A Bandwidth Efficient Pilot Symbol Technique for Coherent Detection of Turbo Codes over Fading Channels*

Matthew C. Valenti
Department of Computer Science and Electrical Engineering
West Virginia University
Morgantown, WV 26505
valenti@mprg.ee.vt.edu

Brian D. Woerner
Mobile and Portable Radio Research Group
Department of Electrical and Computer Engineering
Virginia Polytechnic Institute & State University
Blacksburg, VA 24061-0350
woerner@vt.edu

Abstract- **A method is proposed to implement coherent detection of turbo coded binary phase shift keying (BPSK) signals transmitted over frequency-flat Rayleigh fading channels. After turbo encoding, a subset of the parity symbols is punctured and replaced with known pilot symbols. Because the pilot symbols replace existing parity symbols, no bandwidth expansion is required. However, there is a restriction on the set of possible pilot symbol spacings and interleaver sizes. At the receiver, estimates of the complex channel gain and variance of the additive noise are first computed by filtering observations of the pilot symbols. As the turbo decoder performs its iterations, the channel estimates are refined by using information produced by the decoder. The performance of the proposed technique is assessed by computer simulation, and compared for two normalized fade rates with both perfect coherent detection (corresponding to perfect knowledge of the fading process and noise variance) and differential detection of differentially-encoded binary phase shift keying (DPSK).**

I. Introduction

Turbo codes, originally introduced in [1], can achieve near-capacity performance over Rayleigh flat-fading channels with coherent detection and perfect knowledge of the channel response [2]. However, mobile communication systems are characterized by channel responses with time-varying magnitude and phase. For mobile communications, conventional coherent detection is difficult to implement since the phase must be estimated and tracked. For turbo coded systems, the situation is further complicated because the decoder requires estimates of the magnitude of the fading process and the variance of the additive noise process.

In [3], Hall and Wilson consider turbo coded systems employing frequency shift keying (FSK) with noncoherent detection and differential phase shift keying (DPSK) with differential detection. The receivers for such systems are less complex than their coherent counterparts, primarily because the carrier phase does not need to be estimated. However, due to the noncoherent combining penalty, the performance of these systems is significantly degraded. Compared to coherent binary phase shift keying (BPSK), a loss of 6.1 and 2.7 dB was observed for FSK and DPSK respectively in additive white Gaussian noise (AWGN) with a (2048,1024) turbo code at a bit error rate (BER) of 10^{-5}. In [4], a 4.5 dB performance loss was shown for a similar code with DPSK in Rayleigh flat-fading at a BER of 10^{-5}.

Another solution that we explore in this paper is to incorporate pilot symbols into the transmission. Like pilot tones, pilot symbols are used at the receiver to obtain an estimate of the channel response so that coherent detection can be performed. Following the convention of [5], we use the term *Pilot Symbol Assisted Modulation* (PSAM) to describe systems that incorporate pilot symbols in the transmission. In [6], it is shown that the use of pilot symbols is more power efficient than the use of a pilot tone. In [4], it is shown that additional performance gains can be achieved in a turbo-coded PSAM system by re-estimating the channel after each iteration of decoding. When the channel is re-estimated, the log-likelihoods produced by the turbo decoder are incorpo-

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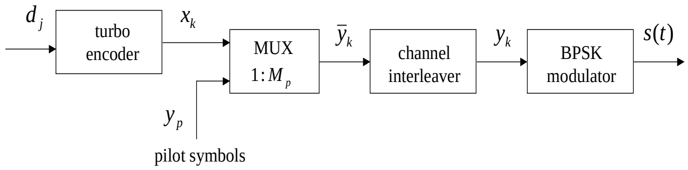

Figure 1: Proposed transmitter.

rated into the channel estimation algorithm. Here, we take a simpler approach by incorporating hard estimates of the code bits into the estimation algorithm in a decision directed manner. A related iterative technique using pilot symbols for convolutional codes is can be found in [7].

