An Optimal Soft-Output Multiuser Detection Algorithm and its Applications

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Abstract: *This paper considers the problem of iterative multiuser detection and decoding of coded asynchronous multiple-access signals. This work differs from previous work in three respects: (1) the soft-output multiuser detection algorithm is implemented using the MAP algorithm rather than a suboptimal approximation, (2) observations from M spatially separated locations can be used, rather than observations from just one location, and (3) the model is general enough to be used for not only CDMA systems, but TDMA systems as well.*

Keywords: multiuser detection, antenna arrays, iterative decoding.

1. INTRODUCTION

Since the introduction of the turbo concept, considerable attention has been given to the problem of jointly detecting and decoding coded multiple-access signals. In general, these "turbo multiuser" detection algorithms have employed suboptimal multiuser detection strategies to accommodate heavily loaded systems (see for example [1], [2], and [3]). The focus on suboptimal multiuser detection was predicated by the fact that the complexity of the optimal multiuser detector is exponential in the number of users and therefore impractical for the large number of cochannel users present in typical CDMA systems. However, there are applications such as TDMA cellular systems, certain wideband CDMA scenarios, and combined CDMA/TDMA systems where the number of interferers is sufficiently small enough to warrant the use of an optimal multiuser detection algorithm.

This paper provides an overview of the optimal soft-input, soft-output (SISO) asynchronous multiuser detection algorithm. The algorithm is a modification of Verdu's optimal hard-output multiuser detector [4] that has been implemented using the MAP algorithm [5] in the log-domain [6]. Due to space limitations, some of the details of the algorithm have been left out. For a more complete description, please see [7].

The proposed algorithm can be applied to the general problem of the joint detection and decoding of coded asynchronous multiple-access networks composed of K transmitters and M receivers. A SISO multiuser detector is used to jointly detect signals at each of the M receivers, and the outputs of the multiuser detectors are combined to form an input to a bank of SISO decoders (one for each of the K users). The soft-output of each of the decoders is fed back to the M multiuser detectors and processing proceeds in an iterative fashion according to the turbo principle.

2. SYSTEM MODEL

Consider a multiple-access network comprised of K transmitters and M receivers. The input to transmitter $k, 1 \leq k \leq K$, is a set $\{m_{k,i}\}, 0 \leq i \leq L-1$, of random binary data $m_{k,i} \in \{0,1\}$. The bits are encoded by a rate r encoder.[1] The code bits $\{x_{k,l}\}, 0 \leq l \leq L/r-1$, are reordered by an interleaving function α_k to form the set of interleaved code bits $\{\bar{x}_{k,l}\}$, where $\bar{x}_{k,\alpha_k(l)} = x_{k,l}$.

The interleaved code bits are passed to a signal mapper, which creates a stream of symbols $\{v_{k,n}\}, 0 \leq n \leq N-1$. To clarify the exposition, we will restrict our attention to the BPSK mapping $v_{k,n} = 2\bar{x}_{k,n}-1$, where $N = L/r$. The symbols are passed through a pulse shaping filter with impulse response $g_k(t)$. Each transmitter k can have its own unique pulse shape[2], but all pulses have energy E_s (we adjust the channel gains to account for different received powers). The output of transmitter k is then

$$s_k(t) = \sum_{n=0}^{N-1} v_{k,n} g_k(t - nT_s). \qquad (1)$$

We adopt a complex multi-input, multi-output channel model. In particular, the impulse response of the channel between transmitter k and receiver m is

$$h_{m,k}(t) = c_{m,k}(t)\delta(t - \tau_{m,k}), \qquad (2)$$

[1] Note that the encoder can be of any type, i.e. we do not restrict our attention to turbo coded systems.

[2] For DS-CDMA, $g_k(t)$ incorporates both the spreading waveform (used to distinguish among users) and the pulse/chip shaping waveform (used to control the power spectrum). For TDMA, $g_k(t)$ is the same for all cochannel users and is merely the pulse shaping waveform.

where $c_{m,k}(t)$ is a complex fading process and $\tau_{m,k}$ is the propagation delay, which is assumed to be less than one symbol period, $0 \leq \tau_{m,k} < T_s$.

The signal at the input to receiver m is the noisy sum of the K transmitted signals convolved with their respective channel impulse responses

$$\begin{aligned} y_m(t) &= \sum_{k=1}^{K} s_k(t) * h_{m,k}(t) + n_m(t) \\ &= \sum_{k=1}^{K} c_{m,k}(t) s_k(t-\tau_{m,k}) + n_m(t), \quad (3) \end{aligned}$$

where $n_m(t)$ is a complex white Gaussian noise process with variance $N_o/2$.

