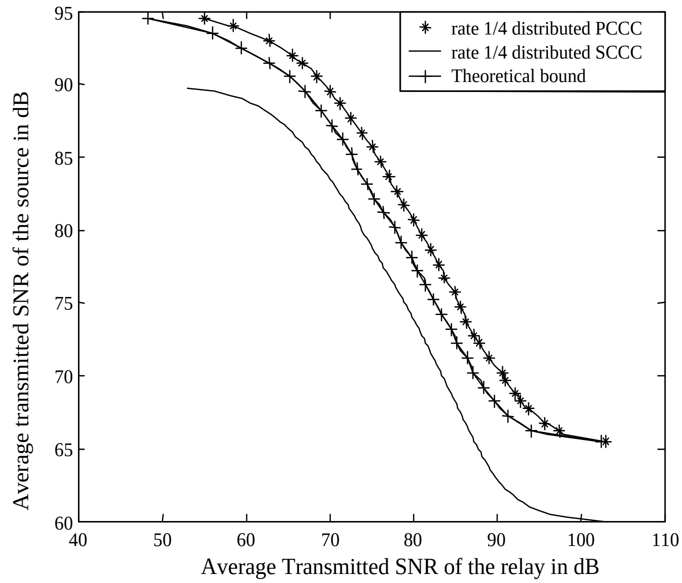

Fig. 3. Minimum transmit SNR at source and relay required to acheive an end-to-end FER of 10^{-2} when the relay is 1 m away from the source and 9 m away from the destination.

least energy efficient case (upper-right curve) is the uncoded BPSK relay channel. The other four systems are coded, and from least to most power efficient are (1) *Repetition coded:* The source and relay each use a (3,2) RSC code and the relay does not interleave (the destination performs MRC combining and Viterbi decoding); (2) *Distributed rate 1/4 PCCC:* Identical to (1) except the relay interleaves the data prior to RSC encoding and the destination performs iterative (turbo) decoding; (3) *Distributed rate 1/3 PCCC:* Identical to (2) except the relay only uses an accumulator (equivalently, the relay's systematic RSC output is punctured); (4) *Distributed rate 1/4 SCCC:* Identical to (3) except the relay re-encodes the data with a (3,2) RSC encoder prior to interleaving and differential encoding.

The RSC coded system (1) asymptotically has a coding gain of about 1.7 dB over the uncoded BPSK system (since the (3,2) RSC code has free distance $d_f = 3$). The distributed PCCC provides an additional 4 dB gain over the RSC code. Note that the rate 1/3 and rate 1/4 PCCC codes achieve nearly the same performance, indicating that the relay does not need to transmit its systematic output. The distributed SCCC provides yet another 2 dB gain over the PCCC. While these gains are somewhat modest for a turbo code, the constituent codes only have two states, and hence these codes may be iteratively detected with reasonable complexity.

When the source's transmit power Γ_s becomes too high or too low, the performance curves of all four coded systems tend to converge and the advantage of distributed turbo coding tends to decrease. At one extreme, when Γ_s is sufficiently high, no relay power is necessary, thus the relay channel is reduced to a regular point-to-point fading channel. At the other extreme, if Γ_s decreases, the relay SNR Γ_r will increase. Thus, the source-relay link becomes less reliable while the relay-destination link becomes more reliable. Eventually, as Γ_s approach its minimum, Γ_r will approach infinity. In this case, the relay channel is transformed into a one transmit two receive antenna system where selective combining is used at the destination. Since those four coded systems will be transformed into identical systems as $\Gamma_r \to \infty$ or $\Gamma_r \to 0$, their corresponding contour curves will asymptotically converge to the same performance. To improve the energy efficiency at low Γ_s, either the source could use a more powerful code or the relay could move closer to the source.

Since the performance relationship between the five strategies has already been revealed in Fig. 2, only distributed SCCC and distributed PCCC are investigated in case (b). From Fig. 3, we observe that when the relay is closer to the source, the Γ_s predicted by the theoretic bound has a wider dynamic range. Also, we note that the curves for distributed SCCC and distributed PCCC are more consistently close to the theoretic bound over a wider range of SNRs than they were when the relay was centrally located. For both topologies, the distributed SCCC is 4.5 dB away from the bound at its closest point.