The primary disadvantage of PSAM is reduced spectral efficiency. The loss in spectral efficiency can be overcome by a process we call *parity-symbol stealing*. Instead of transmitting pilot symbols along with all of the code symbols, the pilot symbols replace a subset of the parity symbols. Alternatively, this process can be viewed as the overpuncturing of the encoder output so that the overall rate of the code with pilot symbols is the same as an equivalent system without pilot symbols. By using this strategy, pilot symbols can be incorporated into the transmission without the need to expand bandwidth.

In this paper, coherent detection of turbo codes using the bandwidth efficient pilot symbol technique and iterative channel estimation is discussed in detail. The system model is presented in Section II. The proposed iterative estimator/decoder is presented in Section III, and simulation results shown in Section IV. Finally a conclusion is given in section V.

II. System Model

A. Transmitter

A block diagram of the proposed transmitter is shown in Fig. 1. A sequence $\{d_j\}, 1 \le j \le L$, of random binary data $d_j \in \{0,1\}$ is first encoded by a rate r turbo encoder. The resulting sequence of code symbols is $\{x_k\}, 1 \le L/r$. At each time k, a multiplexer selects either a code symbol x_k or a known pilot symbol y_p according to

$$\bar{y}_k = \begin{cases} x_k & \text{if } (k + M_p/2) \bmod M_p > 0 \\ y_p & \text{if } (k + M_p/2) \bmod M_p = 0, \end{cases} \quad (1)$$

where M_p is the pilot symbol spacing (i.e. every M_p-th time interval, a pilot symbol is transmitted in place of a code symbol). The symbols $\{\bar{y}_k\}$ are then passed to a M by N block channel interleaver. The channel interleaver is required because turbo encoding may not be sufficient to cope with the burst errors induced by a correlated fading channel. Note that the channel interleaver is different from the nonuniform interleaver required within the turbo encoder.

When replacing parity symbols with pilot symbols, the

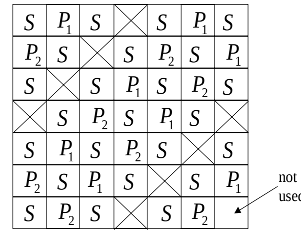

Figure 2: Example parity stealing channel interleaver for $M_p = 6$. S is a systematic bit, P_1 is parity from the upper encoder, P_2 is parity from the lower encoder, and X is a pilot symbol.

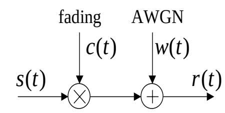

Figure 3: Channel model.

design of the channel interleaver becomes a critical issue. The interleaver must be designed so that: (1) Only parity symbols are deleted (i.e. the systematic information is left intact); and (2) An equal number of parity symbols are deleted from each of the two constituent encoders. For rate $r = 1/2$ turbo codes, M_p must satisfy

$$M_p = 2(2i+1) \quad (2)$$

for some integer $i > 0$, and the dimension of the interleaver must be $mM_p + 1$ by $nM_p + 1$ for integer m and n. Note that the number of storage elements in the interleaver will be odd, but the number of code bits will be even. Thus, the element in the lower right hand corner of the interleaver is not used. An example interleaver for $M_p = 6$ and $m = n = 1$ is shown in Fig. 2.

After interleaving, the sequence $\{y_k\}$ is then fed into a BPSK modulator which produces the output

$$s(t) = \begin{cases} g(t) & \text{if } y_k = 1 \\ -g(t) & \text{if } y_k = 0, \end{cases} \quad (3)$$

where $g(t)$ is an arbitrary pulse shape with energy $E_s = rE_b$.

For comparison purposes, we also consider ideal coherent detection of BPSK. For ideal BPSK we assume that the channel parameters are known precisely at the receiver, and thus no pilot symbols are required (i.e. the multiplexer in Fig. 1 is removed and the code symbols are modulated immediately following the interleaver). Furthermore, we consider differential detection of DPSK. In this case, the multiplexer in Fig. 1 is removed and a differential encoder is placed between the channel interleaver and the BPSK modulator.

