we have evaluated the upper bounds to the bit error probabilities choosing three interleaver lengths, namely N = 100, 1000, 10000 (Fig. 1). The performance is similar to that obtained on the classical AWGN channel since, in this hypothesis, a fully-interleaved channel is assumed. Continuous encoding always yields the best performance and the performance of the truncated encoder is significantly worse than that of the continuous one, whereas that of trellis termination is only slightly worse.

As far as slow fading is concerned, the upper bounds to the bit error probabilities of the above mentioned code have been determined using an interleaver length N = 10000, since in the numerical evaluation of eqn. 9 low values of the abscissas E_b/N_0 are significant, especially when no diversity in reception is used. Therefore, an interleaver length of $N \geq 10000$ has to be chosen in order to achieve an acceptable code performance and to evaluate this performance by means of a sufficiently tight bound. The results are reported in Fig. 2 with respect to continuous and block co-decoding. To achieve a good code performance, space diversity in reception has to be used, where the adopted space diversity technique is maximum-ratio combining, whereas in the fast fading case the use of this technique is not required if a sufficient interleaver length is employed ($N \geq 1000$).

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Refined channel estimation for coherent detection of turbo codes over flat-fading channels

M.C. Valenti and B.D. Woerner

Channel state information is required for the coherent detection and decoding of turbo codes transmitted over flat-fading channels. A channel estimation technique suitable for turbo codes is presented. The technique uses pilot symbols to obtain initial channel estimates, and refines the estimates after each iteration of the turbo decoder.

Introduction: Turbo codes, originally introduced in [1], have been shown to achieve near capacity performance over the additive white Gaussian noise (AWGN) channel. Turbo codes can also achieve remarkable performance over the fully-interleaved Rayleigh flat-fading channel [2]. However, the optimum turbo decoding algorithm requires estimates of the noise variance. In addition, if coherent detection is desired, then estimates of the fading amplitudes and phase are required. In [3], the issue of noise variance estimation was addressed, and a simple but effective noise variance estimator was proposed. In [4], a fading amplitude estimator was introduced, and the performance of turbo codes over unknown channels using the estimator was shown to be slightly inferior to the performance over known channels. In [5], channels with unknown phase were considered, and methods for channel estimation based on the use of pilot symbols and pilot tones were discussed. Alternatively, if fading estimates are not available, turbo codes can be used with noncoherent modulation formats such as differential phase shift keying (DPSK) or noncoherent frequency shift keying (FSK) [6]. However, if noncoherent detection is used, the performance is severely degraded due to the noncoherent combining penalty.

The methods presented in [3 – 5] all perform channel estimation *prior* to turbo decoding. However, turbo decoding is an iterative process which produces bit estimates and their associated reliabilities after each iteration. It is therefore possible to use the decisions at the end of each decoding iteration to refine the channel state estimates used during the next iteration. This strategy can be interpreted as a form of decision-directed estimation, where decisions from one iteration of decoding are used to assist estimation during the next iteration. In this Letter, we propose a technique for integrating the estimation process into the turbo decoder in such a way that the reliability information produced during each iteration is used to assist estimation during the subsequent iteration.

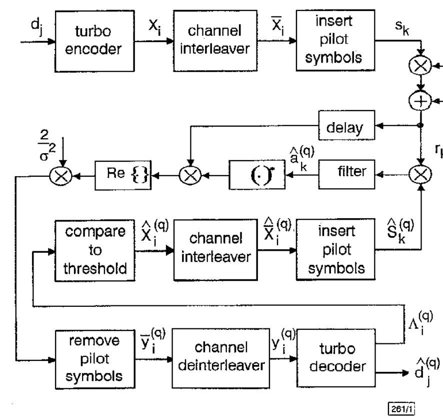

Fig. 1 *System model*

System model: The system model is shown in Fig. 1. A sequence $\{d_j\}$, $1 \leq j \leq L$, of data bits is passed through a rate R turbo encoder with interleaver size L. Binary phase shift keying (BPSK) is assumed, so $d_j \in \{-1, 1\}$. The encoded bits $\{x_i\}$, $1 \leq i \leq L/R$, are interleaved by a channel interleaver, which is implemented by writing the code bits row-wise into an $M \times N$ matrix and reading the interleaved code bits $\{\bar{x}_i\}$ from the matrix column-wise. Next, the sequence $\{\bar{x}_i\}$ is parsed into groups of M_p contiguous bits, where M_p is the pilot symbol spacing [7]. A known pilot symbol is placed in the centre of each group, and the new groups of size (M_p+1) are reassembled into the sequence $\{s_k\}$, $1 \leq k \leq L(M_p+1)/(RM_p)$.

The sequence $\{s_k\}$ is transmitted over a Rayleigh flat-fading channel, correlated according to Clarke's model [8]. Each symbol s_k is multiplied by a fade $a_k = X_k + \sqrt{-1}Y_k$, where X_k and Y_k are independent and identically distributed zero-mean Gaussian processes with autocorrelation $R(k) \propto J_o(2\pi f_d T_s k)$, where f_d is the Doppler frequency and T_s is the symbol period. Samples n_k of an AWGN process are then added to the faded symbols, where the n_ks are independent and identically distributed zero-mean complex Gaussian random variables with variance $\sigma^2 = N_o/(2RE_b)$ (E_b is the energy per bit and N_o is the one-sided noise spectral density).