The front end of receiver m contains a bank of K matched filters. The matched filter output for a particular user k' is

$$\begin{aligned} y_{m,k',n} &= \frac{1}{E_s}\int_0^{T_s} g_{k'}(t) y_m(t+\tau_{m,k'}+nT_s)dt \\ &= V_{m,k',n} + \xi_{m,k',n} + \sum_{k\neq k'} I_{m,k,n}^{(k')}, \quad (4) \end{aligned}$$

where $V_{m,k',n}$ is the contribution due to desired user k', $\xi_{m,k',n}$ is the contribution from the Gaussian noise, and $I_{m,k,n}^{(k')}$ is the contribution from interfering user k.

The notation can be greatly simplified by using a vector and matrix notation. In particular, the output of the bank of matched filters can be expressed as:

$$\mathbf{Y}^{(m)} = \mathbf{R}^{(m)}\mathbf{A}^{(m)}\mathbf{V} + \mathbf{N}^{(m)}. \quad (5)$$

In the above, $\mathbf{Y}^{(m)}$ is a KN column vector that contains all output samples of the bank of K matched filters at receiver m listed in "round-robin" fashion, $\mathbf{Y}^{(m)} = (y_{m,1,0}, ..., y_{m,K,0}, y_{m,1,1}, ..., y_{m,K,N-1})$. $\mathbf{A}^{(m)} = diag(\mathbf{C}^{(m)})$, where $\mathbf{C}^{(m)}$ contains the KN channel gain coefficients between each transmitter and receiver m, and $\mathbf{V}$ contains all KN modulated code bits (also in round-robin order). $\mathbf{R}^{(m)}$ is a $KN \times KN$ autocorrelation matrix whose definition follows the convention of [8]. Finally, $\mathbf{N}^{(m)}$ is a colored Gaussian noise vector with autocorrelation $E\left[\mathbf{N}^{(m)}(\mathbf{N}^{(m)})^H\right] = \sigma^2\mathbf{R}^{(m)}$, where $\mathbf{N}^H$ denotes the Hermitian transpose of $\mathbf{N}$ and $\sigma^2 = N_o/(2E_s)$.

Because the noise samples are colored, it is cumbersome to compute the exact log-likelihood ratios of the symbols using the statistic $\mathbf{Y}^{(m)}$. However, because $\mathbf{R}^{(m)}$ is positive definite, there exists a lower triangular matrix $\mathbf{F}^{(m)}$ with positive diagonal elements such that [8]

$$\mathbf{R}^{(m)} = (\mathbf{F}^{(m)})^T\mathbf{F}^{(m)}. \quad (6)$$

$\mathbf{F}^{(m)}$ is found by performing a Cholesky decomposition of $\mathbf{R}^{(m)}$, and only has nonzero entries in diagonals -K through 0. The whitened matched filter outputs are then

$$\begin{aligned} \bar{\mathbf{Y}}^{(m)} &= (\mathbf{F}^{(m)})^{-T}\mathbf{Y}^{(m)} \\ &= \mathbf{F}^{(m)}\mathbf{A}^{(m)}\mathbf{V} + \bar{\mathbf{N}}^{(m)}, \quad (7) \end{aligned}$$

where $\mathbf{F}^{-T} \equiv (\mathbf{F}^T)^{-1}$ and the autocorrelation of the noise component is $E\left[\bar{\mathbf{N}}^{(m)}(\bar{\mathbf{N}}^{(m)})^H\right] = \sigma^2\mathbf{I}$. Thus, $\bar{\mathbf{N}}^{(m)}$ is a white Gaussian noise process with variance $\sigma^2 = E_s/(2N_o)$.

By taking advantage of the structure of $\mathbf{F}^{(m)}$ and $\mathbf{A}^{(m)}$, which are both sparse matrices, (7) can be expressed more efficiently as

$$\bar{\mathbf{Y}}_i^{(m)} = \sum_{j=0}^{\min(i,K-1)} \mathbf{F}_{i,i-j}^{(m)}\mathbf{C}_{i-j}^{(m)}\mathbf{V}_{i-j} + \bar{\mathbf{N}}_i^{(m)}. (8)$$

A secondary benefit of whitening the matched filter outputs is that the decision statistic $\bar{\mathbf{Y}}_i^{(m)}$ is only a function of the current and past $K-1$ symbols observed through noise. This is in contrast to the unwhitened statistic $\mathbf{Y}_i^{(m)}$, which is a noisy function of not only the current and past $K-1$ symbols, but also the next $K-1$ symbols. This property of the whitened matched filter output will be exploited in the development of the SISO MUD algorithm in the next section.