B. Channel

The transmitted signal $s(t)$ passes through a fading channel with additive white Gaussian noise as illustrated

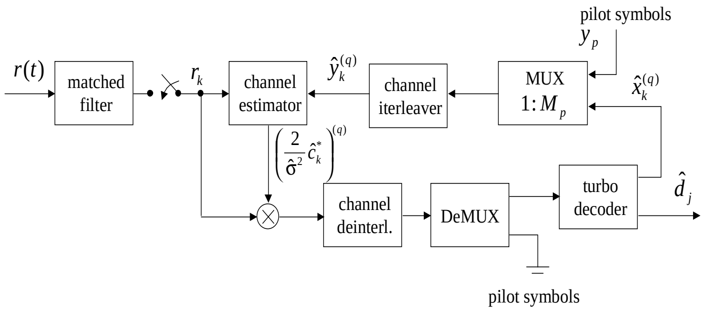

Figure 4: Proposed receiver

in Fig. 3. Here, c(t) is a complex-valued process that describes the multiplicative distortion of the frequency-flat fading channel and $w(t)$ is complex zero-mean white-Gaussian noise. The two-sided noise spectral density of $w(t)$ is $N_o/2$ in each of the real and imaginary directions. The Rayleigh fading is modeled as a zero-mean complex Gaussian low-pass process $c(t)$, where the real and complex components are independent and each have autocorrelation [8]

$$R_c(\tau) \quad = \quad J_o(2\pi f_d \tau), \tag{4}$$

where f_d is the relative Doppler between transmitter and receiver, and $J_o(\cdot)$ is the zeroth order Bessel function of the first kind. Note that the power of $c(t)$ is normalized to unity.

C. Receiver

The proposed receiver is shown in Fig. 4. The received signal $r(t)$ is passed through a matched filter and then sampled at the symbol rate $1/T_s$. We assume that $c(t)$ varies slowly with respect to the symbol period, perfect timing is available, and there is no intersymbol interference. The sequence at the output of the sampler can be expressed as

$$r_k \quad = \quad c_k(2y_k - 1) + n_k, \tag{5}$$

where $c_k = c(kT_s)$ and $\{n_k\}$ is a set of statistically independent complex Gaussian random variables with zero-mean and variance $\sigma^2 = N_o/2E_s$ in each direction. After sampling, r_k is sent to a channel estimation algorithm (described in the next section). The algorithm computes initial estimates $\{\hat{c}_k^{(0)}\}$ of the fading process and $(\hat{\sigma}^2)^{(0)}$ of the noise variance. The statistic $\{r_k(2\hat{c}_k^*/\hat{\sigma}^2)^{(0)}\}$ is formed and passed first through a channel interleaver, and then to a turbo decoder which is implemented with the log-MAP algorithm [9].[1]

After each iteration q, the turbo decoder produces estimates $\{\hat{x}_k^{(q)}\}$ of the code symbols. The symbol estimates are reinterleaved and pilot symbols are reinserted. The resulting estimated symbol sequence $\{\hat{y}_k^{(q)}\}$ is then fed back into the channel estimator in a decision directed manner. The channel is re-estimated prior to the next decoder iteration. New estimates $\{(2\hat{c}_k^*/\hat{\sigma}^2)^{(q)}\}$ are produced by the channel estimation algorithm and used by the turbo decoder during iteration $(q + 1)$.

[1]Pilot symbols must be replaced with erasures prior to decoding, i.e. $r_k = 0$ if $(k + M_p/2) \bmod M_p = 0$.