The received sequence $\{r_k\}$ is multiplied by the complex conjugate of the estimated fade sequence $\{\hat{a}_k^{(q)}\}$, where q indicates the decoder iteration (the delay block is required to synchronise with the filter below it). The initial fading estimates $\{\hat{a}_k^{(0)}\}$ are obtained by training on the pilot symbols only, according to the technique of [5] and [7]. The real part of $\{r_k(\hat{a}_k^{(q)})^*\}$ is then multiplied by the constant $2/\sigma^2$. For ease of exposition, it is assumed that the receiver has perfect knowledge of the variance σ^2. In practice, the methods of [3] and [4] can be used to obtain reliable estimates of σ^2. Next, the pilot symbols are removed and the channel interleaving process is reversed. The resulting sequence $\{y_i^{(q)}\}$ is passed to a turbo decoder implemented using the log-MAP algorithm [9], which executes decoder iteration q and produces two output sequences. The sequence $\{\hat{d}_j^{(q)}\}$ contains estimates of the data bits, while the sequence $\{\Lambda_i^{(q)}\}$ contains the log-likelihood ratio (LLR) of the code bits.

The LLR is compared to a threshold so that $\hat{x}_i^{(q)} = \text{sign}(\Lambda_i^{(q)})$ if $|\Lambda_i^{(q)}| > V_t$, where V_t is a threshold. Otherwise $\hat{x}_i^{(q)} = E$, an erasure symbol. The sequence is then re-interleaved, and the known pilot symbols are reinserted. The resulting sequence $\{\hat{s}_k^{(q)}\}$ is multiplied by the received values $\{r_k\}$, thereby removing the phase modulation of the pilot symbols and the code symbols with reliability above the threshold. The result of the multiplication is passed through a smoothing filter. Optimal fade estimates are obtained by Weiner filtering, although good results can be obtained using a lowpass filter with a cutoff at the Doppler frequency or even just a simple moving average [7]. Erasures at the input to the filter can be handled by replacing them with the value corresponding to the demodulated pilot symbol with location closest to the erasure.

Processing in the receiver is iterative and proceeds until the desired number of decoder iterations is reached.

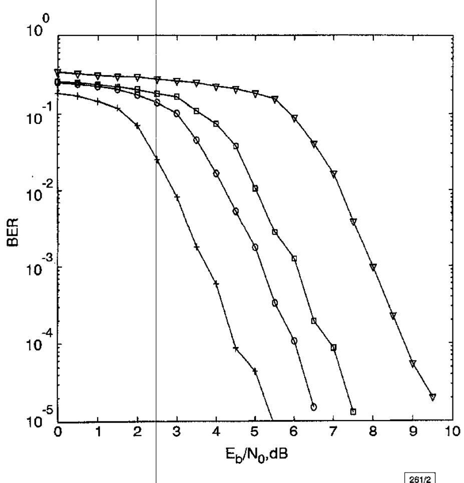

Fig. 2 *BER against E_b/N_o as parameterised by reception and estimation technique*

▼ DPSK with differential detection
□ BPSK with estimation prior to decoding
○ BPSK with refined estimation
+ BPSK with perfect estimates

Simulation results: The potential of the proposed iterative decoding and estimation process is best illustrated by simulation. Consider a rate $R = 1/2$ turbo code composed of a pair of parallel concatenated constraint length $K = 3$ recursive systematic convolutional (RSC) codes separated by an $L = 1024$ bit interleaver. Eight decoder iterations are performed and it is assumed that the trellis of the upper encoder is terminated. The channel interleaver is implemented by a 32 × 64 matrix, and the pilot symbol spacing is $M_p = 16$. The product of the Doppler frequency and symbol period is $f_d T_s = 0.005$. Fig. 2 shows the bit error rate (BER) against the energy per bit to noise spectral density ratio (E_b/N_o) for four reception and estimation techniques. The uppermost curve shows the performance of a DPSK system using the turbo code and the reception technique of [6]. The second curve from the top shows the performance of a coherently detected BPSK system where fading estimation is performed prior to turbo decoding, as suggested in [5]. The third curve from the top shows the performance of a coherently detected BPSK system using the iterative estimation and decoding procedure proposed in this Letter. The fourth (bottom) curve shows the performance of the coherent BPSK system when the fading amplitudes are known precisely at the receiver. Pilot symbols were used for the two cases that involve channel estimation (second and third curves down), but were not used for the other two cases. When channel estimation is performed, the smoothing filter is implemented as a simple moving average with a window size of $N = 83$ symbols, which is approximately equal to the channel coherence time. For the iterative estimation and turbo decoding technique, the threshold was set so that $V_t = 2/\sigma^2$. Note that, when pilot symbols are not used, slightly more energy is available for the transmission of code symbols, a fact accounted for in the simulations.

The case where the fades are known precisely at the receiver exhibits the best bit error performance. However, this case cannot be attained in a practical system and thus serves only as a performance benchmark. The DPSK case shows the worst performance of the three practical methods that were considered. Although the DPSK case performs ~ 4.5dB worse than the ideal case (at a BER of 10^{-4}), it is simple to implement and requires neither pilot symbols nor a channel estimation algorithm. The performance can be greatly improved by using pilot symbols and channel estimation. When channel estimation was performed prior to turbo decoding, the performance was within ~ 2.5dB of the ideal case, while when estimation and decoding are integrated as proposed in this Letter, the performance was within ~ 1.5dB of the ideal case (again at a BER of 10^{-4}).

Conclusions: A technique for channel estimation suitable for coherent turbo coded systems operating over unknown flat-fading channels is presented. Pilot symbols are used to assist channel estimation prior to the first iteration of turbo decoding. During subsequent decoder iterations, the channel is re-estimated using both the pilot symbols and those decoded symbols with reliability above a threshold. A simulation example shows that this technique yields a superior performance to both turbo coded DPSK and turbo coded BPSK without the channel re-estimation procedure.

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