3. OPTIMAL SISO MUD

The Soft-Input, Soft-Output (SISO) MUD algorithm for receiver m computes the log-likelihood ratio

$$\mathbf{\Lambda}_i^{(m)} = \ln\frac{P[\mathbf{V}_i = +1|\bar{\mathbf{Y}}^{(m)}]}{P[\mathbf{V}_i = -1|\bar{\mathbf{Y}}^{(m)}]}. \quad (9)$$

By considering the whitened multiple access interference channel to be a time-varying Markov process, a suitable SISO MUD algorithm can be developed. To see this, first define a one-to-one mapping between the state of the Markov process and the set of $K-1$ past symbols

$$s_i \iff \{\mathbf{V}_{i-1}, \mathbf{V}_{i-2}, ..., \mathbf{V}_{i-K}\}.$$

Likewise, there is a one-to-one mapping between the state transition $s_i \to s_{i+1}$ and the union of the sets containing the past $K-1$ symbols and the set containing the current symbol

$$(s_i \to s_{i+1}) \iff \{\mathbf{V}_i, \mathbf{V}_{i-1}, \mathbf{V}_{i-2}, ..., \mathbf{V}_{i-K}\}.$$

Finally, define a function that reconstructs the noiseless whitened matched filter output, given the state transition

$$f_i^{(m)}(s_i \to s_{i+1}) = \sum_{j=0}^{\min(i,K-1)} \mathbf{F}_{i,i-j}^{(m)}\mathbf{C}_{i-j}^{(m)}\mathbf{V}_{i-j}.$$

Because the noiseless reconstruction of the whitened-MAI channel output depends only on the state transition, we can conclude that it is indeed a Markov process. Note that the irregular nature of $\mathbf{F}^{(m)}$ and $\mathbf{C}^{(m)}$ causes the Markov process to be time-varying, and thus knowledge of the index i associated with the state transition $(s_i \to s_{i+1})$ is implicitly required to reconstruct the signal.

Now define a branch metric

$$\lambda_i(s_i \to s_{i+1}) = \ln P[\bar{\mathbf{Y}}_i^{(m)}|i, s_i \to s_{i+1}] + \ln P[s_{i+1}|s_i]. \quad (10)$$

Note that this branch metric is similar to the metric defined in [6], except now the function can also depend on the index i. The term $P[\bar{\mathbf{Y}}_i^{(m)}|i, s_i \to s_{i+1}]$ is Gaussian with mean $f_i^{(m)}(s_i \to s_{i+1})$ and variance $\sigma^2 = N_o/2E_s$, and thus

$$\ln P[\mathbf{Y}_i^{(m)}|i, s_i \to s_{i+1}] = -\frac{1}{2}\ln\left(\frac{\pi N_o}{E_s}\right) - \frac{E_s}{N_o}\left|\bar{\mathbf{Y}}_i^{(m)} - f_i^{(m)}(s_i \to s_{i+1})\right|^2. \quad (11)$$

The transition probability $P[s_{i+1}|s_i]$ is identical to the probability $P[\mathbf{V}_i]$ of the symbol that causes the transition. In situations that the multiuser detector is working in isolation, without any side information, it is generally assumed that the symbols are equiprobable. However, if there is side information available from another process, such as a channel decoder, then the SISO MUD algorithm can incorporate this information as an a priori input. For BPSK, the a priori input $\mathbf{Z}_i$ can be expressed as a log-likelihood ratio, and is related to the transition probability by

$$\begin{aligned} P[s_{i+1}|s_i] &= P[\mathbf{V}_i : (s_i \to s_{i+1})] \\ &= \begin{cases} \frac{e^{\mathbf{Z}_i}}{1+e^{\mathbf{Z}_i}} & \text{for } \mathbf{V}_i = 1 \\ \frac{1}{1+e^{\mathbf{Z}_i}} & \text{for } \mathbf{V}_i = -1, \end{cases} \end{aligned} \quad (12)$$

and thus

$$\ln P[s_{i+1}|s_i] = \frac{\mathbf{Z}_i\mathbf{V}_i}{2} + \frac{\mathbf{Z}_i}{2} - \ln(1 + e^{\mathbf{Z}_i}). \quad (13)$$

Substituting (11) and (13) into (10) yields

$$\lambda_i(s_i \to s_{i+1}) = \frac{\mathbf{Z}_i\mathbf{V}_i}{2} - \frac{E_s}{N_o}\left|\bar{\mathbf{Y}}_i^{(m)} - f_i^{(m)}(s_i \to s_{i+1})\right|^2 + \eta, \quad (14)$$

with

$$\eta = \frac{\mathbf{Z}_i}{2} - \ln(1 + e^{\mathbf{Z}_i}) - \frac{1}{2}\ln\left(\frac{\pi N_o}{E_s}\right). \quad (15)$$

The log-MAP algorithm [6] can be used to calculate the LLR (9). When used to perform SISO multiuser detection, the branch metric (14) is used, with η set to any arbitrary value (either zero or a normalization constant chosen to improve numerical stability).