III. Channel Estimator

A. Channel Gain Estimation

If the transmitted sequence $\{y_k\}$ were known at the receiver, then the best linear minimum mean-square error (MMSE) estimate of the complex channel gain is found according to

$$\hat{c}_k \quad = \quad \sum_{i=-N_c}^{N_c} w_i \left(2y_{k-i} - 1\right) r_{k-i}, \tag{6}$$

where w_i is a set of filter coefficients found using the Wiener-Hopf equations. However, the only values of $\{y_k\}$ that are known at the receiver a priori are the pilot symbols. Under the assumption of a slowly varying channel, (6) can be approximated by

$$\hat{c}_k \quad = \quad \sum_{i=-N_c}^{N_c} w_i \left(2y_p - 1\right) r_{k-i}^{(p)}, \tag{7}$$

where $r_{k-i}^{(p)}$ is the received value of the pilot symbol located closest to r_{k-i}.

Equation (7) is used to compute the initial estimate $\hat{c}_k^{(0)}$. After the turbo decoder begins producing estimates of the code symbols (i.e. $q > 0$), the channel gain estimate is found using

$$\hat{c}_k^{(q)} \quad = \quad \sum_{i=-N_c}^{N_c} w_i \left(2\hat{y}_{k-i}^{(q)} - 1\right) r_{k-i}. \tag{8}$$

That is, the channel estimator uses a combination of the code bit estimates derived by the decoder and the pilot symbols that are known a priori.

When the normalized fade rate $f_d T_s$ is slow ($f_d T_s << 1$) and the filter order $2N_c + 1$ sufficiently small ($N_c << (f_d T_s)^{-1}$), the filter coefficients are all approximately equal, and the filter is a simple moving average (MA). The benefit of using a moving average is that it is simpler than a Wiener filter and does not require knowledge of the fade rate. In the simulation results, we use a Wiener filter for fast fade rates and a moving average for slow fade rates.

B. Noise Variance Estimation

To determine the noise variance, first assume that the set of channel gains $\{c_k\}$ and transmitted symbols $\{y_k\}$ are known at the receiver. Then form the random variable

$$\begin{aligned} z_k &= (2y_k - 1)r_k - c_k & (9) \\ &= (2y_k - 1)n_k. & (10) \end{aligned}$$

The sequence $\{z_k\}$ is thus a set of independent Gaussian random variables with zero mean and variance σ^2. The best estimate of σ^2 can be found by simply taking the sample variance of z_k. However, $\{c_k\}$ and $\{y_k\}$ are not perfectly known at the receiver. Assuming that the channel is slowly varying, (10) can be approximated by

$$z_k = (2y_p - 1)r_k^{(p)} - \hat{c}_k^{(p)}, \qquad (11)$$

where $\hat{c}_k^{(p)}$ is the estimated channel gain associated with the pilot symbol located closest to r_k.

The sample variance of (11) is used to compute the initial noise variance estimate $(\hat{\sigma}^2)^{(0)}$. For $q > 0$, noise variance estimates are found by taking the sample variance of

$$z_k^{(q)} = (2\hat{y}_k^{(q)} - 1)r_k - \hat{c}_k^{(q)}. \qquad (12)$$

Thus, the variance estimator uses the output of the channel gain estimator along with the tentative decisions of the turbo decoder and knowledge of the pilot symbols.

IV. Simulation Results

Consider a turbo code generated by two rate 1/2, constraint length $K = 4$ recursive systematic convolutional (RSC) encoders concatenated in parallel (the feedback generator is $(15)_o$ and feedforward generator is $(17)_o$). The turbo code's interleaver size is $L = 4,140$, and the last 3 bits of the frame used to terminate the trellis of the upper encoder (the lower encoder's trellis is left open). In the simulations, a new random interleaver is generated for each Monte Carlo trial. Puncturing is used to increase the overall code rate to 1/2 (the even indexed bits from the upper encoder and odd indexed bits from the lower encoder are not transmitted). Pilot symbols overwrite parity symbols with spacing $M_p = 6$, 10, 18, or 30. The channel interleaver is implemented with a 91×91 matrix (with element 91,91 unused). Two normalized fade rates are considered, $f_dT_s = .005$ and $f_dT_s = .02$.[2] We found that when $f_dT_s = .005$ the performance using a moving average was approximately the same as the performance using a Wiener filter. However, for $f_dT_s = .02$ use of a MA is unacceptable. Thus for the slower fade rate we use a moving average, while for the faster fade rate we use a Wiener filter. In both cases $N_c = 30$. Eight decoder iterations are performed using the Log-MAP algorithm, and the simulation proceeds until 40 independent frame errors are generated for each value of E_b/N_o.