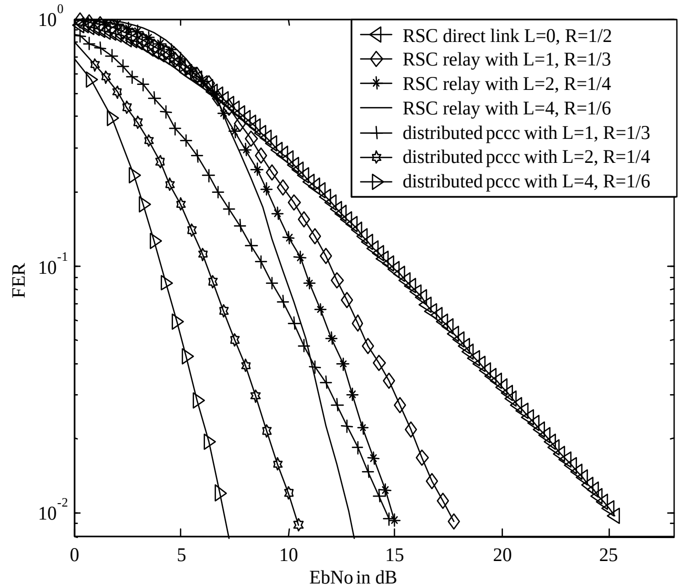

Fig. 4. Frame error rate for distributed multiple turbo codes over the multiple relay channel under the assumption of perfect source-relay links.

B. Multiple Relay Channel

When the concept of distributed turbo coding is extended to the multiple relay channel, the result is a distributed *multiple* turbo code. Thus the code constructions and decoding stategies discussed in [10] can be readily applied. In our simulations, we assume that the broadcast channel from source to the multiple relays are always reliable (which is likely when the relays are clustered close to the source), and that through perfect power control, the destination's average received SNR from the source and multiple relays are identical.

A total of seven scenarios were simulated, one for a direct RSC encoded transmission (no relay), and then a pair of simulations for each of L = 1, 2, and 4 relays. When relays are present, two strategies were simulated, an RSC repetition code (each relay uses the same RSC encoder and no interleaver; MRC combining and Viterbi decoding is performed at the destination) and a parallel multiple turbo code (each relay interleaves the decoded data with a unique interleaving pattern before differential encoding; a turbo decoder with L+1 soft-in/soft-out modules is used at the destination). Because the source broadcasts a rate 1/2 RSC and each relay transmits a rate 1 code (just the parity bit stream of its rate 1/2 encoder), the overall code rate R is related to L according to $R = \frac{1}{L+2}$. The RSC encoded direct transmission could be considered as a special case of multiple relaying with $L = 0$. By varying L, we can benchmark the pure diversity gain due to the multiple relay paths, so that the additional interleaving gain of distributed multiple turbo codes can be easily isolated.

With multiple RSC relaying, a total of 4.7 dB extra diversity gain could be achieved when the number of relays L increases from 1 to 4. With distributed multiple turbo codes, the additional interleaving gain increases from 3 dB for single relay ($L = 1$) to nearly 6 dB for multiple relays ($L = 4$). Compared with RSC encoded direct link transmission, distributed multiple turbo coding ($L = 4$) could achieve a total of 18 dB 'cooperative coding' gain, in which the 6 dB interleaving gain is almost constant in the low to moderate SNR region. Both diversity gain and interleaving gain tend to yield diminishing marginal benefit with each additional relay.

V. Conclusions

A simple, but efficient, coding technique has been developed for the quasi-static relay channel. An appropriate information theoretic bound on capacity was derived, and it was shown that the proposed code comes within 4.5 dB of this bound. This performance gap can be closed by using a much powerful error correcting code at the source and relay. Moreover, the extension of distributed turbo coding to the multiple relay channel has been investigated and a significant performance advantage over the conventional relaying method demonstrated.

For future research, the theoretic limit on the outage probability of the multiple relay channel should be derived and used to benchmark the energy efficiency of the distributed multiple turbo coding schemes. The impact of an unreliable source-relay broadcast channel should be considered when there are multiple relays. Furthermore, amplify-and-forward strategies could be explored, channel estimation issues considered, and effective ARQ protocols devised.

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