4. APPLICATION EXAMPLES

Because of the generality of the system model, there are many applications that are suited for the proposed algorithm. The main limitation of the algorithm is that its complexity is exponential in the number of users and thus it is not recommended for systems with more than perhaps 10 users.

4.1. Turbo-MUD

The algorithm can be used to perform conventional "turbo-multiuser detection" at a single receiver. In this, case $K > 1$ and $M = 1$. The algorithm has been applied to lightly loaded CDMA systems ($K = 5$) in [9] and to TDMA systems ($K = 3$ for 120 degree sectorized antennas at the base station and one tier of interferers) in [10]. In each situation, coded performance close to the single-user bound can be achieved.

4.2. Antenna arrays

The proposed algorithm can be used with an antenna array at the receiver by setting M equal to the number of antenna elements. Care must be taken to incorporate correlation among antenna elements into the channel model. Each antenna element would have a MUD module associated with it, and the soft outputs of the M MUD modules are combined according to[3]:

$$\begin{aligned} \mathbf{\Lambda}_i &= \ln \frac{P[\mathbf{V}_i = +1|\bar{\mathbf{Y}}^{(1)}, ..., \bar{\mathbf{Y}}^{(M)}]}{P[\mathbf{V}_i = -1|\bar{\mathbf{Y}}^{(1)}, ..., \bar{\mathbf{Y}}^{(M)}]} \\ &= \ln \frac{\prod_{m=1}^{M} P[\mathbf{V}_i = +1|\bar{\mathbf{Y}}^{(m)}]}{\prod_{m=1}^{M} P[\mathbf{V}_i = -1|\bar{\mathbf{Y}}^{(m)}]} \\ &= \sum_{m=1}^{M} \ln \frac{P[\mathbf{V}_i = +1|\bar{\mathbf{Y}}^{(m)}]}{P[\mathbf{V}_i = -1|\bar{\mathbf{Y}}^{(m)}]} \\ &= \sum_{m=1}^{M} \mathbf{\Lambda}_i^{(m)}. \end{aligned} \quad (16)$$

That is, the LLR outputs of the M MUD modules are simply added together. For the single user case ($K = 1$), this is equivalent to maximal-ratio combining. For multiuser systems ($K > 1$), then multiuser

[3]Note that (16) assumes that the signals at the input to the M antenna elements are independent which may not be a realistic assumption.

detection is performed prior to combining. If the system is coded, then the combined soft output can be fed into a bank of K SISO channel decoders whose soft-outputs can in turn be fed back to the multiuser detectors.

4.3. Distributed MUD

The channel model and MUD algorithm are not limited to the situation where the M receivers are in close physical proximity (as with an antenna array). Instead, the M receivers could be spatially separated over a wide geographic region. For example, in a cellular communication system, observations from the base stations serving M neighboring cells could be used. Such a technique is particularly effective for TDMA systems where a desired user in one cell is an interferer to a neighboring cell.

Take for example a situation where the base station employs 120 degree sectorized antennas. The base station will receive not only the signal from a desired user from within its cell, but also strong interference from the two cochannel users located in the first tier of cochannel cells. A similar situation exists at the base stations serving the two cochannel users. By placing a SISO multiuser detector at each of three adjacent cell sites, all three users can be jointly detected. Performance is further improved for coded TDMA systems by passing the output of a bank of SISO channel decoders back to the multiuser detectors. The combination of macrodiversity and multiuser detection, also known as "distributed MUD," has been considered for the uncoded case in [11] and [12] and for the coded case in [13].

5. CONCLUSION

An optimal soft-output multiuser detector can be derived from a whitened version of Verdu's optimal hard-output multiuser detector implemented with the MAP algorithm. The algorithm can be applied to a wide variety of problems involving the joint detection and decoding of coded multiple access networks comprised of K transmitters and M receivers. Future work involves the application of the proposed processing architectures to space-time coded systems and to practical situation whereby the channel coeficients might not be known at the receivers.

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