Simulation results are shown for $f_dT_s = .005$ in Fig. 5 and for $f_dT_s = .02$ in Fig. 6. In each figure, the dotted line shows the BER when the receiver has perfect knowledge of the channel gain and noise variance. Note that this

[2]The $f_dT_s = .005$ situation corresponds to a typical digital cellular system operating at 900 MHz with 19.2 kbps symbol rate and a relative mobile velocity of approximately 70 mph. The $f_dT_s = .02$ situation corresponds to a typical PCS system operating at 1.8 GHz with a 9.6 kbps symbol rate and relative mobile velocity of approximately 70 mph.

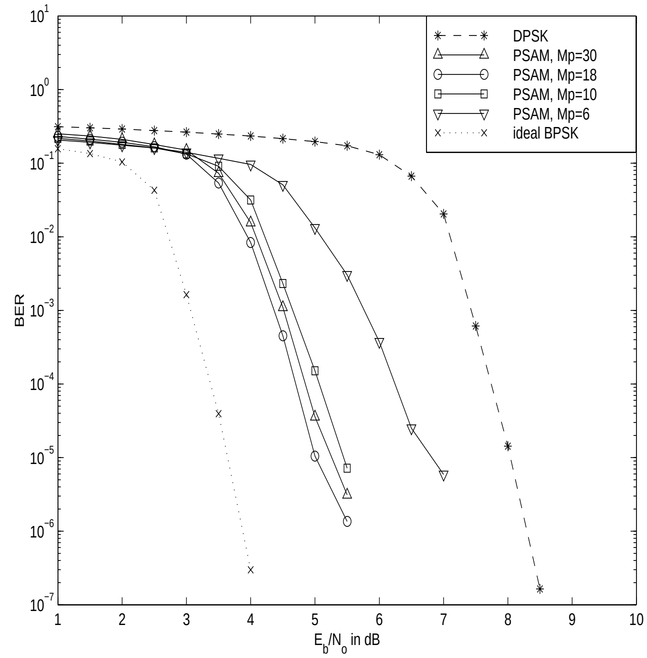

Figure 5: Comparison of coded bit error performance versus E_b/N_o for three reception techniques. The channel is Rayleigh flat-faded with $f_dT_s = .005$ and block channel interleaving of depth 91. A turbo code with rate 1/2, constraint length $K = 4$, and data frame size $L = 4140$ is used. Eight iterations of Log-MAP decoding are performed. The estimator for the PSAM technique is employed using a MA filter with $N_c = 30$.

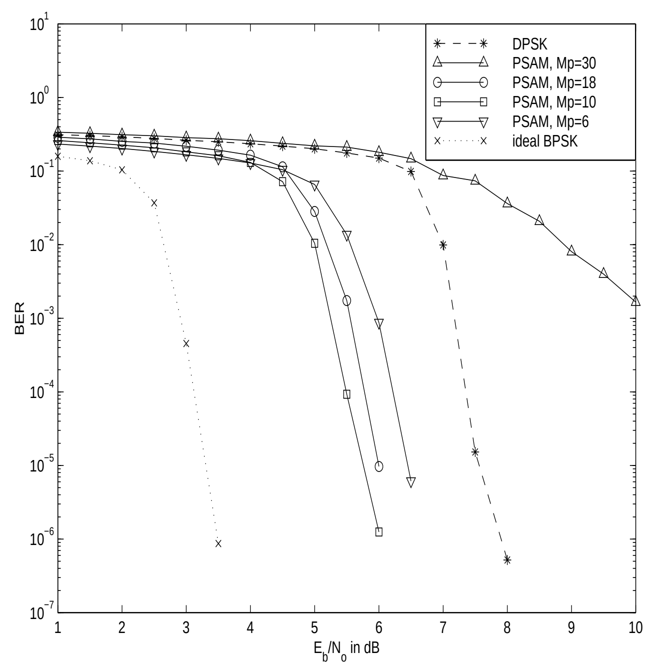

Figure 6: Same as previous figure, except $f_dT_s = .02$ and the estimator is implemented using a Wiener filter.

performance cannot be attained in a practical system and serves only as a benchmark. The dashed line shows the coded performance for DPSK. Note that the performance of ideal BPSK and DPSK for $f_d T_s = .02$ is slightly better than the performance for $f_d T_s = .005$. This is because the channel interleaver is better able to spread out the fades when faster fading is encountered, and thus the interleaved channel gains appear more uncorrelated to the turbo decoder. For $f_d T_s = .005$ and a BER of 10^{-5}, DPSK is 4.4 dB less power efficient than ideal BPSK. For $f_d T_s = .02$, this difference is 4.1 dB. This performance loss was a motivating factor behind this research, as the proposed pilot symbol based techniques bridges the gap between ideal BPSK and DPSK.

For each of Figures 5 and 6, the four solid lines represent different pilot symbol spacings, $M_p = 6, 10, 18$ and 30. When $f_d T_s = .005$, the best non-ideal performance is achieved with $M_p = 18$ (for a BER of 10^{-5}, performance is 3 dB better than DPSK, and within 1.4 dB of ideal BPSK). The performance curves for $M_p = 10$ and 30 are slightly worse than for $M_p = 18$, while performance for $M_p = 6$ is significantly worse. For the smaller pilot symbol spacings $M_p = 6$ and 10, this loss is due to a weakening of the turbo code because too many parity symbols were stolen, while for the larger pilot symbol spacing $M_p = 30$ the spacing was not large enough to track the time variation of the channel.

When $f_d T_s = .02$, the best non-ideal performance is achieved using $M_p = 10$. At a BER of 10^{-5}, performance is still better than DPSK, but only by 1.7 dB. Performance is 2.4 dB worse than ideal BPSK. This implies that the proposed method tends to work better for slower fading, and that perhaps other alternatives should be sought for faster fading. The performance curves for $M_p = 6$ and 18 are slightly worse than for $M_p = 10$, while for $M_p = 30$ the performance is actually worse than for DPSK. The extremely poor performance for $M_p = 30$ can be attributed to the fact that the pilot symbol spacing is not sufficient to track the channel. Note that in all cases no additional bandwidth is required to achieve this level of performance, although the channel estimation algorithm increases the receiver complexity (compared to DPSK).

V. Conclusion

In this paper, a method for detecting turbo codes over time-varying channels has been presented. At the transmitter, pilot symbols replace a fraction $1/M_p$ of code symbols. Because the pilot symbols replace existing symbols, no bandwidth expansion is required. The pilot symbols are used at the receiver to obtain initial estimates of the channel gain and noise variance. After each iteration of turbo decoding, estimates of the code bits are fed back to the channel estimation algorithm. The channel estimator uses a Wiener filter, although a simple moving average can be used for slow fade rates. Simulation results show that the proposed pilot symbol method is more power efficient than DPSK. The results are less promising for faster fade rates, although still superior to DPSK provided that the pilot symbols are sufficiently close together.

For ease of exposition, this paper has focused on the use of BPSK modulation. The proposed system can be easily extended to accommodate other modulation formats, particularly M-ary phase shift keying (PSK). Also, the results shown in this paper were for one particular turbo code and two particular values of normalized fade rate $f_d T_s$. A natural extension of this work would be to consider other normalized fade rates, particularly faster fade rates, and other turbo codes.

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